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ON SOME BASIC PROBLEMS OF STOCHASTIC MODELLING

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Abstract

The development of computerized digital measuring and simulating equipment opened the way to the practical use of stationary stochastic process modelling. It raised also new problems in PSD records smoothing, space-time conversion and individual phase difference simulation. These were solved by generalization of the natural-parameter method and by introduction of complex specimen eigenspectra. A four-parameter spectrum family and the NAPA4 modelling system have been developed for the practical utilization of these innovations.

Keywords: natural parameters, stochastic input modelling, stochastic surface.

1. Introduction

Correct analysis, calculation and simulation of vehicle dynamics is possible only by using stochastic process modelling. A key issue in it is the true description of stochastic environmental influences (road unevenness, rail position errors, sea waves, atmospheric turbulence) on moving vehicles. Stochastic input-output relationships can be analysed by spectral methods (see e.g.: BENDAT and PIERSOL [1, 2]). It is unnecessary if not wrong to follow every little local peak or fold in these calculations; a good smoothing analytical function would do much better. Environmental load spectra have a characteristic negative power-law like appearance over the measured frequency range like *Fig. 1*. There are several analytical function types giving acceptable formal fit over the measured frequency range but not allowing a logical insight into the character of the process. So is, e.g. the simple power-law type function

$$G_x(n) = \frac{C}{n^\alpha} \qquad \text{resp.} \qquad G_x(f) = \frac{C'}{f^\alpha}\,.$$

Several fundamental as well as practical problems associated with this and similar formulae induced us to develop a family of spectrum-smoothing formulae based on the so-called natural stochastic process parameters.

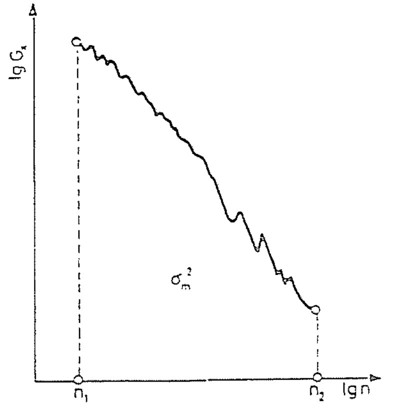

Fig. 1. Raw input spectrum

The first one of the natural parameters is the well-known standard deviation σ indicating the magnitude of the random deviations. The second and a most important one is the integral scale parameter

$$L = \lim_{\zeta\to\infty} \frac{1}{\sigma_x^2} \int_0^{\zeta_1} R_x(\zeta)\mathrm{d}\zeta \qquad \text{(See } \textit{Fig. 2}\text{)} \tag{1}$$

coming from flow-turbulence work. The fact that it is not only a special parameter of turbulence but a universal constant for all stationary stochastic processes has been given by Prof. KOVÁSZNAY [3] giving also the zero value for the eigenspectra. A full list of natural parameters with their definition regarding practical calculation formulae can be seen in *Table 1.*

$$G_x(n) : \quad G_x(0) = 4L\sigma_x^2; \quad n_{\max} = \frac{1}{\lambda}$$
$$G_x(\Omega) : \quad G_x(0) = \frac{2}{\pi}L\sigma_x^2; \quad \Omega_{\max} = \frac{2\pi}{\lambda}.$$

From among these, Taylor's scale parameter seems to be of secondary influence in vehicle dynamics work, its possible influence being overshadowed by other effects.

In 1982 the author has initiated the three-parameter road/rail spectrum formula:

$$G_x(n) = \sigma_x^2 \frac{4L}{\left(1 + \frac{4}{\alpha-1}Ln\right)^\alpha}. \tag{2}$$

Table 1
Natural parameters of stationary stochastic processes

Parameter:	Calculation formula using $x(\xi)$ $(\overline{x}=0)$	autocorrelation func. $R_x(\zeta)$	spectral density func. $G_x(n)$ resp. $G_x(\Omega)$
Sandard deviation σ_x	Def.: $\sigma_x = \lim\limits_{S\to\infty}\left[\frac{1}{S}\int\limits_0^S x^2(\zeta)\mathrm{d}\zeta\right]^{\frac{1}{2}}$	—	$\sigma_x^2 = \int\limits_0^\infty G_x(n)\mathrm{d}n = \int_0^\infty G_x(\Omega)\mathrm{d}\Omega$
Scale parameter (integral) L	—	Def.: $L = \lim\limits_{\zeta_1\to\infty}\frac{1}{\sigma_x^2}\int\limits_0^{\zeta_1} R_x(\zeta)\mathrm{d}\zeta$	Regression analysis
Taylor's scale parameter λ	—	Def.: $\lambda = \frac{\sqrt{2}\sigma_x}{\left[-\left(\frac{\mathrm{d}^2 R_x(\zeta)}{\mathrm{d}\zeta^2}\right)_{\zeta=0}\right]^{\frac{1}{2}}}$	$\lambda = \frac{\sigma_x}{\sqrt{2}\pi}\left[\int\limits_0^\infty n^2 G_x(n)\mathrm{d}n\right]^{-\frac{1}{2}}$ $\lambda = \sqrt{2}\sigma_x\left[\int\limits_0^\infty \Omega^2 G_x(\Omega)\mathrm{d}\Omega\right]^{-\frac{1}{2}}$
Exponent α	—	—	Regression analysis

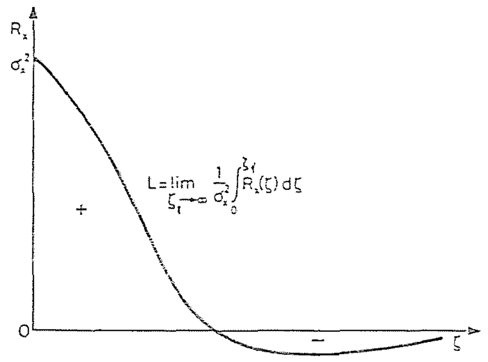

Fig. 2. Definition of the integral scale parameter

Space-time conversion is based on the time scale

$$T = \frac{L}{V} \tag{3}$$

giving for the input time-spectra in the wheels:

$$G_x(f) = \sigma_x^2 \frac{4L}{\left(1 + \frac{4}{\alpha-1}Tf\right)^{\alpha}} . \tag{4}$$

Detailed description, development and cases for practical application were given in the years 1982 – 90 by the author [5 – 10]. It has been used successfully in a rough terrain mobility model, too, [11] showing that results obtained by it though on simple linear vehicle models can give very good approximations to measured field data.

This paper intends to give a short report on further research done in the last three years.

2. Four-Parameter Spectrum Formula

We were quite happy with the fits obtained over the frequency band used in practical calculations but there were also signs indicating the need of further refinements. First of all, integral scale parameter values giving the best fit over the measured and used frequency range diverged significantly from those obtained by the definition formula (1). This indicated *Eq.* (2) to be deficient in the — unutilized — low frequency range. Furthermore, stochastic process theory indicates power spectra to be even functions of the frequency which functions (2) and (4) resp. are not by themselves.

To cover both deficiencies, a new four-parameter formula has been introduced. It reads as follows:

$$G_x(n) = 4L\sigma_x^2 \frac{1 + A(CLn)^2}{[1 + (CLn)^2]^\alpha}, \tag{5}$$

or:

$$G_x(f) = 4T\sigma_x^2 \frac{1 + A(CTf)^2}{[1 + (CTf)^2]^\alpha}, \tag{6}$$

respectively. In a sense, it can be said to be a generalization of the Kármán turbulence spectrum. The new parameter introduced in the new formula is the peak factor A. The influence of the individual parameters on the shape of the spectrum curve, drawn on logarithmic scale, are pictured in *Figs. 3 - 6.*

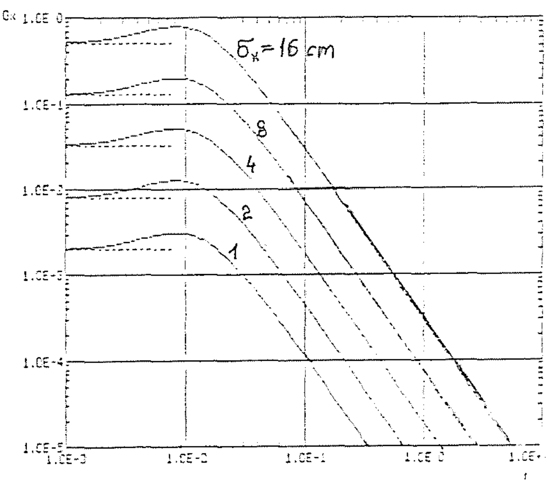

Fig. 3. Influence of the standard deviation

Calculations done so far using the new power-spectrum function proved to be a very useful and versatile research/development tool. Besides giving substantial improvements in road/terrain, rail and atmospheric turbulence modelling, it is expected to be adaptable to the correct description of various sea wave conditions, too.

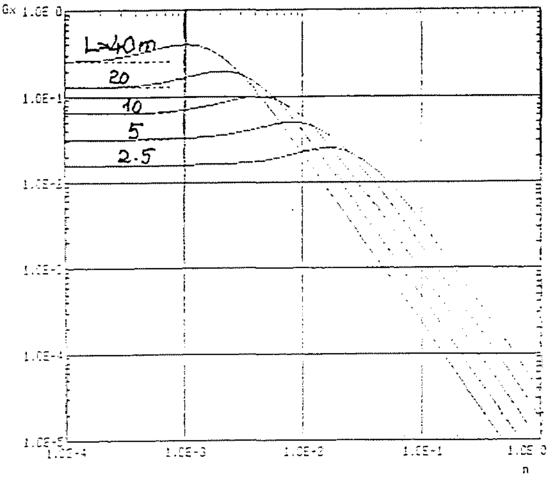

Fig. 4. Influence of the scale parameter

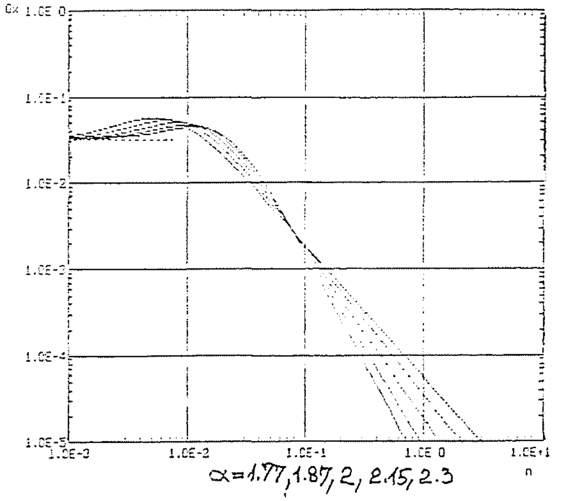

Fig. 5. Influence of the exponent

3. Linear Vehicle Models

Ride comfort and fatigue load calculations are among the first choices for the introduction of natural-parameter spectral methods. Our experience in this line is that while two-dimensional (fore- and aft wheels) models do not give even the rough trends correctly, simple three-dimensional linear models

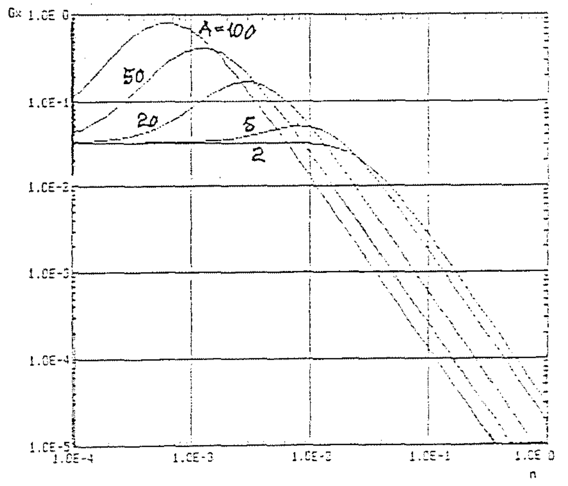

Fig. 6. Influence of the peak factor

will do very well. Ref. [10] has shown some results of such calculations. Driver comfort as function of speed shows the right trends experienced in practice. The three-parameter spectrum given by *Eqs.* (2) and (4) has been used, for which the influence of the single parameters on ride comfort are also presented. The application of these methods to off-road work has given quite acceptable results. Actual tests on terrain/dirt road [11] reproduced the calculated mean terrain-crossing speeds to a few per cent.

An essential part of the method is the introduction of complex eigenspectra for the road unevenness. The original PSD eigenspectrum as defined by Rayleigh does not contain any information on phase angle relationships. But these are essential for wheel track simulation as well as for multiple input-output calculations. The published road spectra (see e.g. [3] and [12]) are all of this type. Time functions for individual wheel motion input generation are made from these by adding randomly generated phase angles to the single Fourier components [3]. For multiple input-output calculations on an n-wheeled vehicle, an $n \times n$ spectrum matrix of all eigen- and cross-spectra is necessary. The output spectrum matrix $\mathbf{G}_{yy}(f)$ reads then as follows:

$$\mathbf{G}_{yy}(f) = \mathbf{H}^*_{yx}(f)\mathbf{G}_{xx}(f)\mathbf{H}^T_{yx}(f)\,. \tag{7}$$

It has been proved in Appendix 2 of Ref. [9] that complex eigenspectra are correct for all finite base-length random data analysis and that an n element complex spectrum vector composed of them contains all and more information given by the traditional $n \times n$ spectrum matrix. Moreover, the input-output equation giving the output spectral vector as

$$[G_y(f)]^{1/2} = \mathbb{H}_{yx}(f)\,[G_x(f)]^{1/2} \tag{8}$$

requires much less computer time and occupies much less memory space than the equivalent *Eq.* (7). *Figs. 7* and *8* are showing the output spectrum of a right-forward spring and a left-forward damper force, respectively.

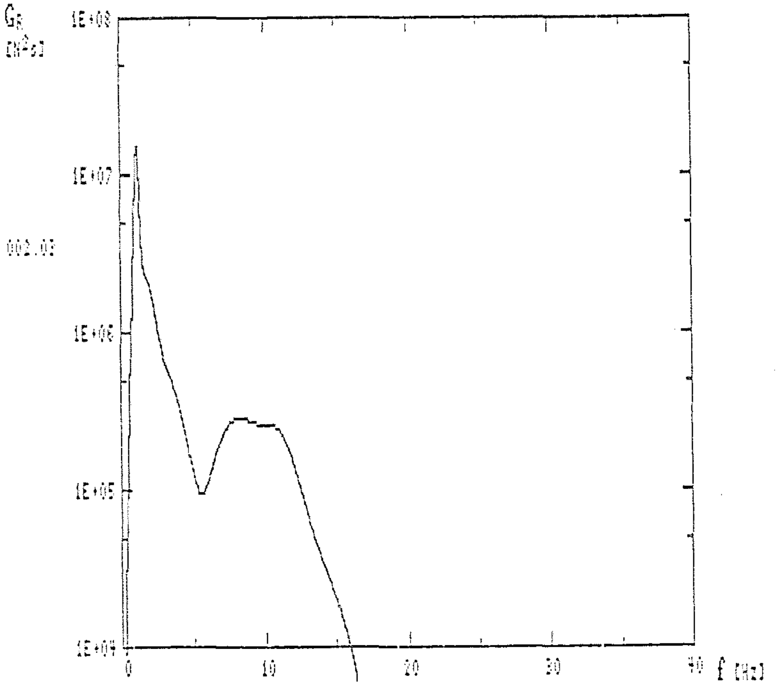

Fig. 7. Spectrum of the right-forward spring force

4. Road/Terrain Surface Generation

To tell it frankly, our knowledge of the true two-dimensional character of stochastic surfaces like a road or a meadow is very limited indeed. There are some authors specifying 'isotrope' or 'orthotrope' road surfaces for their calculations and that is that. There is not much speculation if and how these assumptions could be met by the roads or terrains. It seemed us therefore advisable to investigate into the problem a little closer.

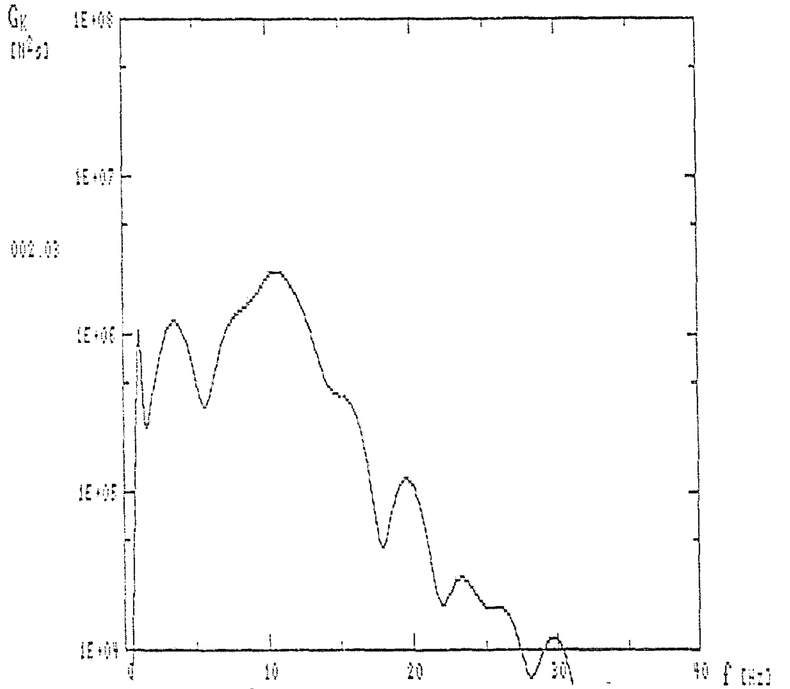

Fig. 8. Spectrum of the left-forward damper force

The starting point was that every square of (regular) stochastic surface can be interpreted as the resultant of two single-dimensional, say perpendicular, stochastic processes. Let us mark the generating spectrum in the direction of the vehicle motion by 1, at right angles to it by 2. All the four parameters of both, parallel as well as perpendicular, spectra and phase angle relationships are systematically varied in the investigations.

Only some preliminary results can be given at present. First of all, some doubts arose if true isotropic surfaces are existent at all. There is no strict mathematical proof against them as yet, but neither a process for their generation. Nevertheless, further research might discover an opportunity.

Orthotropic surfaces are doing well and fulfilling the expectations. Phase angle relationships turned out to be all important and offering innumerable variations. A sample of computer results, depicted on an 18 m × 16 m surface can be seen in *Figs. 9 - 13*. The spectrum constants are as follows:

$\sigma_{x1} = 3\,\text{cm}$, $L_1 = 20\,\text{m}$, $\alpha_1 = 2.2$, $A_1 = 5$;
$\sigma_{x2} = 2.4\,\text{cm}$, $L_2 = 22\,\text{m}$, $\alpha_2 = 2.09$, $A_1 = 5.5$.

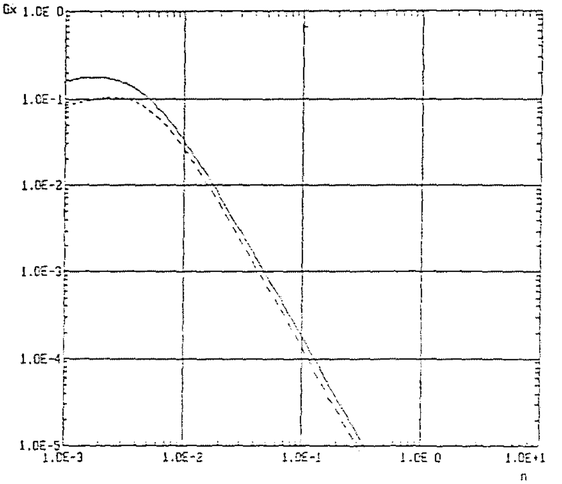

Fig. 9. Surface unevenness spectra
(The dotted line is for the generating spectrum in the direction of vehicle motion, the full line is for the resultant spectrum in the same direction)

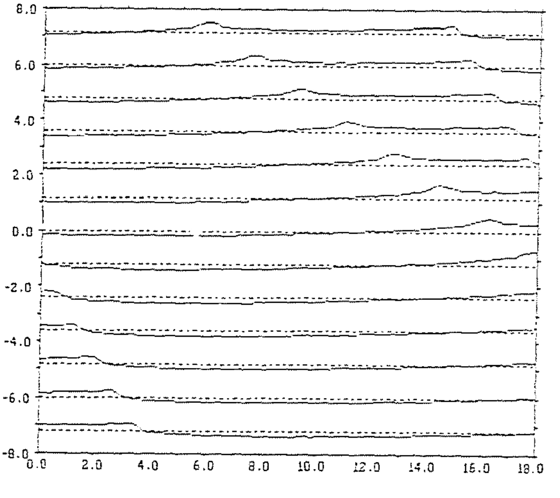

Fig. 10. Longitudinal sections of the road

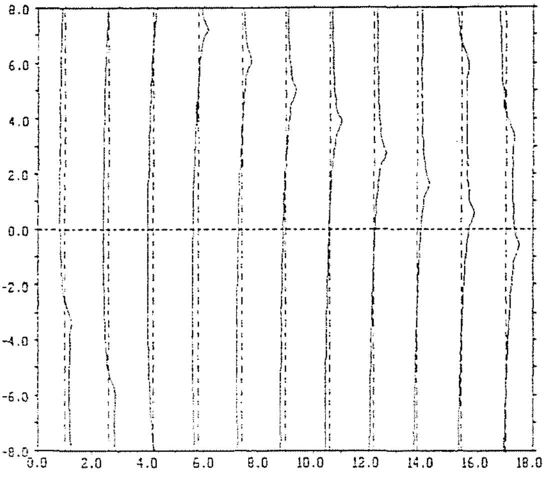

Fig. 11. Transversal sections of the road

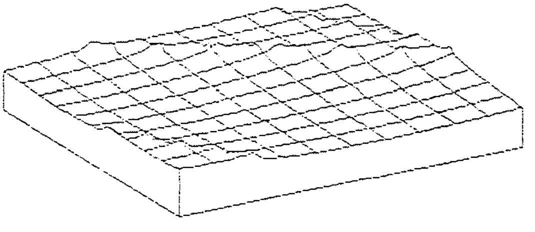

Fig. 12. Wire model of the surface

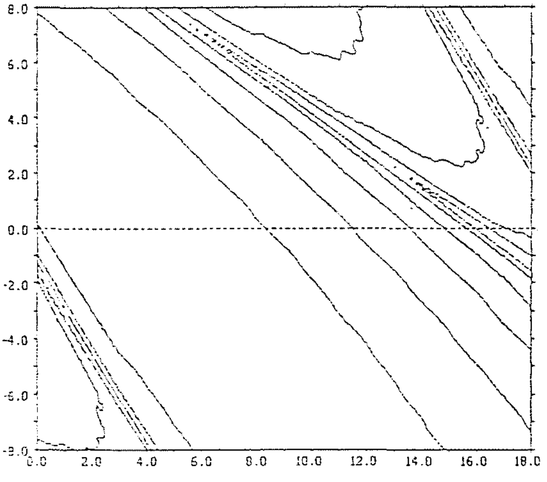

Fig. 13. Level lines on the surface

Figs. 12 and *13* intend to give a general picture of the surface unevennesses. The former is giving something like a perspective in the form of a wire model. The latter depicts level lines in 1 cm steps.

Notations

f — frequency [1/s]
n — wave number [1/m]
x — road unevenness elevation [m]
y — output variable
A — peak coefficient
C — constant
G_x — power spectral density function
H_{yx} — frequency response function
L — (integral) scale parameter [m]
T — time scale [m]
V — vehicle speed [m/s]
α — exponent
ξ — space lag [m]
λ — Taylor's scale parameter [m]
σ — standard deviation [m]

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