A TECHNIQUE FOR OPTICAL DIFFERENTIATION

By

Z. FÜZESSY

Department of Physics, Institute of Physics, Technical University, Budapest

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1. Introduction

Although holography has found applications in a wide variety of fields, among which we might mention three-dimensional display, data processing, pattern recognition and image enhancement, it is generally conceded that one of its most dramatic and important applications has come from its combination with interferometry.

The first problem to be attacked by means of holographic interferometry was vibration analysis [1]. The fringes on holographic images indicate loci of equal amplitudes of vibration if measurement is carried out by time average holographic interferometry.

The other significant method of holographic interferometry is the double-exposure technique. Its application results in recording two nearly identical wave-fronts on the same photosensitive film. One of the wave-fronts corresponds to the undeformed state of an object, and the other one — to the deformed state [2, 3]. The resulting interference fringes locate points along which equal changes of optical paths between the source and the observer have occurred due to the deformation.

Practical applications of holographic interferometry in most cases leads to the problem of determining the stress/strain state of different construction elements. The present paper proposes a technique for directly measuring the quantities characterizing the strain distribution on the basis of interference fringes.

2. Analysis of the method

The fringes observed in holographical interferometry are generally contours of constant displacement of object points, in the direction of a sensitivity vector. In mechanics, however, it is very often the strain of an object surface that is of interest. In order to obtain strain distributions, spatial derivatives of displacement must be determined. This can be done either by estimating the fringe frequency, rather than the fringe number, or by differentiating numerically, optically or graphically the displacement values.

All the parameters of the mechanical deformation, such as strain, shear, bending and torsion, can be expressed in terms of the derivatives mentioned above. On the other hand, elements of the deformation matrix are proportional to the derivatives. So the main problem of the experimental mechanics can be formulated as the problem of determining the deformation matrix.

In the vicinity of a given point x, y, z the displacement of the points caused by deformation alone can be given in terms of these matrix elements as follows:

$$(L_i) = \left(\frac{\partial L_i}{\partial k}\right)(i) \tag{1}$$

where $i, k = x, y, z$; L_i is the component of displacement vector.

Many works have been published on deformation measurements, recommending various methods and techniques for determining matrix elements. One of them [4] deals with the problem how to use the form of the fringes to calculate directly the deformation of the object without the intermediate step of calculating the object displacement. The omission of this intermediate step would improve the over-all accuracy and practicability of these calculations.

The technique of optical differentiation can prove to be useful in this respect. In order to obtain the derivatives on the basis of an interference pattern related to object displacement and observed directly, it is useful to define the fringe locus function as:

$$\Omega = [S_x S_y S_z] \begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix}, \tag{2}$$

where S_x, S_y, S_z are the rectangular components of the sensitivity vector.

A differentiation of Eq. (2) with respect to spatial co-ordinates gives

$$\frac{\partial \Omega}{\partial i} = \sum_k S_k \frac{\partial L_k}{\partial i}.$$

Taking the derivatives we have assumed that the sensitivity vector does not change over the surface. It seems that the derivatives of Ω contain the elements of deformation matrix indeed.

There are many purely optical methods for deriving lines of spatial derivatives from the displacement pattern. They are based upon the fact that if two identical copies of a set of curves Ω (x, y, z) are slightly shifted in direction s, the resulting Moiré-fringes are loci of points showing the same value of partial derivative [5].

For the complete determination of matrix elements, however, two interferograms placed rectangularly must be recorded (Fig. 1). By reason of fringe locus functions from interferogram H_1, the spatial derivatives of the displacement vector components with respect to x and y can be determined, and so can be the derivatives with respect to z from interferogram H_2.

During recording, the object is illuminated from three different directions and observed from two ones in order to get the six sensitivity vectors S_j, $j = 1 \dots 6$. The corresponding fringe locus functions are Ω_j $j = 1 \dots 6$.

Let us denote the sensitivity matrices by $[S_I]$ and $[S_{II}]$ for the interferograms I and II, respectively:

$$[S_I] = (S_{ij}) \begin{array}{l} i = 1, 2, 3 \\ j = x, y, z \end{array} \quad \text{and} \quad [S_{II}] = (S_{kj}) \begin{array}{l} k = 4, 5, 6 \\ j = x, y, z \end{array}.$$

Accordingly the derivatives of fringe locus functions can be expressed in matrix form as

$$\begin{bmatrix} \frac{\partial \Omega_1}{\partial x} & \frac{\partial \Omega_1}{\partial y} & 0 \\ \frac{\partial \Omega_2}{\partial x} & \frac{\partial \Omega_2}{\partial y} & 0 \\ \frac{\partial \Omega_3}{\partial x} & \frac{\partial \Omega_3}{\partial y} & 0 \end{bmatrix} = \begin{bmatrix} S_{1x} & S_{1y} & S_{1z} \\ S_{2x} & S_{2y} & S_{2z} \\ S_{3x} & S_{3y} & S_{3z} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial L_x}{\partial x} & \frac{\partial L_x}{\partial y} & 0 \\ \frac{\partial L_y}{\partial x} & \frac{\partial L_y}{\partial y} & 0 \\ \frac{\partial L_z}{\partial x} & \frac{\partial L_z}{\partial y} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & \frac{\partial \Omega_4}{\partial z} \\ 0 & 0 & \frac{\partial \Omega_5}{\partial z} \\ 0 & 0 & \frac{\partial \Omega_6}{\partial z} \end{bmatrix} = \begin{bmatrix} S_{4x} & S_{4y} & S_{4z} \\ S_{5x} & S_{5y} & S_{5z} \\ S_{6x} & S_{6y} & S_{6z} \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & \frac{\partial L_x}{\partial z} \\ 0 & 0 & \frac{\partial L_y}{\partial z} \\ 0 & 0 & \frac{\partial L_z}{\partial z} \end{bmatrix}.$$

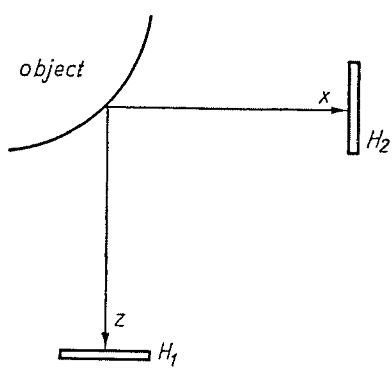

Fig. 1. Arrangement of photographic plates relative to the object surface

The complete deformation matrix can be determined in terms of sensitivity matrices and derivatives of fringe locus functions; since the sensitivity matrices are nonsingular:

$$\begin{bmatrix} \frac{\partial L_x}{\partial x} & \frac{\partial L_x}{\partial y} & \frac{\partial L_x}{\partial z} \\ \frac{\partial L_y}{\partial x} & \frac{\partial L_y}{\partial y} & \frac{\partial L_y}{\partial z} \\ \frac{\partial L_z}{\partial x} & \frac{\partial L_z}{\partial y} & \frac{\partial L_z}{\partial z} \end{bmatrix} = [S_I]^{-1} \begin{bmatrix} \frac{\partial \Omega_1}{\partial x} & \frac{\partial \Omega_1}{\partial y} & 0 \\ \frac{\partial \Omega_2}{\partial x} & \frac{\partial \Omega_2}{\partial y} & 0 \\ \frac{\partial \Omega_3}{\partial x} & \frac{\partial \Omega_3}{\partial y} & 0 \end{bmatrix} + [S_{II}]^{-1} \begin{bmatrix} 0 & 0 & \frac{\partial \Omega_4}{\partial z} \\ 0 & 0 & \frac{\partial \Omega_5}{\partial z} \\ 0 & 0 & \frac{\partial \Omega_6}{\partial z} \end{bmatrix}.$$

3. Experimental demonstration

To get the fringes belonging to equal values of fringe locus function derivatives (partial derivatives of the displacement vector) we used a conventional Michelson-type interferometer (Fig. 2) to produce two overlapping sets of fringes. Shifting in a well-defined direction was accomplished by tilting the mirror in one of the arms of the interferometer. This technique has an advantage over the other ones, i.e. the formation of Moiré-fringes can be observed in real time and so the present method is free from supplementary determination of fringe order number. Overlapping of the two sets of fringes results in terms of different spatial frequencies at the output of the interferometer. The useful information about spatial derivatives can be obtained from the Moiré-fringes of lower spatial frequencies. The high spatial frequency fringes can be filtered out in the usual way.

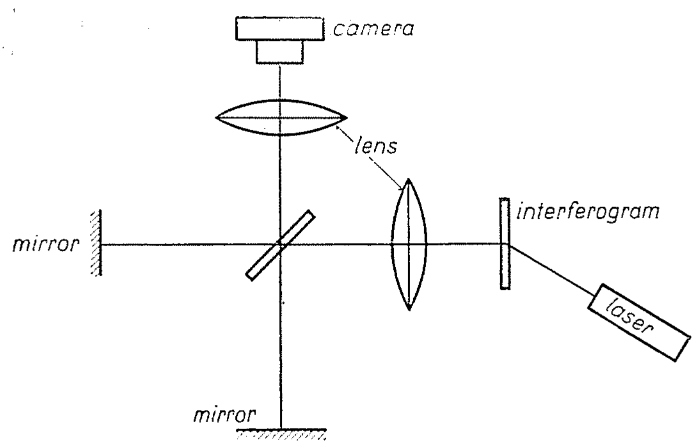

Fig. 2. Scheme of shifting by Michelson-interferometer

To demonstrate the usefulness of the technique reported we have introduced to the input of the interferometer the wave-fronts from the double-exposed hologram of a cantilever clamped at one end and loaded at the other one. The lower spatial frequency fringes are characterized by the term $\cos\left[\frac{\Omega - \Omega(\Delta s)}{2}\right]$ in Fig. 3, where Ω is the fringe locus function and Δs is the value of shifting, moreover

$$\frac{\Omega - \Omega(\Delta s)}{\Delta s} \sim \frac{\partial \Omega}{\partial s}.$$

These results suggest that on the basis of the technique described above, the Michelson-interferometer may be a really useful device for directly measuring partial derivatives of displacement vector components.

4. Conclusions

It was shown that lines of constant values of spatial derivatives could be derived from displacement pattern. In the general case, complete determination of matrix elements needs recording of two interferograms placed rectangularly. This procedure will yield all the elements of the deformation matrix. Optical differentiation can be realized by Michelson-interferometer.

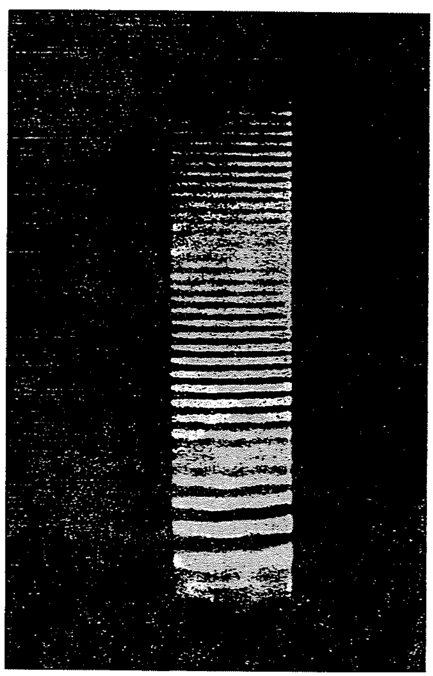

Fig. 3. Moiré-fringes on the image of a cantilever

5. Acknowledgement

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Summary

There are many purely optical methods for deriving lines of spatial derivatives from displacement pattern. An expression has been derived for a complete deformation matrix in terms of sensitivity matrices and derivatives of fringe locus function. A technique for optical differentiation is presented which utilizes the Michelson-interferometer. The differentiation is carried out in the particular one-dimensional case.

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Zoltán FÜZESSY, H-1521 Budapest