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CHAOS AND ASYMMETRY IN RAILWAY VEHICLE DYNAMICS

Hans TRUE[1,2] and Jens Christian JENSEN[1,3]

[1] Institute of Mathematical Modelling and MIDIT
The Technical University of Denmark, Bldg. 321
DK–2800 Lyngby, Denmark
[2] ES Consult, Staktoften 20, DK–2950 Vedbaek, Denmark
[3] The Danish State Railways, Pilestraede 58[3],
DK–1112 Copenhagen, Denmark

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Introduction

The dynamics of a rolling railway vehicle can be described as a mechanical dissipative multi-body system, which is fundamentally nonlinear. The fundamental nonlinearity stems from the stress-strain relation in the wheel-rail contact surface. In many vehicle constructions nonlinearities may also arise through impacts between bodies, dry friction between bodies in contact or nonlinear characteristics of springs or dampers — either through nonlinear dependence of the forces on the deformation and/or the rate of deformation. The dissipation in the system is provided not only by the dampers but also by the friction forces in the wheel-rail contact surface.

General nonlinear parameter dependent dynamical dissipative systems very often have parameter ranges, in which the dynamics is chaotic — it has even been claimed that chaos is the natural state of dynamical systems. Chaotic behaviour, in the sense of a fully deterministic evolution of the system in time, yet erratically looking behaviour, bounded in phase space with sensitive dependence on initial conditions, might therefore be expected to occur also in railway vehicle dynamical systems. In order to secure a calm ride in spite of the unavoidable disturbances on the real railway, the attenuation of dampers in railway cars and locomotives is quite large. Therefore it might be expected that chaos in realistic models will be observed only in case of large external forces, and when it occurs, the corresponding attractor will be of a low spatial dimension in the phase space.

Discontinuities in the forcing function have been found often to be sources of chaotic behaviour. In contrast the effect of discontinuous derivatives of first or higher order on the development of chaos is hardly explored at all. Since the simple mathematical model of the dynamics of a rolling wheelset does have a discontinuity at zero in the second derivative of the contact force in the wheel-rail contact zone, and since the contact zones may jump depending on the wheel-rail contact geometry several possible discontinuities exist. The theoretical investigations of the dynamics of vehicles therefore serve as examples of unexplored features in the mathematical theory of dynamical systems in general.

The theoretical vehicle models posses several symmetries. In parameter dependent nonlinear dynamical systems such symmetries often break at certain parameter values. A train moving along the track is generally modelled with a reflexion symmetry about the center line of the track. It is generally assumed that lateral oscillations will have the center line as a neutral line, and this is indeed found to be true in many cases. However, the symmetry is broken in certain parameter ranges — often as a prelude to chaos — whereby two equally probable modes develop. One is a reflexion of the other in the track center line, but they are both oscillations around an off-center neutral line. Such asymmetric oscillations give rise to an asymmetric wear of the wheelsets, and under certain circumstances such an effect may become self-amplifying.

The seemingly erratic component of the lateral motion of a vehicle rolling along a track is normally caused by the disturbances in the track geometry. They are generated by fluctuations within the tolerances of manufacture, by errors generated by wear, by uncontrollable variations in the flexibility of the track structure, weather conditions and more. It may be very difficult — perhaps impossible to detect the deterministic chaotic oscillations on a background of such high noise level. The chaotic oscillations are therefore mainly of academic interest, unless situations can be determined, where chaos constitutes a safety hazard.

The asymmetric oscillations, however, may be detected through the long term effect of the wear of the wheelsets. Such — hitherto unexplained — asymmetric wear has indeed been detected on certain unit freight trains and suburban train sets limited to run on a certain line. The connection with an asymmetric oscillation has not yet been established, but it remains an interesting possibility that deserves attention.

We shall briefly describe two theoretical models of railway bogies. In the first model — the so-called Cooperrider bogie — the nonlinearities are purely dynamical. One arises from the strain-stress relation in the wheel-rail contact zone. It is better known as the creep force-creepage relation. The other nonlinearity arises as a result of the modelling of the flange as a very stiff, linear spring with a dead band. When the amplitudes of the wheelset in the lateral direction are sufficiently large, impact occurs between the wheel and the spring. In the second model we consider a wheel running on a rail with realistic profiles. Then the flange contact takes place 'naturally' as a consequence of another nonlinearity, which replaces the impact in the first model. This other nonlinearity stems from the nonlinear kinematic conditions determined by the contact geometry.

The Model

Linear Constraints

We examine the COOPERRIDER model [1] of a bogie running with constant speed V on a perfect, stiff, level and straight track. The displacements are measured relative to a coordinate system moving along the straight track with the constant speed of the vehicle. In this moving reference frame the displacements are assumed small. The wheels, axles and the bogie frame are stiff, and friction is only included in the wheel-rail forces.

In the first model flange forces are described as stiff linear springs with a dead band, the wheels are assumed conical, and the railheads are arcs of a circle so gravitational stiffness is not included. The model is described in detail with all parameter values in [2]. It is shown on *Fig. 1.*

Realistic Wheel and Rail Profiles

The other model consists of the same bogie but now it has wheels with a DSB 82–1 profile running on UIC60 rails with nominal gauge 1435 mm and a cant of 1/40. The equations of constraint must then be added to the system.

These equations define the wheel and rail profiles and are used simultaneously with the dynamic system to determine the points of contact between the wheelsets and the rails — as long as they touch each other. When a normal force at a contact point becomes negative, the constraint equations are substituted by dynamical equations for the motion of the wheelset under the action of gravity and inertial forces on that wheel until contact happens again.

Fig. 2 shows the point of contact in dependence on the lateral displacement of the wheelset.

Our models have seven degrees of freedom. They are: Lateral and yaw motion of each of the two wheelsets and the bogie frame and roll motion of the bogie frame. The only nonlinearities in the models arise from the creepage-creep force relationship and in the second model also from the equations of constraint. The seven second order ordinary differential equations are transformed into an autonomous dynamical system consisting

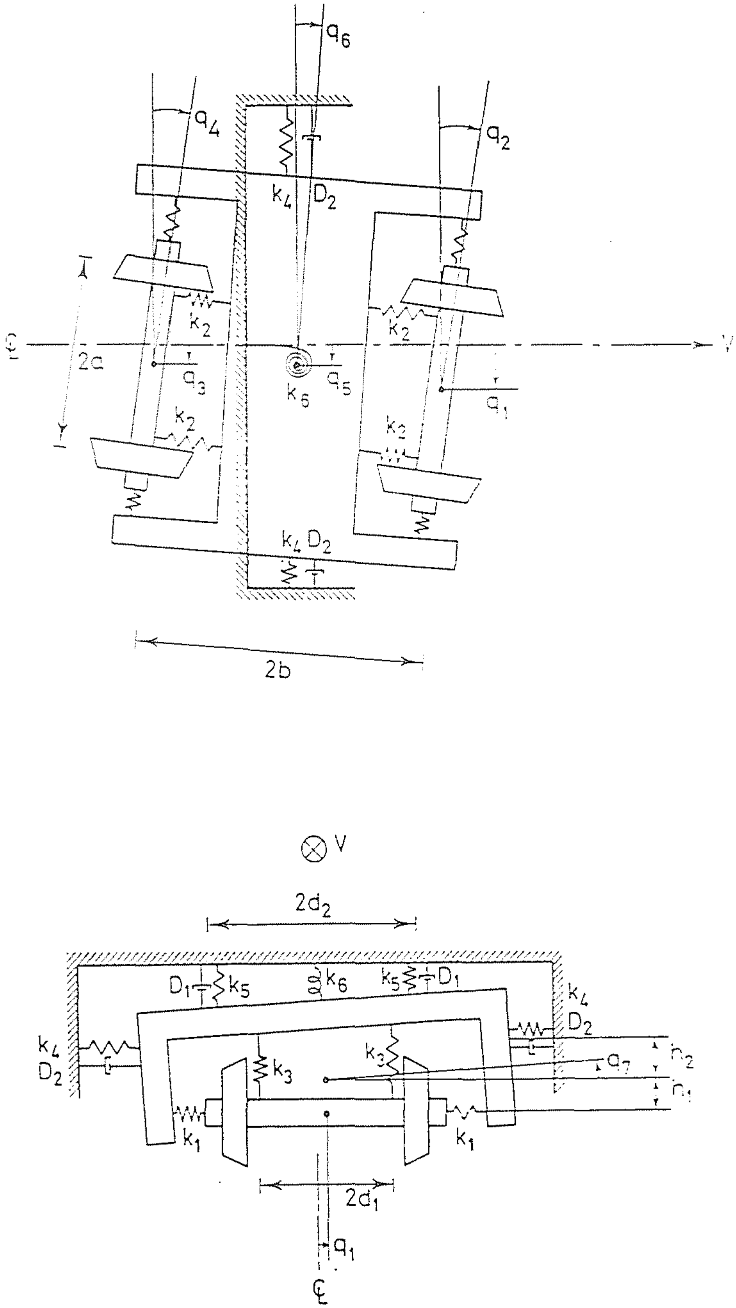

Fig. 1. The bogie model

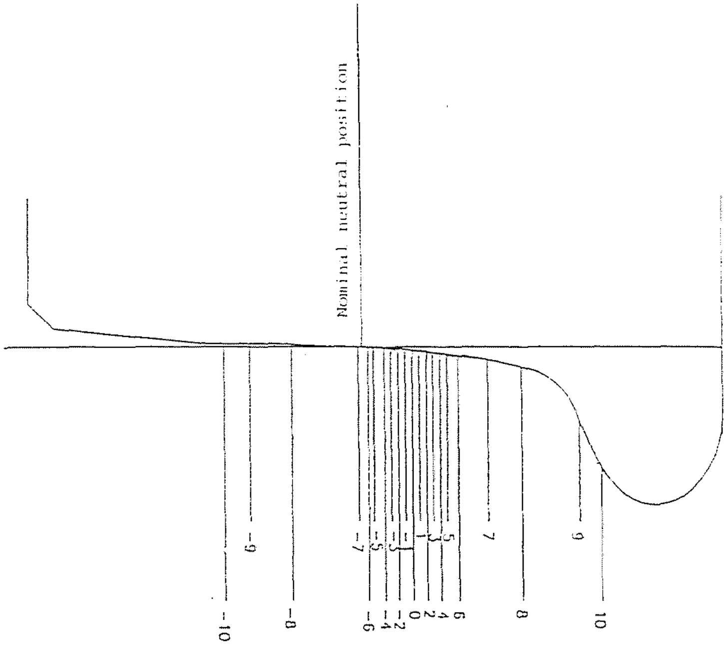

Fig. 2. Position of the contact point on the wheel profile as a function of the lateral displacement (mm) from the centred contact position calculated by means of W. Kik's RSGEO

of fourteen first order ordinary differential equations with the speed V as the control parameter through the substitution: $q_n = x_{2n-1}$, $\dot{q}_n = x_{2n}$, $n = 1, \cdots, 7$. See KAAS – PETERSEN [2].

The Method of Investigation

We use numerical methods for the investigation of the mathematical model. Primarily the program 'PATH' is used to follow the solutions in phase-parameter space in dependence on the control parameter, the speed of the vehicle V. The application of 'PATH' in vehicle dynamics is described earlier [2], [3]. When we find chaos, we display it by numerically determined time series and phase space projections.

The Results

Bifurcation Diagram for the Cooperrider Bogie, Linear Constraints

The bifurcation diagram for the Cooperrider bogie from [4] is shown on *Fig. 3*.

On the diagram we have plotted the amplitude of the — stationary or time dependent — lateral displacement of the rear axle versus the speed V. It must be noted that $x_5 = 0$ is a solution for all $V \geq 0$. This solution is asymptotically stable below A ($V = 65.4$ m/s). Beyond A the solution is unstable, and in A a periodic solution bifurcates away from the zero solution. This periodic solution is unstable, and it has a growing amplitude for decreasing speed until B ($V = 63.6$ m/s), where the amplitude is so big that the flange hits the rail.

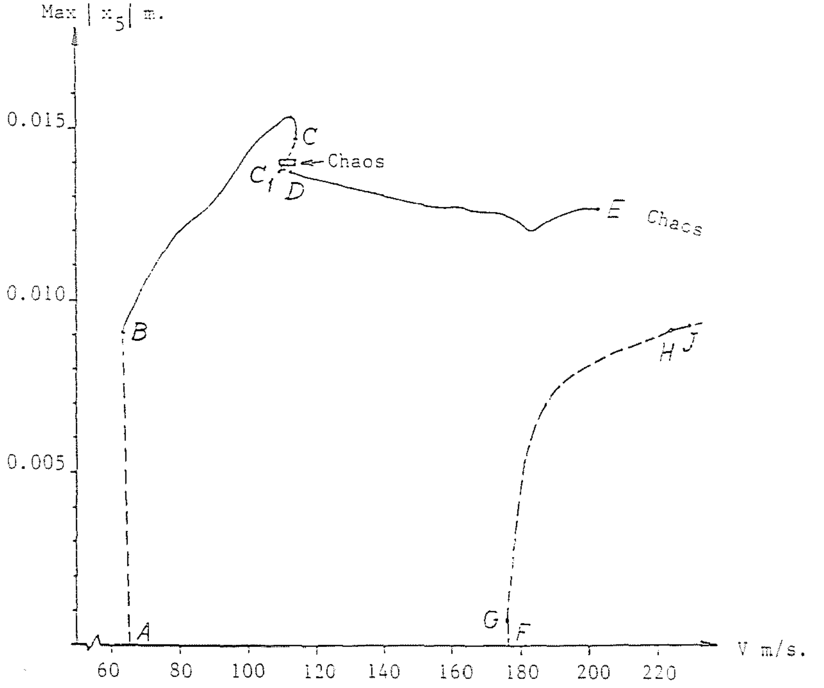

Fig. 3. Bifurcation diagram for linear kinematic conditions. Dotted line indicates an unstable solution.

The flange contact stabilizes the oscillation, and its amplitude continues to grow — now with growing speed — until C ($V = 114.6$ m/s), where the solution 'bends back' into another unstable oscillation.

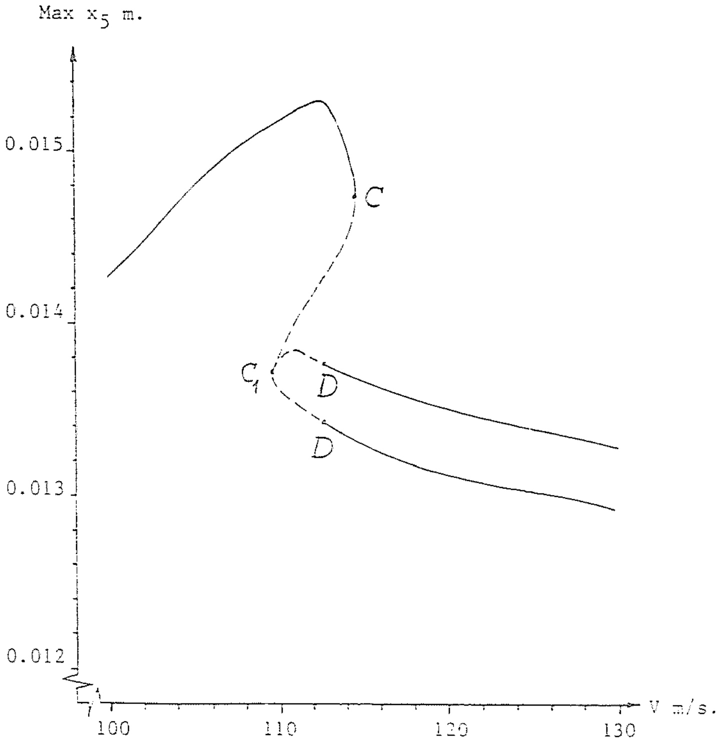

Fig. 4. Blow up of a part of the bifurcation diagram of *Fig. 3*

In the speed range $109.7 < V < 114.6$ m/s a complicated sequence of bifurcations occur, and we show a blow up of this range on *Fig. 4*. The interesting feature is the TWO asymmetric solutions that are created at C_1 and which turn stable at D ($V = 112.7$ m/s) and remain stable up to $V = 203.3$ m/s (point E on *Fig. 5*). The curve from D to E on *Fig. 3* therefore symbolizes two stable asymmetric oscillations. For a given V the amplitude for positive y of one oscillation lies on the upper curve (see *Fig. 4*) and the amplitude for positive y of the other oscillation lies on the lower curve. The two asymmetric oscillation modes are reflexions of one another in the track centre line. The asymmetry in this case is less than one millimeter. The hunting motion, however, is so violent that the usual disturbances will have no influence on the behaviour. If the asymmetric oscillation lasts long enough or occurs repeatedly over the same piece of the line, then the wheelset will wear unevenly. Above point E ($V = 203.3$ m/s) in *Fig. 3* a chaotic attractor exists, and the oscillations become chaotic. The asymmetry is preserved as shown on *Fig. 5*. *Fig. 6*

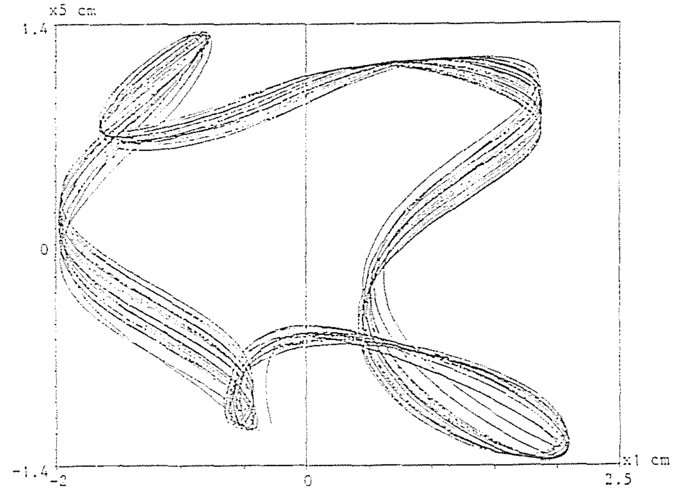

Fig. 5. Phase plane projection of the chaotic attractor at 204 m/s

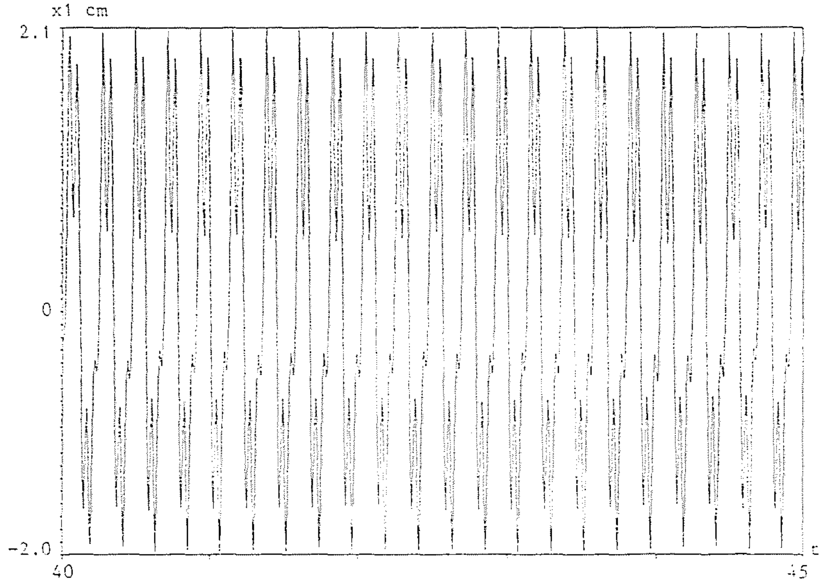

Fig. 6. Chaotic oscillations at 204 m/s

shows x_1 versus time, and the erratic behaviour is hardly visible, but on *Fig. 5* the broad band structure of the attractor reveals its true nature. We have computed the maximal Liapunov exponent for a sequence of steps in time, and *Fig. 7* shows that the Liapunov exponent decreases fast to

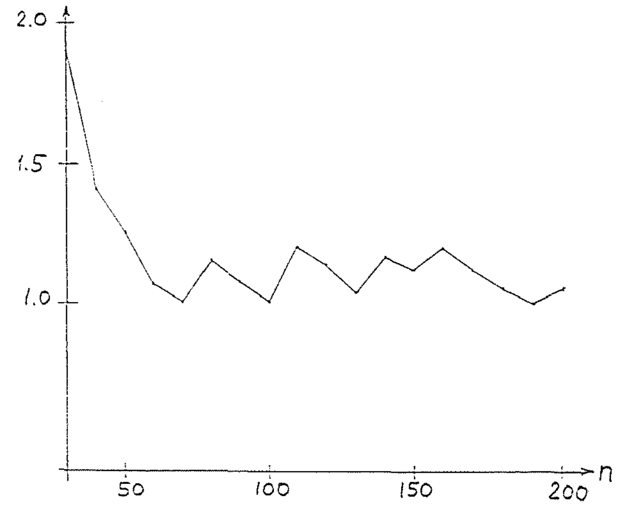

Fig. 7. The approximation to the maximal Liapunov exponent versus number of time steps at 204 m/s

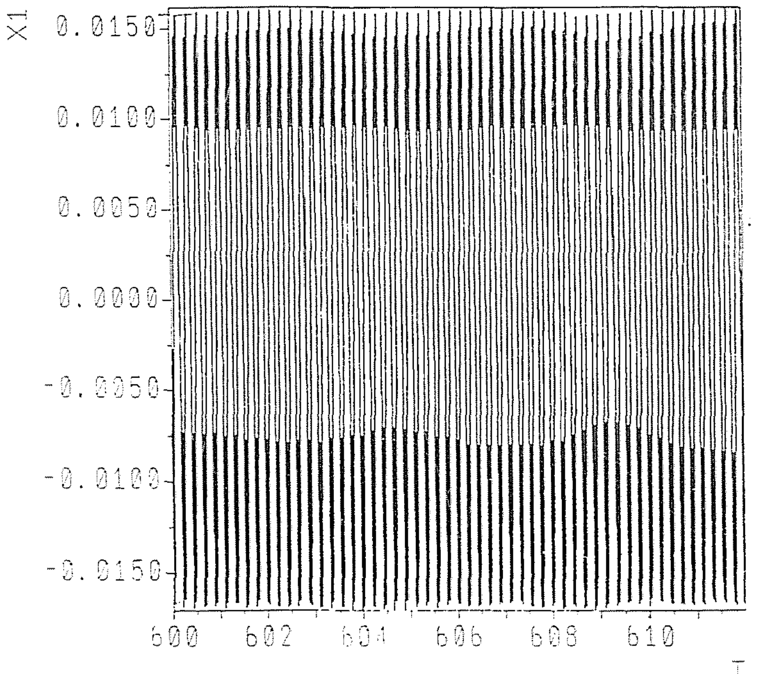

Fig. 8. Chaotic motion at 111.8 m/s

a value around 1 and then remains in that neighbourhood. A positive Liapunov exponent indicates chaos.

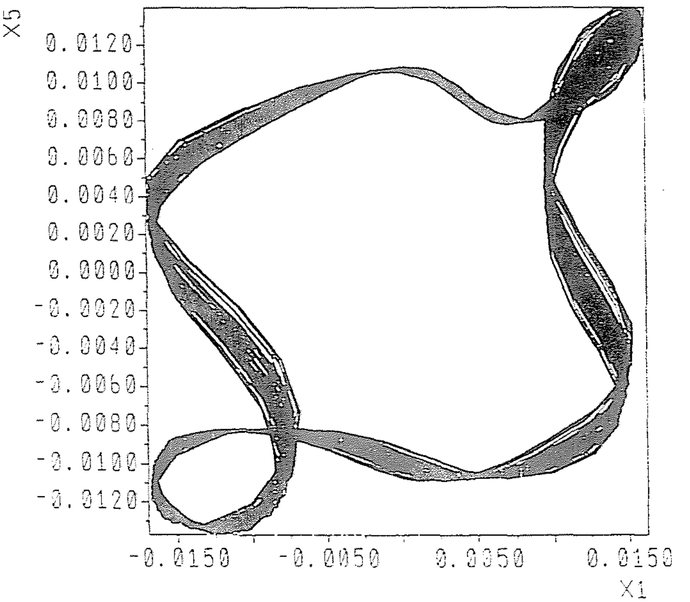

Fig. 9. Phase plane projection of chaos at 111.8 m/s

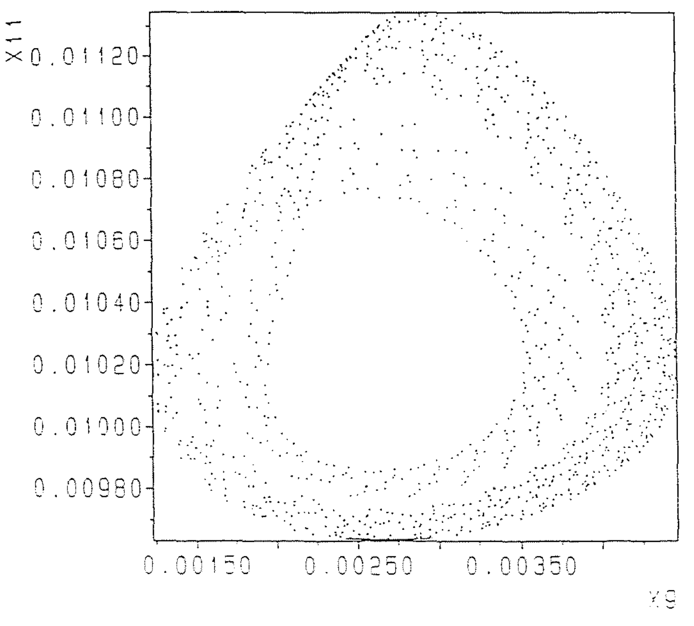

Fig. 10. Poincare section of the chaotic attractor at $V = 111.8$ m/s

The complicated structure of solutions between $V = 109$ m/s and 114 m/s may also give rise to the development of chaos. It is often a consequence of 'competing attractors'. We have indeed found chaos in this speed range. As an example we show on *Fig. 8* x_1 versus time (the time sequence starts at $t = 10$ min). On *Fig. 9* which shows x_5 versus x_1, the broad band structure much more clearly reveals the aperiodic nature of the oscillations. Carsten Nordstrøm Jensen produced the interesting *Fig. 10*. It shows a Poincaré section of the chaotic attractor, and the points seem to be distributed on a Rössler band. The Rössler band is a chaotic attractor, and we thus conclude that the motion we found is also chaotic.

Bifurcation Diagram for the Cooperrider Bogie, Realistic Wheel and Rail Profiles

On *Fig. 11* the amplitude of the front axle versus the speed is shown. Again the steady motion is asymptotically stable only up to A ($V = 50.2$ m/s). The bifurcation point A is at a lower speed here than in *Fig. 3*.

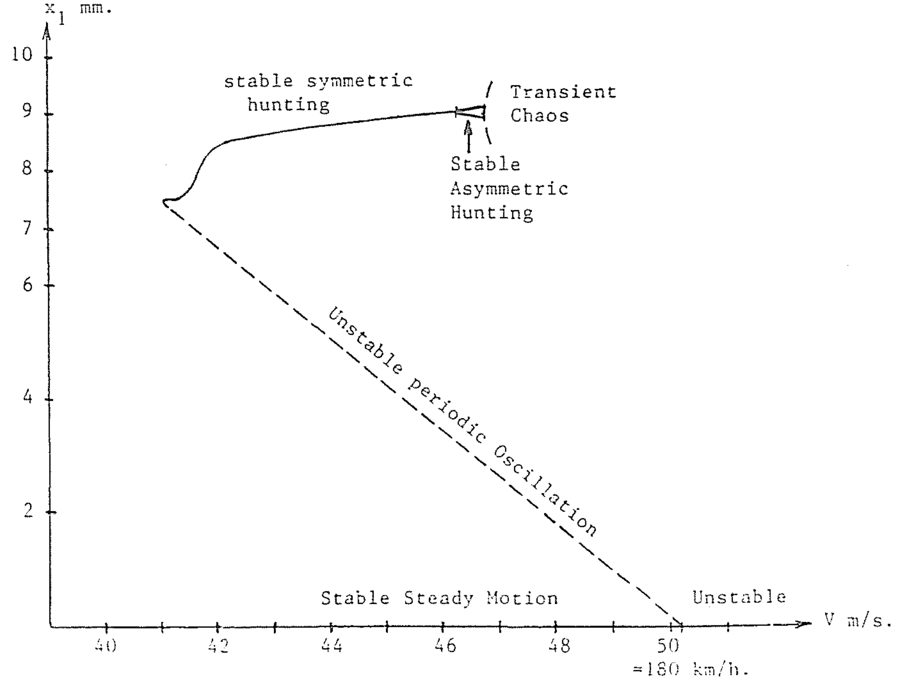

Fig. 11. Bifurcation diagram for realistic wheel and rail profiles

The unstable periodic oscillation on *Fig. 11* exists all the way down to B ($V = 41.1$ m/s), which is at a much slower speed than B on *Fig. 3*. The change is typical of the influence of the added nonlinearities from the kinematic contact relations. From B the symmetric periodic oscillations grow in amplitude with speed, and the oscillations are asymptotically stable up to D ($V = 46.2$ m/s). In D symmetry breaking bifurcation occurs, and two stable asymmetric periodic oscillations develop. They only exist up to E ($V = 46.7$ m/s), where the motion turns chaotic.

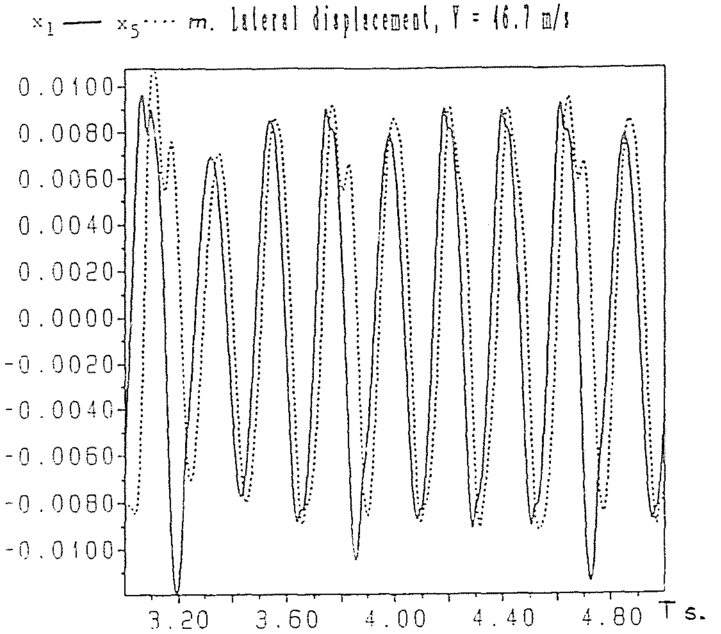

Fig. 12. Chaotic oscillations at 46.7 m/s with realistic wheel and rail profiles

This chaotic motion is so violent, see *Fig. 12* that the assumption for the validity of the model no longer holds, and a more complicated model will be needed to simulate the dynamics numerically. The oscillations will, however, certainly be very violent, and it is remarkable, that they may occur at a lower speed than the linear critical speed at 50.2 m/s.

Actually the chaotic motion may develop at any speed above 46.7 m/s if the disturbance in the track is of the 'right size', i.e. if it is in the domain of attraction for the chaotic motion. The domain of attraction is of a finite size, and it grows with speed. Above $V = 50.2$ m/s, it is the only possible attractor we know.

Conclusions

In both vehicle models we found speed ranges with chaotic motion and with asymmetric periodic oscillations. In the Cooperrider model with linear constraints the chaos appears as a pair of asymmetric attractors. The amplitudes of the chaotic motion are not larger than the amplitudes of the periodic oscillations in neighbouring speed ranges. It merely adds an erratic component to an otherwise regular oscillation. The large speed range of asymmetric oscillations is more noteworthy. Indeed asymmetric motion is the only form of a stable mode above $V = 114.6$ m/s. Notice also that chaos exists only is speed ranges, where asymmetric attractors exist.

The asymmetric motion occurs in speed ranges where the amplitude of the lateral oscillation is so large — larger than 9.1 mm — that impact with the stiff spring, which simulates the restoring force of the wheel flange, takes place. The modelling is somewhat unrealistic, but other discontinuities in the force function may have the same effect. We therefore conjecture that discontinuities in the forcing in vehicle dynamics may give rise to the development of asymmetric motion and chaos.

The bifurcation at $V = 65.4$ m/s is subcritical, but the nonlinear critical speed is only a little smaller $V_{\text{crit}} = 63.6$ m/s. In reality a jump from a calm ride to a large amplitude hunting motion will occur at a speed between 63.6 and 65.4 m/s. The hunting will be stable with an amplitude growing with speed. If the vehicle slows down, the hunting will abruptly cease at $V_{\text{crit}} = 63.6$ m/s. This happens at a 'safe distance' from the speed $V = 109.7$ m/s, where the chaotic motion may develop. It is another matter that the large amplitudes seen in the speed range above say 95 m/s in reality would correspond to derailment, a situation our dynamical system does not model correctly.

The Cooperrider bogie with nonlinear constraints shows a different behaviour in dependence on the speed of the vehicle. The linear critical $V = 50.2$ m/s is lower than in the previous case, and the nonlinear critical speed $V_{\text{crit}} = 41.1$ m/s is significantly lower than $V_{\text{crit}} = 63.6$ m/s in the previous example. Again we find that the amplitude of the oscillation increases with the speed. The oscillation is time periodic and symmetric around the track center line up to $V = 46.2$ m/s. Notice that there exist two stable solutions in the speed range 41.1 m/s $\leq V \leq$ 50.2 m/s — one oscillating and another steady solution. At $V = 46.2$ m/s the ideal point of contact between wheel and rail jumps to the flange at the amplitudes of the oscillation. This causes the normal force in that point to jump discontinuously in magnitude and direction, and the symmetric oscillation becomes unstable. In the short speed range 46.2 m/s $< V <$ 46.7 m/s two stable, asymmetric oscillations coexist with the stable steady motion, but at $V = 46.7$ m/s the asymmetric oscillations lose stability and chaos develops.

The chaotic motion has such large amplitudes that our theoretical model ceases to be valid. It is therefore meaningless to continue the integrations in time in order to find out whether the phase space trajectories eventually will approach an attractor. We thus only claim to have found transient chaos with large amplitudes.

The interesting feature in the development of chaos in the Cooperrider bogie with realistic rail and wheel profiles is that large amplitude transient chaos coexists with the steady motion at speeds BELOW the linear critical speed, which still is the only stability limit most manufacturers determine. Since the transient chaos leads to large amplitude oscillations, and since it is more and more likely to happen the closer the speed gets to the linear critical speed, this example illustrates how inadequate the linear critical speed is for the description of the stability of the vehicle. A truly nonlinear analysis of the stability should therefore always be performed by the manufacturers of railway vehicles.

Acknowledgement

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