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MODELLING FOR VEHICLE STABILITY ENHANCEMENT

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Abstract

This paper presents a vehicle model built in the Matlab/Simulink environment to support the qualitative analysis of integrated control design methods through simulation case studies. The vehicle model contains the brake, the steering and the suspension systems. For illustration purposes a braking control for avoiding rollover is applied to the model.

Keywords: vehicle dynamics, nonlinear modelling, steering, braking, suspension, wheels.

1. Introduction

These days there is a growing demand for vehicles with ever better driving characteristics, in which efficiency, safety, and performances, such as passenger comfort, road holding, rollover stability, yaw stability, suspension working space and energy consumption, are ensured. Several individual active control mechanisms are applied in road vehicles to solve different control tasks. The braking system influences the longitudinal dynamics, however it also modifies the yaw dynamics during vehicle maneuvers. Basically, the role of the suspension system is to improve passenger comfort, i.e. to reduce the effect of harmful vibrations on the vehicle and passengers. In addition, by using active dampers it is able to enhance the road holding. Using additional steering angle generated by a controller the yaw angle is modified and the harmful effect of the side slip angle is reduced to improve the vehicle stability. Since these components affect the same vehicle dynamics interactions may occur that may result in conflicts. If the controllers are designed independently and balance between them is not taken into consideration, they may attenuate the effects of other controllers as they were disturbances. However, the aim of the integrated control methodologies is to combine and supervise all controllable subsystems affecting vehicle dynamic responses, [6, 7]. The effects of one subsystem are not disturbances but known effects on the other subsystems and belong to the model for control design.

Since the control design methods developed in our project focus on the brake, the steering and the suspension systems, the vehicle model developed in Matlab/Simulink environment contains these components. The vehicle structure is presented in *Fig. 1.* The complexity of the model is developed so that interactions and conflicts between the subsystems can be analised. Actuators applied in the

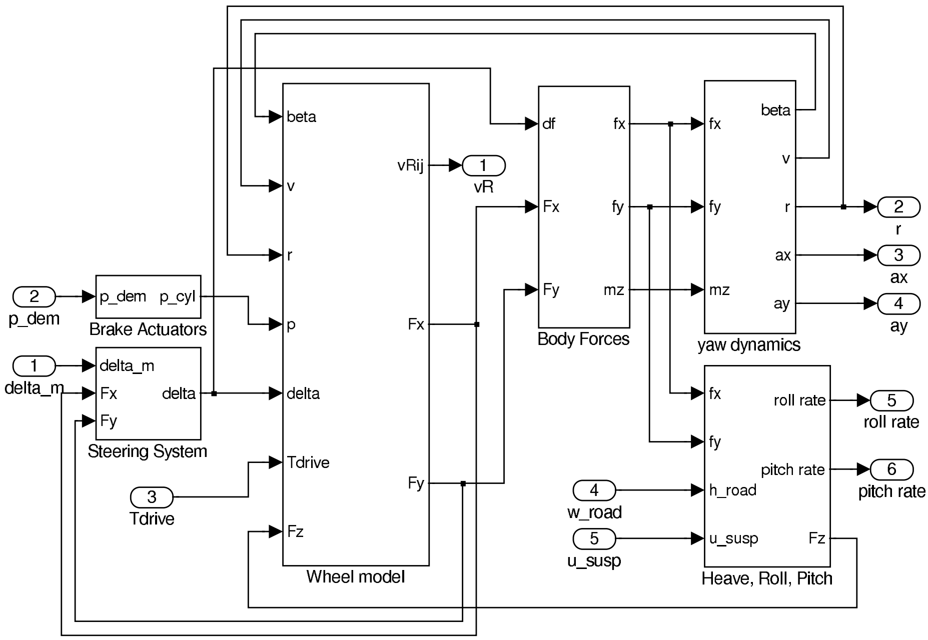

Fig. 1. The vehicle structure in Matlab/Simulink

model are approximations of the real ones. The brake dynamics is a linear model with delay time. The suspension hydraulic actuators have nonlinear dynamics. The tire dynamics consists of a static empirical nonlinear characteristics and the dynamics of the rotating wheel. The qualitative features of the developed integrated control methods and the appropriation of the theoretical methods can be analysed through several simulation case studies.

The layout of the paper is the following. In Section 2 the structure of the vehicle model which is built in Matlab/Simulink is summarized. In Section 3 the model validation process is illustrated. Finally, Section 4 contains some concluding remarks.

2. The Structure of the Vehicle Model

In this chapter a three-dimensional full-car vehicle model is built. The derivation of the dynamics assumes rigid bodies for which force and torque balance equations are formalized. Modelling simplifications are applied that leads to inconsistency in the system of modelling assumptions. The inconsistency comes from the partial decoupling of yaw, pitch, roll and heave motions. For example deriving the yaw dynamics the whole vehicle is considered to be a single rigid body. Positions of external forces are fixed, inertia parameters are constants. The roll, pitch and heave motions influence it only through the change of wheel loads. During the derivation of the other components, however the wheels are separate rigid bodies. The displacement of the wheels is only vertical. The sprung mass is able to rotate around fixed horizontal roll and pitch axles and affects the heave dynamics.

These modelling simplifications are customary in the literature when designing control systems, see for example in [3].

First, the vehicle yaw dynamics is derived based on *Fig. 2*. Horizontal road is assumed and the air-drag, rolling resistance and wheel caster are neglected. Positions of external forces and the center of gravity are fixed. The mass moment of inertia around the vertical z-axis is constants. The force and torque balance equa-

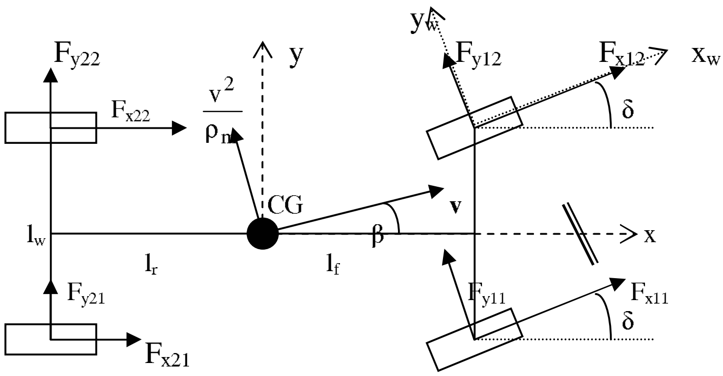

Fig. 2. The yaw dynamics

tions can be easily written. The momentaneous radius of motion can be expressed as $\rho_m = \dfrac{v}{r+\beta}$, where r is the yaw rate, β is the side slip angle and v is the forward and absolute velocity. Reshaping the equations we arrive to the following system of equations:

$$
\begin{aligned}
\dot{v} &= \frac{\cos\beta}{m} f_x + \frac{\sin\beta}{m} f_y \\
\dot{\beta} &= -\frac{\sin\beta}{vm} f_x + \frac{\cos\beta}{vm} f_y - r \\
\dot{r} &= \frac{m_z}{J_z} \\
f_x &= F_{x21} + F_{x22} + (F_{x11} + F_{x12})\cos\delta - (F_{y11} + F_{y12})\sin\delta \\
f_y &= F_{y21} + F_{y22} + \big(F_{y11} + F_{y12}\big)\cos\delta + (F_{x11} + F_{x12})\sin\delta \\
m_z &= -(F_{y21} + F_{y22})l_r + l_f\,\big((F_{y11} + F_{y12})\cos\delta + (F_{x11} + F_{x12})\sin\delta\big) \\
&\quad + \frac{l_w}{2}\,\big((F_{y12} - F_{y11})\sin\delta + (F_{x11} - F_{x12})\cos\delta + F_{x21} - F_{x22}\big)
\end{aligned}
$$

and δ is the steering angle, J_z is the inertia, f_x and f_y are the forces in longitudinal and lateral directions, m_z is the torque, F_{xij} and F_{yij} are the ground contact forces.

The wheel model computes the tire-road contact forces when the steering angle, wheel loads, brake pressures and the states of yaw motion are given. The wheel model consists of three parts.

The wheel-center velocity vectors are computed from the state variables r, v and β of the yaw motion. For more details see [3]. The absolute value of the velocity vector and the wheel slip angle are denoted by v_{wij} and α_{ij}, respectively.

The second part of the wheel model describes the rotational motion of the wheel. Here the inertia of the wheel $J_w \dot{v}_R$ keeps balance with the moment of the longitudinal contact force, $F_x r_{eff}$, the driving moment, T_{dr}, and the braking moment, T_{br}. The v_R is the rotation equivalent wheel velocity, J_w is the mass moment of inertia and r_{eff} is the wheel radius. The dynamic equations are the following:

$$\begin{aligned}\dot{v}_{R,ij} &= -F_{xij}\frac{r_{eff}^2}{J_w} - T_{br,ij}\frac{r_{eff}}{J_w} + T_{dr,ij}\frac{r_{eff}}{J_w}\\ T_{br} &= (p - p_0)c_{brc} \text{ if } p > p_0, 0 \text{ otherwise}\end{aligned}$$

The third part of the wheel model computes the contact forces. It is the empiric and static nonlinear adhesion model depending on the slip angle α_{ij} and the longitudinal slip $\lambda_{ij} = \dfrac{v_{Rij}}{v_{wij}}$. The model is similar in form for all wheels, therefore the ij indexing is omitted for brevity.

$$\begin{aligned}F_x &= F_z\frac{\mu(s)}{s}\frac{\lambda - \cos(\alpha)}{\max(1, \cos(\alpha)\lambda)} = F_z C(\lambda, \alpha)(\lambda - \cos(\alpha))\\ F_y &= F_z\frac{\mu(s)}{s}\frac{\sin(\alpha)}{\max(1, \cos(\alpha)\lambda)} = F_z C(\lambda, \alpha)\sin(\alpha)\\ \mu(s) &= c_1(1 - e^{-c_2 s}) - c_3 s\\ s &= \frac{\sqrt{1 + \lambda^2 - 2\lambda\cos(\alpha)}}{\max(1, \cos(\alpha)\lambda)},\end{aligned}$$

where $C(\lambda, \alpha)$ is the cornering stiffness function.

The dynamics of the brake actuator is modelled by a linear time invariant system using a delay and a linear time-invariant component W_{br}. The dynamics in frequency domain is the following:

$$p_{ij}(s) = W_{br}(s)e^{-sT_h}p_{dem,ij}(s)$$

where p_{ij} is the brake cylinder pressure and $p_{dem,ij}$ is the required pressure demand. The dynamics of the braking system is modelled by several papers, see e.g. [2, 5].

The modelling of the suspension system includes the vertical, pitch and roll dynamics. In *Fig. 3* the model with four independent suspension systems is plotted to describe effects of road vibrations. This model is extended by the inertia forces $-f_x\dfrac{m_s}{m}$ and $-f_y\dfrac{m_s}{m}$ and some fictitious dampers (with coefficients denoted by $b_{s\chi}$

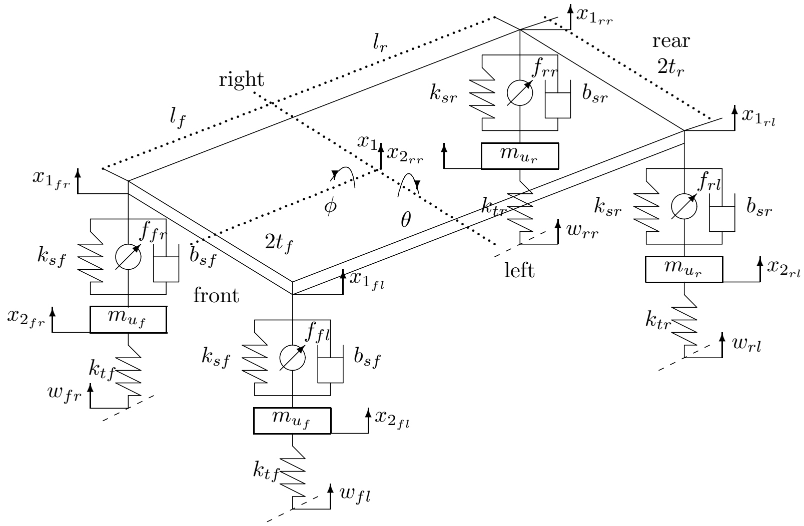

Fig. 3. The suspension model

and $b_{s\varphi}$) and springs (with $k_{s\chi}$ and $k_{s\varphi}$) in order to damp the roll and pitch dynamics. The sprung-mass is denoted by m_s. The actuators of the suspension system generate forces u_{ij}.

The dynamic motion equations are defined for the vertical, pitch and roll dynamics of the sprung mass and the vertical dynamics of the unsprung masses. For illustration purposes a part of the Lagrange equations of the suspension model is presented.

$$
\begin{aligned}
\ddot{\chi} J_y &= 2\left(k_{sr}l_r - k_{sf}l_f\right) z - \left(k_{s\chi} - m_s g h_\chi + 2k_{sr}l_r^2 + 2k_{sf}l_f^2\right)\chi \\
&\quad + k_{sf}l_f\left(z_{fl} + z_{fr}\right) - k_{sr}l_r\left(z_{fr} + z_{fr}\right) + 2\left(-b_{sf}l_f + b_{sr}l_r\right)\dot{z} \\
&\quad - \left(b_{s\chi} + 2b_{sr}l_r^2 + 2b_{sf}l_f^2\right)\dot{\chi} + b_{sf}l_f\left(\dot{z}_{fl} + \dot{z}_{fr}\right) - b_{sr}l_r\left(\dot{z}_{fr} + \dot{z}_{fr}\right) \\
&\quad + J_y\frac{m_s}{m}h_\chi f_x - l_f\left(u_{sfl} + u_{sfr}\right) + l_r\left(u_{srl} + u_{srr}\right) \\
\ddot{z}_{fl}m_{uf} &= k_{sf}z + k_{sf}l_f\chi + k_{sf}t_f\varphi - \left(k_{sf} + k_{tf}\right)z_{fl} + \\
&\quad b_{sf}\dot{z} + b_{sf}l_f\dot{\chi} + b_{sf}t_f\dot{\varphi} - b_{sf}\dot{z}_{fl} + u_{sfl} + k_{tf}w_{fl}
\end{aligned}
$$

For more details see [1].

In contrast to the derivation of yaw dynamics the wheel caster r_s cannot be neglected in the steering model, since it generates the self-aligning torque of the front wheels.

Next a simple model of a stock steering system is derived (see *Fig. 4*) in order to describe the connection between the steering wheel angle δ_m, the tire forces and the steering angle δ. The dynamics of the power assist unit is ignored. The torque

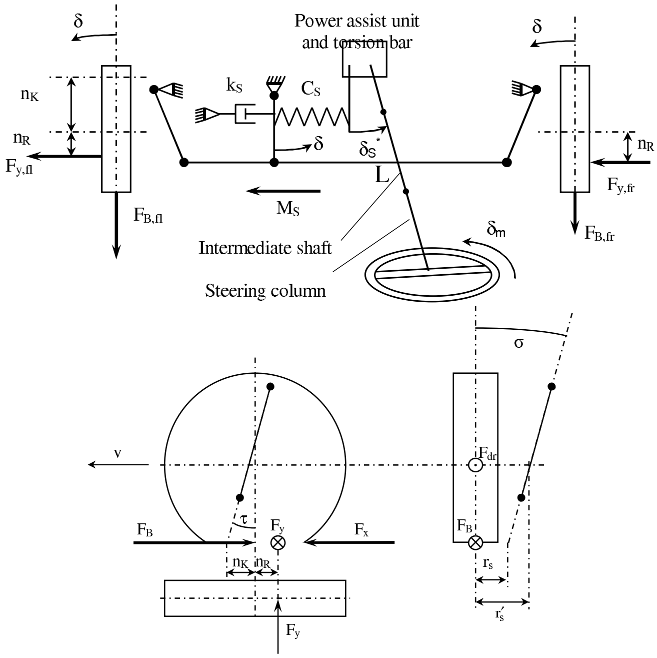

Fig. 4. Left: a simple stock steering system.
Right: forces on the wheel in side and front view

balance equations according to *Fig. 4* are

$$\begin{aligned} J_m\ddot{\delta}_m &= -C_S(k\delta_m - \delta) - k_m\dot{\delta}_m + T_d \\ M_S + J_S\ddot{\delta} &= C_S(k\delta_m - \delta) - k_S\dot{\delta} \\ M_S &= r_s'(F_{xfl} - F_{xfr}) + r_s(F_{Bfr} - F_{Bfl}) + (n_R + n_K)(F_{yfl} + F_{yfr}) \end{aligned}$$

where J_m, k_m are inertia and damping of the upper steering column, J_S, k_S are lumped inertia and damping of the steering system below the torsion bar, C_S is a spring coefficient of the torsion bar, k is the steering ratio, r_s', r_s are geometric constants, F_{dr} is driving force, F_{Bfr}, F_{Bfl} are braking forces, M_S is the aligning torque, T_d is the torque of the driver on the upper steering column. There are several papers that are concerned with different modelling approaches that develop steering systems, see e.g. [4].

3. The Model Validation

In this section steering and braking maneuvers are presented to show simulation results and to explore the plausibility of the state signals and internal variables.

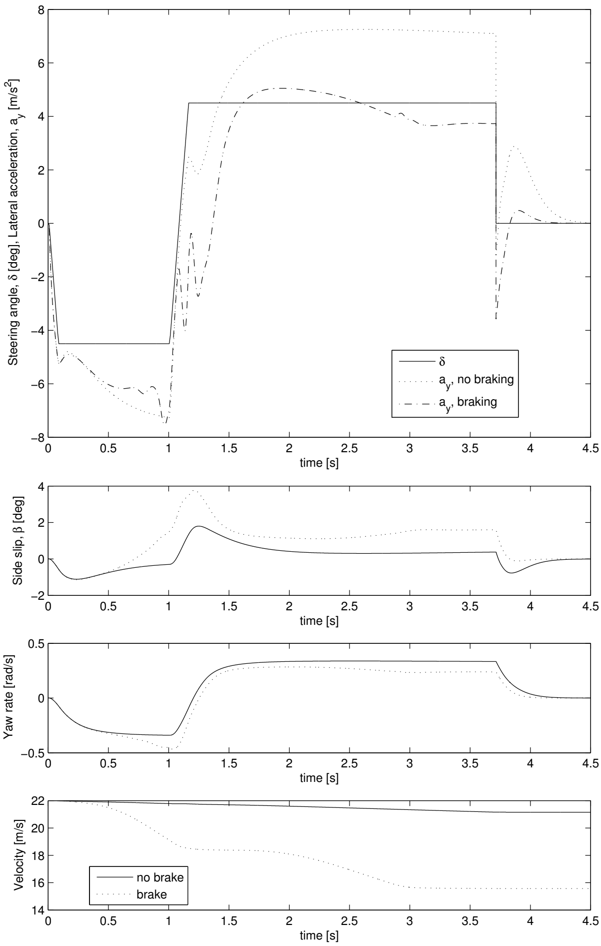

Fig. 5. Up: Steering angle (solid) and lateral accelerations in case of braking (dash-dot) and no braking (dotted). Down: side slip, yaw rate and velocity

On 22 m/s velocity a sharp left steering was applied with about 4.5 grad of steering angle for one sec. (first stage), after that a sharp right turn for a 2.5s period (second stage) followed by a zero steering angle stage. Two simulations were applied: this steering manoeuvre with and without anti-roll braking control. There was no driving, no rolling resistance and no air-drag.

The steering angle and the lateral acceleration of the two cases are shown on the up of *Fig. 5* and the vehicle slip angle, yaw rate and the velocity on the downt. The lateral acceleration a_y achieved a dangerous level if no control worked. In the second experiment the outer front and rear wheels were braked by a controller that

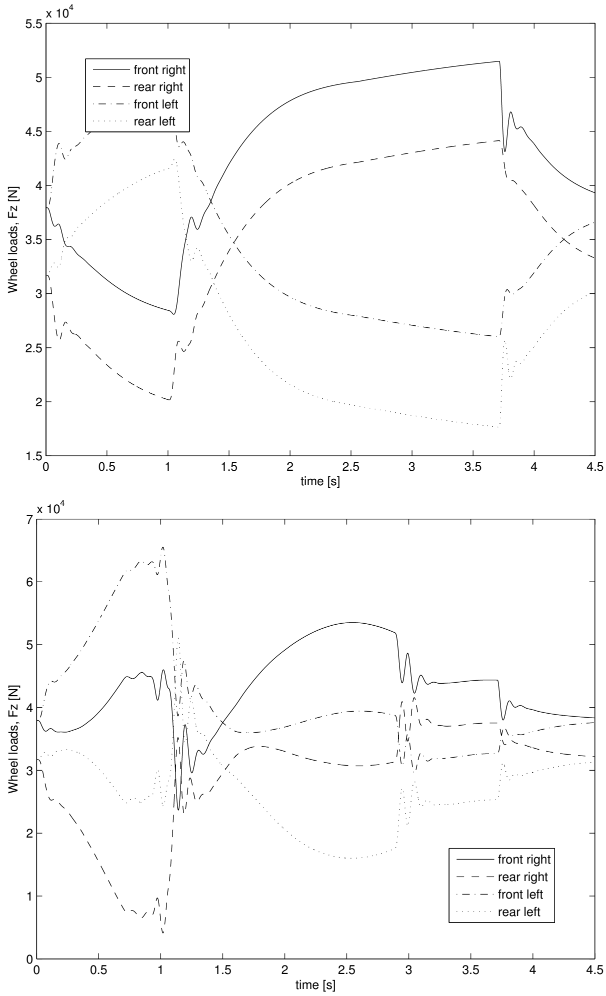

Fig. 6. Wheel loads without braking (up), with braking (down)

was switched on whenever $|a_y| > 4\text{m/s}^2$. The brake controller aimed at decreasing the lateral acceleration. It can be seen that the lateral acceleration was considerably decreased in the second stage, due to the braking.

The wheel loads are plotted in *Fig. 6*. During the up turn the up wheels had larger load and because of the steering resistance the load of the first wheel was larger than the rear. It can be seen from the comparison of the two pictures of the figure that the braking translated the weight from the rear to the front wheels. At about 3s the lateral acceleration became less then 4 m/s^2 and the controller was switched off. The pitch angle (solid line in *Fig. 7*) started decreasing and so the difference between the front and rear wheel loads.

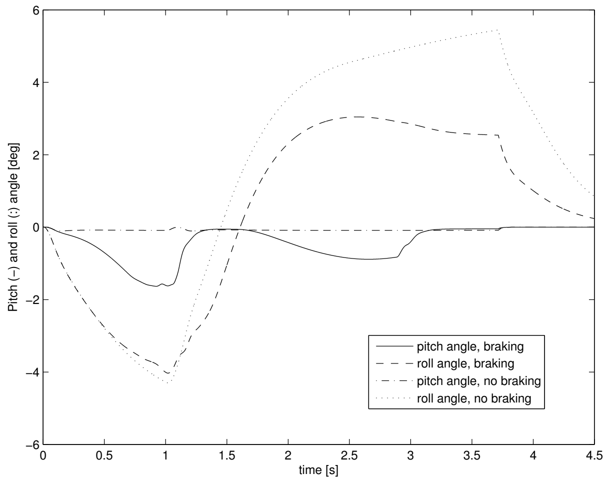

Fig. 7. Pitch and roll angles in case of braking and no braking.

During an overtake manoeuvre, which consists of two turns of opposite directions, the vehicle is more easily rolling over, than during a simple turn. At the first turn the body inclines towards the outer side then in the opposite turn it inclines hard to the opposite direction. The goal of the control, i.e. the avoidance of rollover can be well followed in *Fig. 7.* In the second stage the roll angle was much greater than in the first, when no brake action was allowed (dotted line). In the second stage the roll angle became much smaller due to the braking.

It can be concluded based on the down of *Fig. 5* that the brake control should be improved. The yaw rate increased instead of decreasing. The reason is shown in *Fig. 8.* The slip control could not increase the longitudinal slip to the reference, because the actuator saturated. Indeed, this is realistic: on high velocity and large load on the front wheel the 10 bar brake pressure cannot block the wheel. Meanwhile the longitudinal slip of the rear wheel oscillated around the reference. The side force decreased and the rear of the vehicle slipped sideways. This is the very behaviour we await from the model, because the brake control was realized such that equal slips were prescribed on the braked side, and the demandable slip was limited at the peak of the longitudinal adhesion function i.e. at the boundary of the stable and instable region of the slip.

The controller has been analised based on the model constructed in Matlab/Simulink. The results suited the physical expectations. This shows that the model is appropriate for design and analise controllers.

The developed vehicle model was also compared with a high-performance vehicle simulator applied in industry. Tested with some typical steering and braking excitations the measured signals showed good fit in wide range of operation.

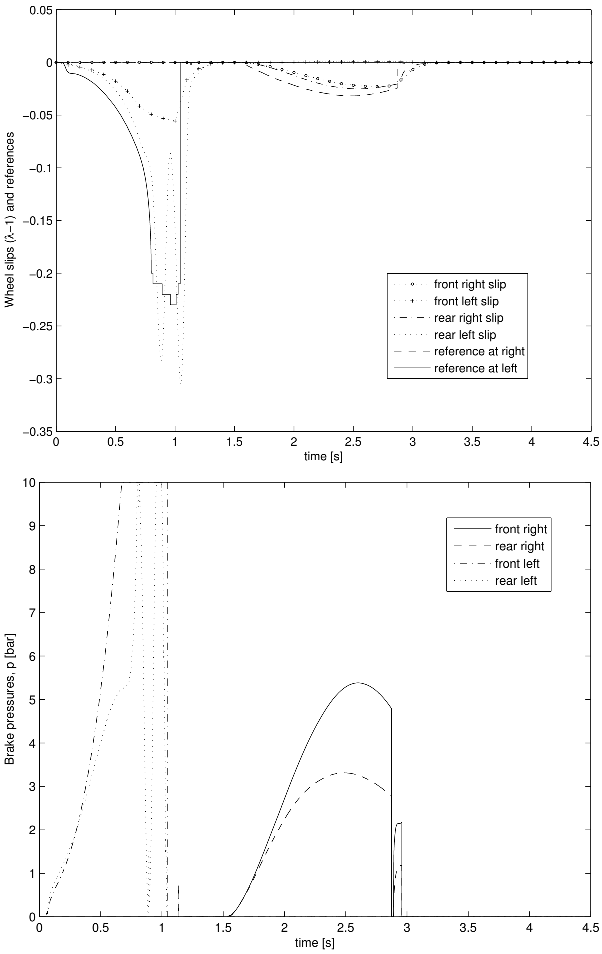

Fig. 8. Up: longitudinal slips, $\lambda_{ij} - 1$ and its references; down: brake pressures

4. Conclusion

In the paper a vehicle model built in the Matlab/Simulink has been presented. It is a full vehicle model including steering, braking, suspension and wheel models. Although the model is built based on simple modelling principles, the simulation results show that the vehicle maneuvers are similar to the real ones. This model is a good tool for the analysis of the control design.

5. Acknowledgments

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