STRENGTH OF THICK-WALLED TUBES AND CYLINDRICAL VESSELS EXPOSED TO AXISYMMETRICAL LINE LOADS

(PART I)

By

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Introduction

In numerous fields of industry, for example, in industrial chemistry, increasing economy aspects and the production of various new products require the large-scale application of high-pressure technology. In high-pressure technology, cylindrical vessels, tubes closed by elements of different shapes are generally used. Besides vessels of chemical industry, the reactor bodies used in nuclear technique, the pneumatic and hydraulic cylinders belong to this group.

These units are generally loaded by uniformly distributed internal or external pressure; strength calculation for this type of loading is generally known. Also symmetric line loading occurs in practice. The additional stresses due to point loads are superimposed upon the stresses produced by uniform loads. Axisymmetric local stresses develop at the joints of closing devices (Fig. 1), at axisymmetric supports, etc. In many cases the line loads are negligible as compared to the internal pressure; but in certain cases, e.g. for the self-sealing cover in Fig. 1/a, the additional load may be decisive. In spite of this, in practice, experimental methods prevail in shaping vessel parts able to bear additional stresses, since the strength analysis methods found in literature are complicated and difficult to apply.

In the following, a simplified method for strength analysis is presented.

Application of the modified method

The modified method starts from common ones for stresses in thin-walled tubes and cylindrical vessels, generalizing these to thick-walled tubes. The advantage of this method applied for thick-walled tubes and cylindrical vessels exactly is not to require from specialists any knowledge beyond the strength calculation of thin-walled vessels. Its application, on the other hand, relies on the practical fact that the thickness of tube or vessel walls generally corresponds to a ratio of radii:

$$k_0 = \frac{r_1}{r_2} \geq 0.4 \tag{1}$$

this ratio being determined in the first line by the strength characteristics of the structural material. Similarly, in case of laminated walls used for higher pressures, the ratio of radii corresponding to the thickness of the individual layers is not lower than 0.4.

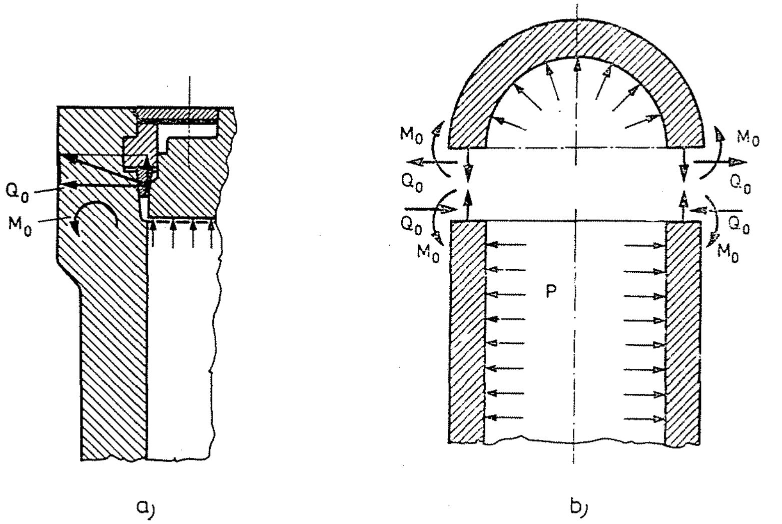


Fig. 1. Axisymmetrical line load at the junction of closing elements

Further it can be stated that the consideration of additional stresses has an importance for tubes and cylindrical shells with an internal diameter of

$$2r_1 \geq \sim 100 \text{ mm} \tag{2}$$

With regard to the defined application field of thick-walled tubes, the modified method can be used with an accuracy acceptable for practice. The correctness of the method has been justified by experiments on test pieces and by comparisons to other methods known from literature [1].

Also the cylindrical shells of devices can be regarded as tubes, therefore in the following the denomination "thick-walled tube" will be used as generalization.

Axisymmetrical bending of thick-walled tubes

Joints in structural elements and other axisymmetrical line loads produce transversal forces or moments which bend the thick-walled tube. Relationships describing the axisymmetrical bending are the simplest to derive on the basis of the theory of elastically bedded beams used with thin shells. This theory with certain modifications is suitable for a relatively simple determination of flexural stresses in thick-walled tubes.

Let us cut out from the thick-walled tube a beam belonging to the central angle

$$\Delta\varphi = \frac{1}{r_s} \tag{3}$$

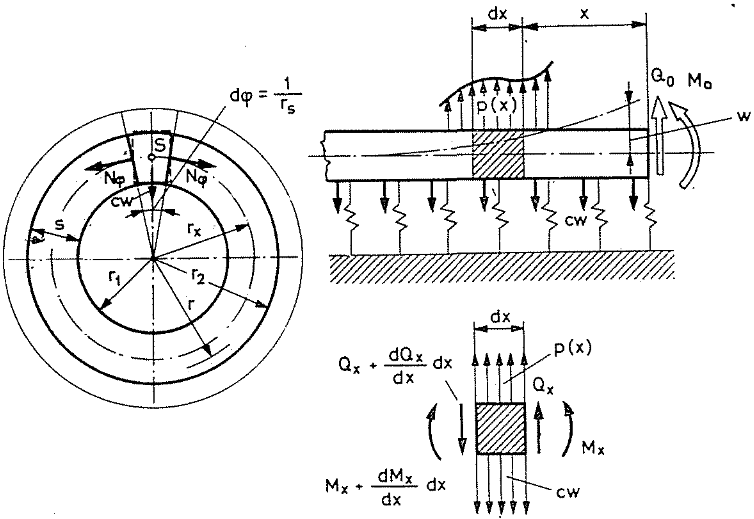


Fig. 2. Thick-walled tube exposed to bending load

(Fig. 2). The effect of elements adjacent to the beam is represented by the "spring force" cw.

For writing the differential equation of bending, first the equilibrium equation of forces and moments acting upon the elementary cube cut out from the beam will be established.

The equilibrium equation of forces according to Fig. 2 is:

$$p(x)dx - cwdx + Q_x - \left(Q_x + \frac{dQ_x}{dx}\,dx\right) = 0 \tag{4}$$

After reduction:

$$\frac{dQ_x}{dx} = p(x) - cw. \tag{5}$$

The equilibrium equation of moments, according to Fig. 2:

$$M_x - \left(M_x + \frac{dM_x}{dx}\,dx\right) - Q_x dx = 0 \tag{6}$$

After reduction:

$$Q_x = \frac{dM_x}{dx} \tag{7}$$

A further relationship is needed which may be the functional relationship between displacement and moment. In case of a beam

$$\frac{d^2 w}{dx^2} = w^{11} = \frac{M_T}{B_T} \tag{8}$$

Since in our case M_T can be understood as edge moment $-M_x$ along the generatrix, B_T has to be substituted by the flexural rigidity B of the cut-out beam, expression (8) can be brought to the form:

$$M_x = Bw^{11} \tag{9}$$

where

$$B = k_B \frac{Es^3}{12(1-\mu^2)} \tag{10}$$

The factor k_B in expression (10) is the ratio of the moments of inertia of the bent thin-walled to thick-walled tube, taken for the gravity center axis, and of the cross-sectional factors:

$$k_B = \frac{K}{K_v} = \frac{J}{J_v} \tag{11}$$

For the given geometry the approximation $\sin \Delta\varphi \approx \Delta\varphi$ can be used

$$\Delta\varphi = \frac{1}{r_s} \approx \frac{1}{r_k} < 10^0$$

and k_B can be calculated from the relationship:

$$k_B = \frac{0.5 + k_0 - 3k_0^2 + k_0^3 + 0.5k_0^4}{1 - k_0 - k_0^3 + k_0^4} \tag{12}$$

The two equilibrium equations above and the deformation relationship are sufficient for determining the radial displacement w, transverse force Q_x and the M_x sought for. On the basis of relationships (7) and (9),

$$\frac{dQ_x}{dx} = B\frac{d^4w}{dx^4} \tag{13}$$

In this way, Eq. (5) can be brought to the following form:

$$B\frac{d^4w}{dx^4} + cw = p(x) \tag{14}$$

In case of axisymmetrical line loads:

$$p(x) = 0 \tag{15}$$

Thus, bending of the thick-walled tube is characterized by the homogeneous, linear differential equation of fourth order:

$$B \frac{d^4 w}{dx^4} + cw = 0. \tag{16}$$

Previous to the solution of the differential equation, let us determine the spring constant c.

A cross-section of the tube will be displaced in radial direction by a value w. The displacement produces peripheral stresses in the tube of the order:

$$\sigma_{\varphi r} = E \varepsilon_{\varphi r} = E \frac{w}{r} \tag{17}$$

The resultant of stresses vayring along the thickness is given by the relationship:

$$N_\varphi = \int_{r_1}^{r_2} E \frac{w}{r} dr = Ew \ln \frac{r_2}{r_1} = Ew \ln \frac{1}{k_0}\ . \tag{18}$$

In modelling the elastically bedded beam, the radial resultant N_φ of the forces per unit length is substituted by the spring force cw. Accordingly,

$$N_\varphi \Delta\varphi = cw. \tag{19}$$

Considering the expression (18) for N_φ, substituting

$$\Delta\varphi = \frac{1}{r_s} \approx \frac{1}{r_k}$$

and reducing, the value of the spring constant will be

$$c = \frac{E}{r_k} \ln \frac{r_2}{r_1} \tag{20}$$

Write relationship (20) in the form:

$$c = k_c E \frac{s}{r_k^2} \tag{21}$$

where k_c represents the relationship between spring constants now derived, and valid for the thin-walled tube [2], expressed as:

$$k_c = \frac{1}{2} \frac{1 + k_0}{1 - k_0} \ln \frac{1}{k_0} \tag{22}$$

Using relationships (20) and (21), the differential equation describing the axisymmetrical bending is brought to the form:

$$\frac{d^4w}{dx^4} + 4\,\beta^4 w = 0, \tag{23}$$

where the shell constant is

$$\beta = k_\beta \sqrt[4]{\frac{3(1-\mu^2)}{r_k^2 \cdot s^2}} \tag{24}$$

The encountered factor k_β is the quotient of the shell constants of the thick-walled to thin-walled tube, to be calculated as:

$$k_\beta = \sqrt[4]{\frac{k_c}{k_B}} \tag{25}$$

In case of a relatively long cylinder ($l \geq 3.1\sqrt{r_k\, s}$) the solution of Eq. (23) is known to be transformable as [3, 4, 5]:

$$w = e^{-\beta x}(C_1 \cos \beta x + C_2 \sin \beta x). \tag{26}$$

The value of the integration constants C_1 and C_2 can be determined with edge loads related to the given load cases, to yield the internal forces and moments.

Stresses arising in the thick-walled tube

The internal forces and moments needed for determining the stresses are seen in Fig. 3. All of them are expressed as functions of the radial displacement w.

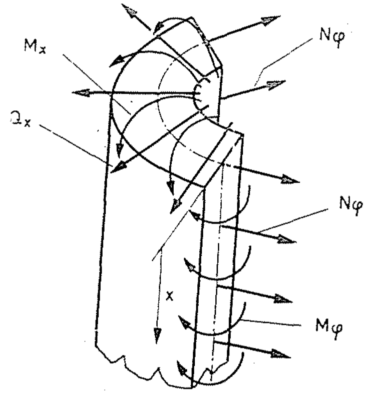


Fig. 3. Line forces and line moments acting on the thick-walled tube

The tangential line load:

$$N_\varphi = E \cdot s \frac{w}{r} \tag{27}$$

The shear line load:

$$Q_x = B \frac{d^3 w}{dx^3} \tag{28}$$

The axial moment per unit length:

$$M_x = B \frac{d^2 w}{dx^2} \tag{29}$$

The tangential moment per unit length:

$$M_\varphi = \mu B \frac{d^2 w}{dx^2} \tag{30}$$

In the knowledge of the internal forces and moments per unit length caused by a given external load, the stresses can be calculated by the following known relationships:

Axial stress:

$$\sigma_x = \pm \frac{12 M_x}{k_B \cdot s^3} z = \pm \frac{12 B}{k_B \cdot s^3} \frac{d^2 w}{dx^2} z, \tag{31}$$

Tangential stress:

$$\sigma_\varphi = \frac{N_\varphi}{s} \pm \frac{12 M_\varphi}{k_B \cdot s^3} z = E \frac{w}{r} \pm \frac{12 B}{k_B \cdot s^3} \mu \frac{d^2 w}{dx^2} z, \tag{32}$$

where z is the radial distance from the center line.

The edge load may be a transverse line load Q_0 and a line moment M_0.

For analyzing stresses in the thick-walled tube it is expedient to introduce dimensionless stress factors.

In case of edge load Q_0:
the axial stress factor

$$\bar{\sigma}_{xQ} = \frac{\sigma_{xQ}}{\beta Q_0}, \tag{33}$$

the tangential stress factor

$$\bar{\sigma}_{\varphi Q} = \frac{\sigma_{\varphi Q}}{\beta Q_0}, \tag{34}$$

In case of edge load M_0:
the axial stress factor

$$\bar{\sigma}_{xM} = \frac{\sigma_{xM}}{\beta^2 M_0}, \tag{35}$$

the tangential stress factor

$$\bar{\sigma}_{\varphi M} = \frac{\sigma_{\varphi M}}{\beta^2 M_0}. \tag{36}$$

Denotations applied for the attenuation functions are [6]:

$$\begin{aligned} H_1(\beta x) &= e^{-\beta x} \cos \beta x \\ H_2(\beta x) &= e^{-\beta x} \sin \beta x \\ H_3(\beta x) &= e^{-\beta x} (\cos \beta x + \sin \beta x) \\ H_4(\beta x) &= e^{-\beta x} (\cos \beta x - \sin \beta x) \end{aligned} \tag{37a—d}$$

Further, let the following factors be introduced:
The ratio of an arbitrary radius to the external radius:

$$k = \frac{r}{r_2} \tag{38}$$

The ratio of the radius belonging to the center of gravity to the external radius:

$$k_s = \frac{r_s}{r_2} = \frac{2}{3} \frac{1 - k_0^3}{1 - k_0^2} \tag{39}$$

Line load factor:

$$k_N = - \frac{1 + k_0}{\ln k_0} \tag{40}$$

Line moment factor:

$$k_M = \frac{6(1 + k_0)}{k_\beta^2 \cdot k_B \sqrt{3(1 - \mu^2)} \cdot (1 - k_0)^2} \tag{41}$$

Factors k_N and k_M have been determined for the case of edge loads Q_0 and M_0. Their introduction simplifies the relationship for stress factors and stresses.

The stress factors calculated for the edge load Q_0 and M_0 (Fig. 2) are the following:

At an arbitrary point along the radius of the thick-walled tube exposed to edge load Q_0 the values of stress factors are
in axial direction

$$\bar{\sigma}_{xQ} = k_M(k_s - k) H_2 \tag{42}$$

in tangential direction

$$\bar{\sigma}_{\varphi Q} = \frac{k_N}{k} H_1 + \mu \bar{\sigma}_{xQ} . \tag{43}$$

At an arbitrary point along the radius of the thick-walled tube exposed to edge load M_0 the values of the stress factors are:

in axial direction

$$\bar{\sigma}_{xM} = k_M(k_s - k)H_3, \tag{44}$$

in tangential direction

$$\bar{\sigma}_{\varphi M} = \frac{k_N}{k} H_4 + \mu \bar{\sigma}_{xM} \,. \tag{45}$$

To facilitate actual calculations, the diagrams in Figs 4 through 10 have been plotted where all intervening factors are included versus k_0.

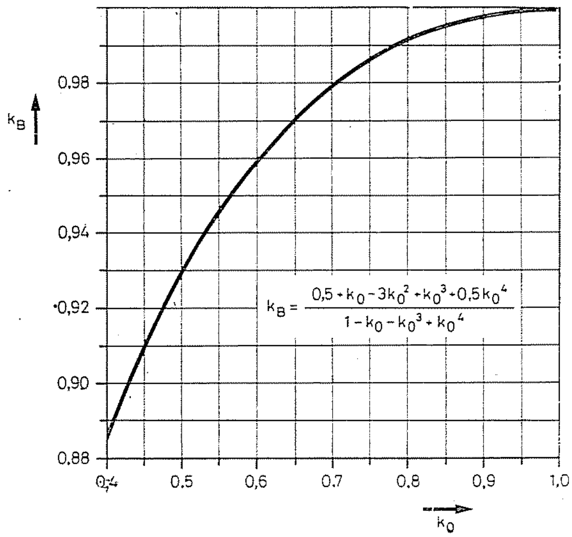


Fig. 4

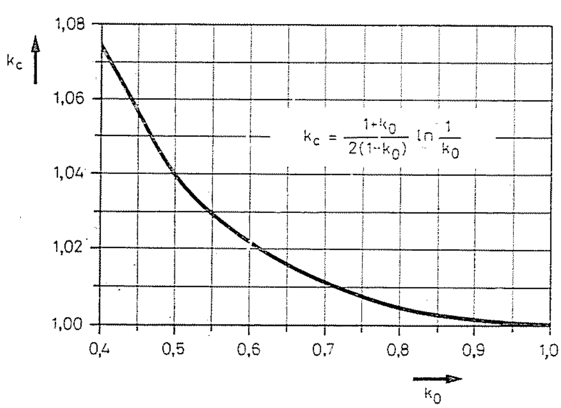


Fig. 5

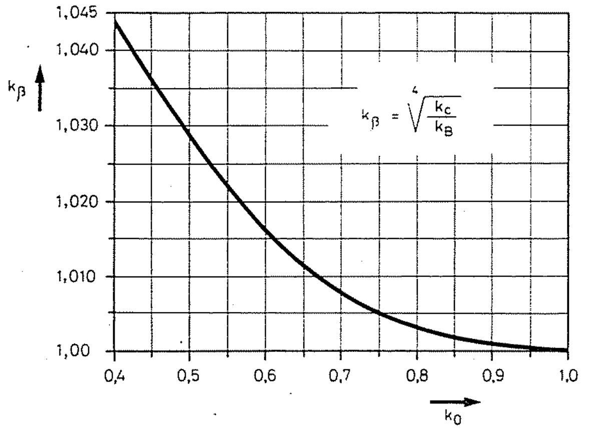

1,045
1,040
k_β
1,030
1,020
1,010
1,00
0,4
0,5
0,6
0,7
0,8
0,9
1,0
k_0
$k_\beta = \sqrt[4]{\frac{k_c}{k_B}}$

Fig. 6

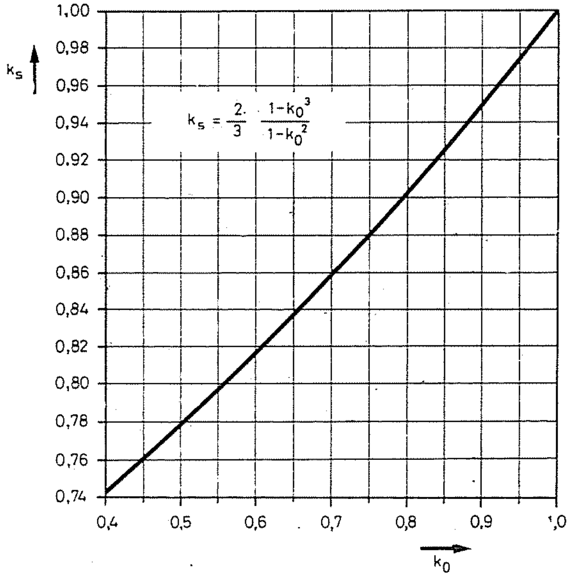

1,00
0,98
0,96
0,94
0,92
0,90
0,88
0,86
0,84
0,82
0,80
0,78
0,76
0,74
k_s
$k_s = \frac{2}{3} \cdot \frac{1-k_0^3}{1-k_0^2}$
0,4
0,5
0,6
0,7
0,8
0,9
1,0
k_0

Fig. 7

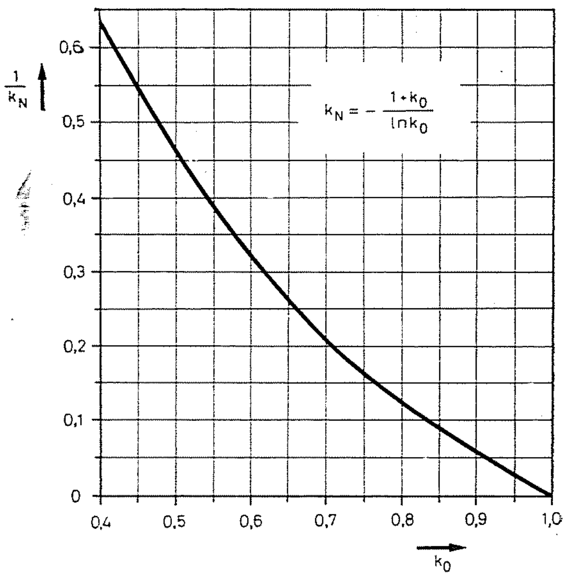
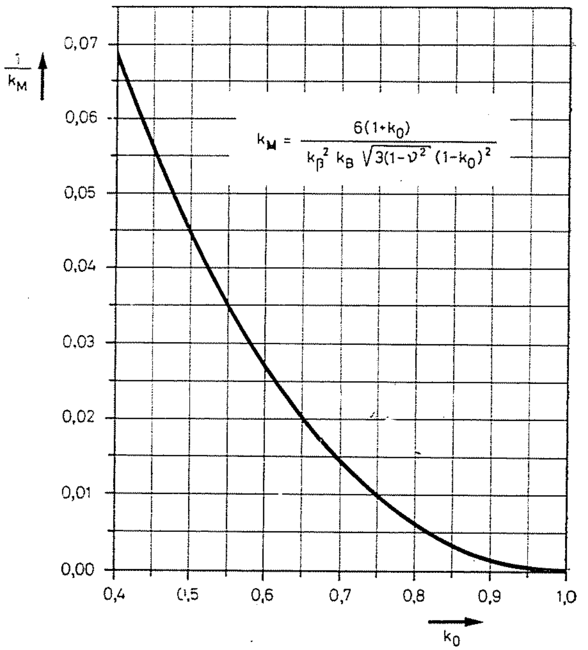

$\frac{1}{k_N}$
0,6
0,5
0,4
0,3
0,2
0,1
0
$k_N = -\frac{1+k_0}{\ln k_0}$
0,4
0,5
0,6
0,7
0,8
0,9
1,0
k_0

Fig. 8


$\frac{1}{k_M}$
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0,06
0,05
0,04
0,03
0,02
0,01
0,00
$k_M = \frac{6(1+k_0)}{k_\beta^2 \, k_B \sqrt{3(1-\nu^2)} \, (1-k_0)^2}$
0,4
0,5
0,6
0,7
0,8
0,9
1,0
k_0

Fig. 9

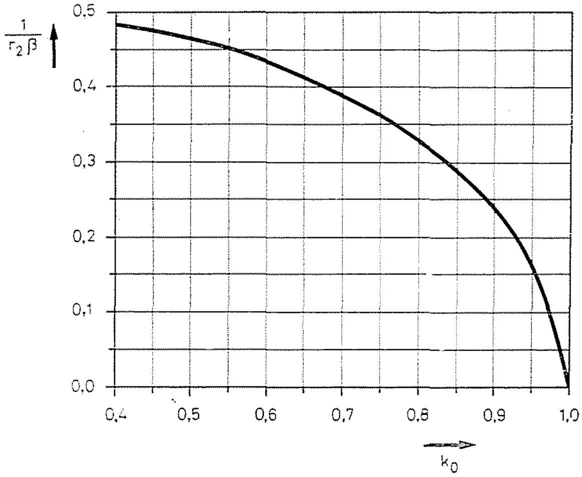

Fig. 10

Relationships (42) through (45), yield additional stresses produced by line load and line moment, Q_0 and M_0 resp., acting on the edge.

The stress formulae, comparison to other methods, as well as the experimental results will be described in the second part of this paper.

Summary

A strength calculation method has been developed for thick-walled tubes and cylindrical vessels exposed to axisymmetrical line loads. The additional stresses in operating high-pressure devices arising at the junction of closing elements in case of axisymmetrical seating may be quite considerable, to be absolutely included in strength calculations.

The method for determining additional stresses is based on the principle of elastically bedded beams valid for thin shells, and it is an extension of this method to thick-walled tubes and cylindrical vessels. The modified theory has led to relationships for the determination of stresses in tubes with a radius ratio of

$$k_0 = \frac{r_1}{r_2} \geq 0.4$$

most generally used in practice.

NOTATIONS

B cm^3 $\frac{\text{kp}}{\text{cm}^2}$	flexural rigidity of unit shell element
C_1; C_2 cm	integration constants
c $\frac{\text{kp}}{\text{cm}}\frac{1}{\text{cm}^2}$	spring constant of elastic bedding
E $\frac{\text{kp}}{\text{cm}^2}$	Young's modulus

J	cm^4	moment of inertia
K	cm^3	cross-section factor
k	—	radius ratio, factors
M	cmkp, $\frac{\mathrm{cmkp}}{\mathrm{cm}}$	moment, edge moment
N	kp, $\frac{\mathrm{kp}}{\mathrm{cm}}$	force, edge force
p	$\frac{\mathrm{kp}}{\mathrm{cm}^2}$	pressure
Q	kp, $\frac{\mathrm{kp}}{\mathrm{cm}}$	transverse force, edge force
r	cm	radius
s	cm	wall thickness
w	cm	radial displacement
z	cm	distance of extreme fibre from the gravity axis
x	cm	distance from the rim
β	$\frac{1}{\mathrm{cm}}$	shell factor
ε	—	specific strain
μ	—	Poisson's ratio
σ	$\frac{\mathrm{kp}}{\mathrm{cm}^2}$	stress

Subscripts

b	referring to the internal side
k	referring to the external side, but as to radius, central radius
M	in case of line moment
r	radius-dependent
Q	in case of line load
s	in centre of gravity
x	axial
v	referring to thin shells
φ	tangential

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