PERIODICA POLYTECHNICA SER. TRANSP. ENG. VOL. 21, NO. 2, PP. 137–147 (1993)

TRACK MAINTENANCE MACHINES MODELLING FOR RIDING QUALITY ASSESSMENT

Daniel KALINČÁK

Department of Railway Vehicles
Engines and Lifting Equipment
University of Transport and Communications, Žilina
Slovakia

Received: Nov. 9, 1992

Abstract

The paper deals with a simple dynamic and mathematical model of a two-axle track maintenance machine for riding assessment in vertical direction. The aim was to develop an user-friendly system of computation programs enabling the evaluation of wide range of dynamic characteristics. The excitation of the model are stochastic rail irregularities described by their power spectral density.

Keywords: track maintenance, dynamic model, stochastic excitation.

1. Introduction

A track maintenance machine can be considered as a special kind of railway vehicle. In spite of the required maximum speed of up to 100 km/h, sufficient attention has not been paid to the riding quality of track maintenance machines.

When running gear of the new track maintenance machines was designed, it turned out that at least a simple mathematical model should be used in order to ensure convenient riding quality.

2. Dynamic and Mathematical Model

A significant number of track maintenance machines have two-axle running gear.

A relatively simple dynamic and mathematical model of two-axle track maintenance machines for the solution of vertical dynamics has been built [1, 2]. A sketch of mentioned dynamic model is shown in *Fig. 1.*

The linear model of a vehicle consists of a rigid main frame and an arbitrary number of rigid bodies placed on the main frame. Both the axle suspension and the suspension of rigid bodies consist of springs and dampers. This dynamic system is excited by stochastic vertical track irregularities described by their power spectral density.

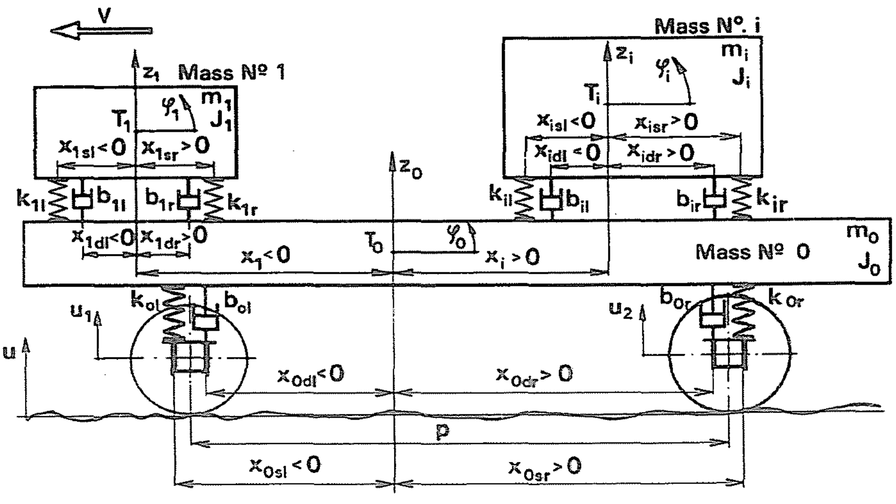

Fig. 1. The mechanical model

This dynamic model can be described by a system of linear differential equations in the matrix form:

$$\mathbf{A} \cdot \ddot{\mathbf{z}} + \mathbf{B} \cdot \dot{\mathbf{z}} + \mathbf{K} \cdot \mathbf{z} = \mathbf{g}(t)\,, \tag{1}$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{K}$ are the matrices of mass, damping and stiffness, $\mathbf{z}$, $\dot{\mathbf{z}}$, $\ddot{\mathbf{z}}$ are the vectors of unknown displacements, velocities and accelerations, vector $\mathbf{g}(t)$ is the vector of excitation.

By a suitable choice of the coordinate system, it is possible to obtain the above mentioned matrices in the form enabling their fully automated compiling and evaluation by a computer for an arbitrary number of bodies placed on the main frame of a track maintenance machine. If vector $\mathbf{z}$ has the shape $\mathbf{z} = [z_0, \varphi_0, z_1, \varphi_1, \ldots, z_n, \varphi_n]$ and n is the number of masses placed on the main frame, matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{K}$ have suitable form. The dimension of all matrices is $2 \cdot (n+1)$.

The mass matrix $\mathbf{A}$ is a diagonal matrix with elements $m_{i,i}$, $\mathbf{J}_{i,i}$, where $i = 0, 1, \ldots, n$.

Because matrices $\mathbf{B}$ and $\mathbf{K}$, are symmetric, and have formally the same shape, only the elements of stiffness matrix $\mathbf{K}$ will be shown.

The elements of the first row of matrix $\mathbf{K}$ are:

$$k_{1,1} = \sum_{i=0}^{n} (k_{il} + k_{ir}) \ , \tag{2}$$

$$k_{1,2} = k_{0l} \cdot x_{0sl} + k_{0r} \cdot x_{0sr} + \sum_{i=1}^{n} \left[k_{il} \cdot (x_i + x_{isl}) + k_{ir} \cdot (x_i + x_{isr}) \right] \quad (3)$$

and

$$k_{1,2i+1} = -\left(k_{il} + k_{ir}\right) , \quad (4)$$

$$k_{1,2i+2} = -\left(k_{il} \cdot x_{isl} + k_{ir} \cdot x_{isr}\right) , \quad (5)$$

for $i = 1, 2, \ldots, n$.

The second row contains elements:

$$k_{2,1} = k_{1,2} , \quad (6)$$

$$k_{2,2} = k_{0l} \cdot x_{0sl}^2 + k_{0r} \cdot x_{0sr}^2 + \sum_{i=1}^{n} \left[k_{il} \cdot (x_i + x_{isl})^2 + k_{ir} \cdot (x_i + x_{isr})^2 \right] , \quad (7)$$

$$k_{2,2i+1} = -k_{il} \cdot (x_i + x_{isl}) - k_{ir} \cdot (x_i + x_{isr}) , \quad (8)$$

$$k_{2,2i+2} = -k_{il} \cdot x_{isl} \cdot (x_i + x_{isl}) - k_{ir} \cdot x_{isr} \cdot (x_i + x_{isr}) , \quad (9)$$

for $i = 1, 2, \ldots, n$.

All other rows contain only four non-zero elements of which the first two ones are already known (due to the symmetricity of the matrix), and the other two elements are:

in the row $2 \cdot i + 1$:

$$k_{2i+1,2i+1} = k_{il} + k_{ir} , \quad (10)$$

$$k_{2i+1,2i+2} = k_{il} \cdot x_{isl} + k_{ir} \cdot x_{isr} , \quad (11)$$

and in the row $2 \cdot i + 2$:

$$k_{2i+2,2i+1} = k_{2i+1,2i+2} , \quad (12)$$

$$k_{2i+2,2i+2} = k_{il} \cdot x_{isl}^2 + k_{ir} \cdot x_{isr}^2 , \quad (13)$$

for $i = 1, 2, \ldots, n$.

The vector of excitation $\mathbf{g}(t)$ can be written in the form:

$$\mathbf{g}(t) = \mathbf{g}_0 \cdot u \cdot e^{i \cdot \omega \cdot t} , \quad (14)$$

where vector $\mathbf{g}_0$ has only the first two non-zero elements:

$$g_{10} = k_{0l} + k_{0r} \cdot \exp \cdot \left[-\frac{i\omega p}{\nu} \right] + i \cdot \omega \cdot \left(b_{0l} + b_{0r} \cdot \exp \cdot \left[-\frac{i \cdot \omega \cdot p}{\nu} \right] \right) , \quad (15)$$

$$g_{20} = k_{0l}x_{0sl} + k_{0r}x_{0sr}\exp\cdot\left[-\frac{i\omega p}{\nu}\right] + i\omega p\left(b_{0l}x_{0dl} + b_{0r}x_{0dr}\exp\cdot\left[-\frac{i\omega p}{\nu}\right]\right). \tag{16}$$

It is evident that the shape of all matrices and the formulas for evaluation of their elements enable relatively easy algorithmization of computing these matrices for an arbitrary number of rigid bodies placed on the rigid main frame of the machine.

The vector of frequency responses of excitation can be obtained by solving *Eq.* (17) for $\mathbf{z}_0$ if the amplitude of excitation u_0 will be equal to one:

$$\left(-\omega^2\cdot\mathbb{A} + i\cdot\omega\cdot\mathbb{B} + \mathbb{K}\right)\cdot\mathbf{z}_0 = \mathbf{g}_0\cdot u_0\,. \tag{17}$$

In this case, vector $\mathbf{z}_0$ becomes the vector of frequency responses $\mathbb{F}_u(i\cdot\omega)$. The frequency response between excitation $\mathbf{u}$ and the arbitrary output $\mathbf{z}_i$ (not only for the coordinates defined in the sketch of dynamic model) can be obtained from the elements of this vector. Such outputs can be, for example, acceleration at an arbitrary point of the main frame, or the body placed on it, the deformations of arbitrary spring, the force in arbitrary element of suspension, etc.

It is possible to compute the power spectral density of the arbitrary output $\mathbf{z}_i$ from the formula:

$$S_{z_i}(\omega) = \left|F_{uz_i}(i\cdot\omega)\right|^2\cdot S_u(\omega)\,, \tag{18}$$

where $F_{uz_i}(i\cdot\omega)$ is the frequency response between input $\mathbf{u}$ and output $\mathbf{z}_i$,
$S_u(\omega)$ is the power spectral density of vertical rail irregularities.

In our mathematical model, power spectral densities of following shapes have been used:

$$S_u(\Omega) = A\cdot\Omega^{-n} \qquad \left[\mathrm{m}^3;\quad \mathrm{m}^{-1}\right], \tag{19}$$

$$S_u(\Omega) = \frac{A}{(B+\Omega)^3} \qquad \left[\mathrm{m}^3;\quad \mathrm{m}^{-1}\right], \tag{20}$$

or

$$S_u(\Omega) = \frac{A}{\Omega\cdot(1+B\cdot\Omega^3)} \qquad \left[\mathrm{m}^3;\quad \mathrm{m}^{-1}\right], \tag{21}$$

where Ω is track circular frequency $\left[\mathrm{m}^{-1}\right]$.

Expression (21) gives the best agreement with the experimentally obtained power spectral density (PSD) at the Railway test circle in Cerhenice, if $A = 4.2\cdot10^{-6}$ and $B = 5.75$ [3]. The comparison between experimental

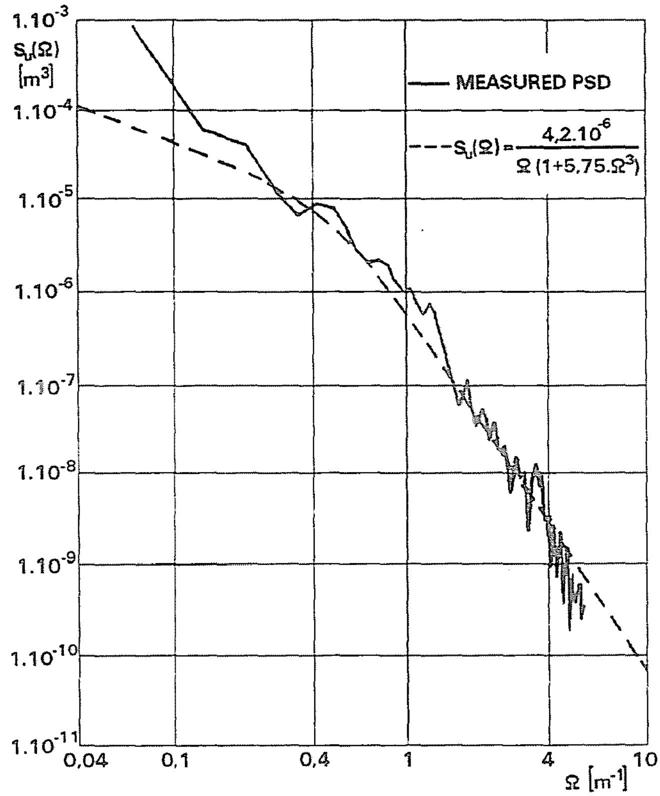

Fig. 2. Experimental and approximate values of the PSD

PSD of vertical track roughness and its approximation by formula (21) is shown in *Fig. 2.*

A certain deviation of approximated PSD from the measured one for large wavelengths is justified by distorsion of PSD calculated from measured data caused by leakage through side lobes of spectral window in this frequency area [4].

PSD $S_u(\omega)$ should be used in calculations which can be expressed as:

$$S_u(\omega) = \frac{S_u(\Omega)}{\nu}; \qquad \Omega = \frac{\omega}{\nu}, \tag{22}$$

where ν is the velocity of vehicle.

If ride index value $\mathbf{W}_z$ is evaluated in accordance with [5], the following formula can be used:

$$\mathbf{W}_z = 3.66 \cdot (D[\ddot{z}])^{0.15}, \tag{23}$$

where $\mathbb{D}[\ddot{z}]$ is the dispersion variance of vertical acceleration of human body.

The sensitivity of human body to the vibrations of various frequencies can be taken into account by using the frequency response of human body [5, 6]:

$$F_{\ddot{z}_f\ddot{z}}(i \cdot \omega) = \frac{1 + i \cdot 0.02135 \cdot \omega}{1 - (0.02135 \cdot \omega)^2 + i \cdot 0.02135 \cdot \omega}, \tag{24}$$

defined between the acceleration of floor $\ddot{z}_f$ of the vehicle and the acceleration of human body $\ddot{z}$.

Dispersion variance $\mathbb{D}[\ddot{z}]$ can be evaluated from the formula:

$$D[\ddot{z}] = \int_{\omega_1}^{\omega_2} S_{\ddot{z}}(\omega) \cdot d\omega, \tag{25}$$

where

$$S_{\ddot{z}}(\omega) = \left|F_{\ddot{z}_f\ddot{z}}(i \cdot \omega)\right|^2 \cdot S_{\ddot{z}_f}(\omega). \tag{26}$$

3. The System of Computation Programs

The system of computation programs enabling the evaluation of a wide range of dynamic characteristics has been developed. The program is user-friendly. It requires only all basic data about the dynamic model of track maintenance machine in a dialogue form and the user does not care about the derivation of the system of differential equations or matrices, etc.

The program-set consist of the short main program which controls subsequent two programs.

The first part of the program-set evaluates all basic transfer functions in the chosen range of frequencies and frequency steps, and their storage for the following treatment. Because this dynamic model does not consider the bending of the main frame, the range of frequencies in which frequency response is to be calculated will be limited.

The second part of the program-set enables the evaluation of arbitrary derived frequency responses, PSD and other characteristics, e. g. dispersion variance, ride index value W_z from the basic frequency responses evaluated by the first part of the program-set. This program enables also the output of evaluated characteristics and functions to be displayed or printed, at the request of the user.

The main aim of the program is to evaluate the ride index value, but it can also be used for the evaluation of many other characteristics, e. g. power spectral densities of displacements, velocities, accelerations of arbitrary points of each body of the given dynamic system, power spectral densities of deformations, forces, etc. of the elements of suspension, all transfer functions, etc.

4. Some Results of Calculations of the Dynamic Properties of Ballast Regulator KP-900

The described computation program was used for the design of the ballast regulator KP-900. The dynamic three-mass model of the ballast regulator KP-900 is shown in *Fig. 3*.

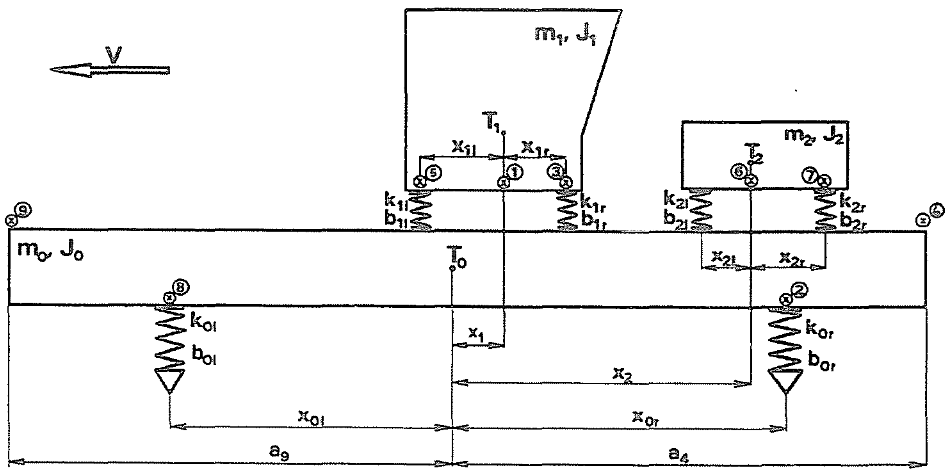

Fig. 3. The three-mass model

It turned out that it was sufficient to take one-mass model if calculations are used for optimization of the parameters of primary suspension. Some results of optimization of dampers in primary suspension are shown in *Fig. 4*. The optimal damping constant b_0 for given stiffness of the primary suspension is approximately in the range from 40,000 to 120,000 [$\mathrm{Ns \cdot m^{-1}}$].

If the aim of calculation is to obtain the vibration or ride index value in the cab of the machine, it is necessary to consider at least two- or better three-mass model, because elastic suspension of the driving cab has influence on the previously mentioned characteristics.

The influence of the parameters of elastic suspension of the cab on PSD of the vertical acceleration on the floor (at the point No. 1 according to *Fig. 3*) of the cab is shown in *Fig. 5*.

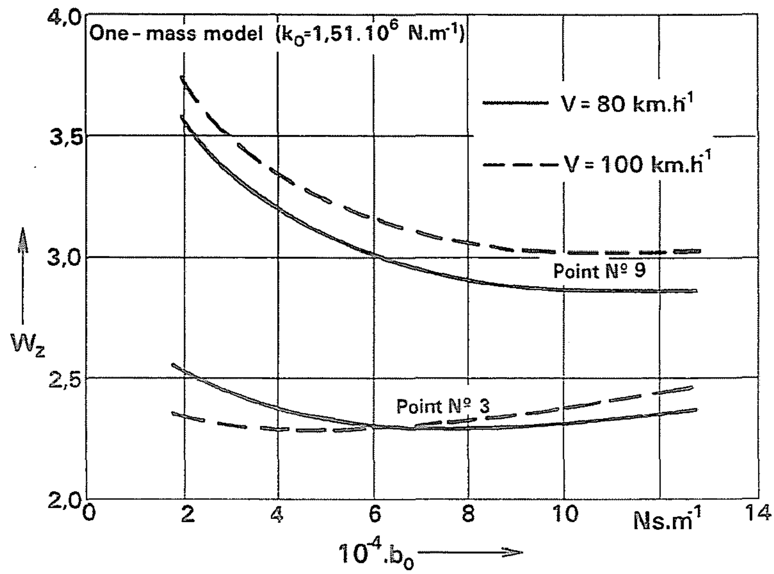

Fig. 4. Results of damper's optimization

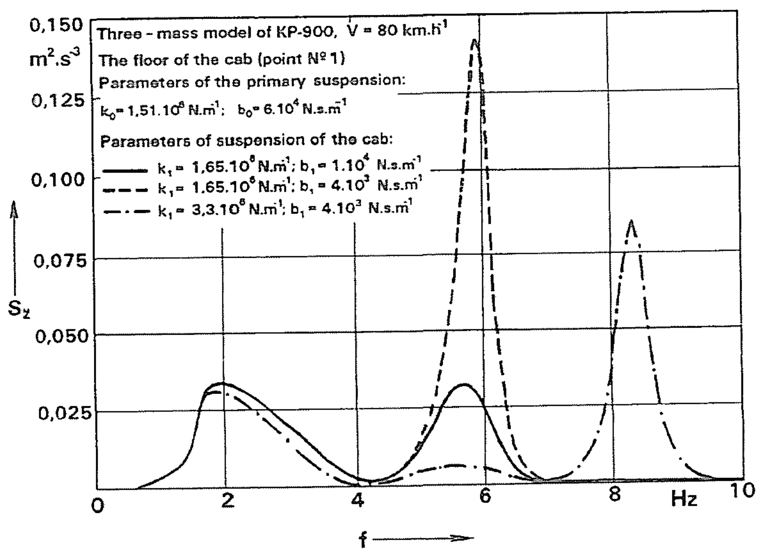

Fig. 5. Vertical acceleration curves

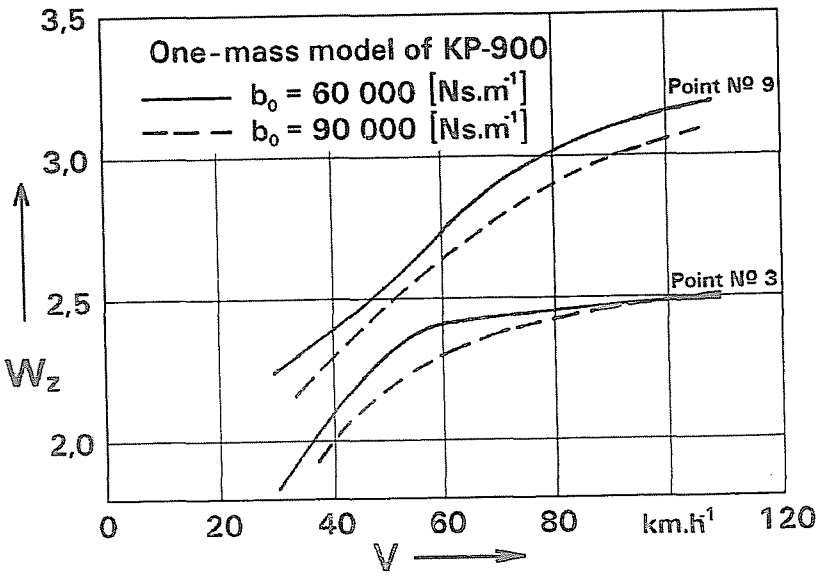

Fig. 6. Ride index versus velocity

The calculated dependence of the ride index value on the velocity at points No. 3 and 9 of the ballast regulator is shown in *Fig. 6.*

5. Some Results of the Tests on KP-900

The measurements of dynamic properties were carried out on the prototype of the ballast regulator KP-900 [7].

The tests were carried out on the railway line between stations Pribovce and Diviaky. Values of the vertical and lateral acceleration at the floor of the cab (approximately at point No. 3 in *Fig. 3*) and at the buffer beam of the main frame (at point No. 4), as well as the lateral and longitudinal motion of the axle-boxes towards the main frame and the deformations of the primary suspension were measured and registered.

PSD of vertical acceleration on the floor of the cab (point No. 3) is shown in *Fig. 7*, as an example of the results of the tests.

The running tests of this track maintenance machine confirmed the correct evaluation of the parameters of springing and the good riding quality of the ballast regulator.

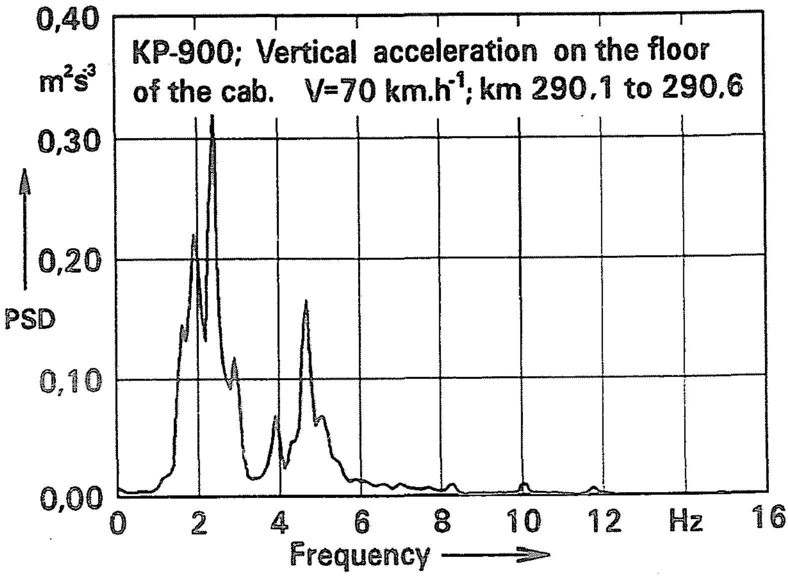

Fig. 7. PSD values for vertical acceleration

6. Conclusion

The results of tests confirmed relatively a good agreement with the results obtained by the described dynamic and mathematical model of the track maintenance machine. Such a simple model and the user-friendly set of computer programs are useful for the designers involved in the development of new track maintenance machines (or two-axled railway vehicles).

We intend to include the axle-drive into the described dynamic model. In this case, the simplicity of the program should be preserved also. We intend to develop a new dynamic model with the bending flexible main frame of machine. This model will use the finite element method, so that its usage will require specialist knowledge.

References

1. KALINČÁK, D. – IZER, J. – JANDA, J.: Štúdia pojazdu koľajového pluhu KP-900. VHČ HZ č.: 73/2019/88. Report No. KV-1-89, VŠDS Žilina 1989.
2. KALINČÁK, D.: Matematický model pre riešenie zvislého chodu traťového stroja. In: Zb. konf.: 'Dynamika kolejových vozidel a železničních tratí'. DT ČSVTS Praha, 1990.
3. KALINČÁK D.: Prvky a sústavy vypruženia s hlavným zreteľom na použitie prvku vypruženia negel. Thesis, VŠDS Žilina 1977. .
4. KALINČÁK, D.: Vplyv šírky spektrálneho okienka na skreslenie priebehu spektrálnej výkonovej hustoty zvislých nerovností koľaje. In: Práce a štúdie VŠDS, séria strojnícka, No. 13 (1983).
5. FREIBAUER, L. a kol.: Výzkum stochastických dejů při jízdě vozidel. Research Report III–6–8/14, No. KKV 1–72. VŠD Žilina 1972.
6. FREIBAUER, L. – RUS, L. – ZAHRÁDKA, J.: Dynamika kolejových vozidel. NADAS Praha 1991.
7. KALINČÁK, D. – IZER, J. – FITZ, P. – POLÁCH, O. – SMIEŠKO, J. – ŽAKOVÁ, M.: Meranie chodových vlastností prototypov OV-2 a KP-900. VHČ HZ č.: 19/199/91. Report No. KKVMZ-01-91/DMZ, VŠDS Žilina 1991.