UP-TO-DATE INVESTIGATION METHODS OF PROCESS ANALYSIS*

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Received May 29, 1984

The theory of engineered systems has developed slowly in spite of the fact that, as compared with other systems, such systems can be discussed in the most exact way. For a long time, the design of engineered projects has been stagnant, the resulting system obtained being a more or less successful synthesis of system components — machines and devices — dimensioned by the use of static state deterministic characteristics. The dynamic behaviour came into the limelight as the classic control theory developed, first of all for systems consisting of components of linear behaviour. Another limitation was that the methods applicable in practice were confined to the description of systems consisting of lumped parameter components.

The investigations were made in the time domain most becoming the engineering approach. As a result, the dynamic system models complying with the above limitations were described by linear differential equations of invariable coefficients. As the next step, a further constraint was introduced in the limitation that only harmonic signals were used as the input signals of the system and therefore investigations were made only in the frequency range instead of the time domain. In this way, the description of the phenomena was less demonstrative but the way to the ultimate result became shorter. The use of the signals's Laplace transforms instead of the signals themselves released somewhat the constraint applied to the test signals, because in the first step periodic signals (Fourier transform) could be used instead of harmonic signals and then arbitrary deterministic signals by increasing the period beyond any limit. This approach describes the system components by their transfer function and produces the transfer function of the resultant system by relatively simple methods.

* Based on the author's report presented at the scientific session held on the occasion of the 200-year anniversary of the Technical University Budapest.

The advent of up-to-date computers permitted a return to the time domain most becoming the engineering approach. The system behaviour is described in the space of states the fundamental independent variable of which being time. A consistent application of this approach requires that the characteristics describing the system be consistently systematized. The descriptive characteristics are usually grouped as characteristics determining the behaviour and those determining the state (Fig. 1).

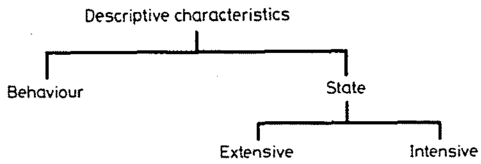

Fig. 1

The characteristics expressing the behaviour are usually unambiguous, they include data on geometry and shape as well as a wide range of material characteristics (specific heat, viscosity, electric conductivity etc.).

Thermodynamically, the state characteristics (state determinants) can be categorized as extensive and intensive characteristics. The extensive characteristics usually express, or are proportional to, some dimension, measure, or quantity, and act as energy carriers. Such characteristics are the mass, volume, and of course also the energy itself. It is a fundamental fact that the value of extensive characteristics for the entire system is identical with the sum of values relating to a subsystem each. Applying to the extensive characteristics are the conservation laws.

Intensive characteristics express the intensity of some effect, these effects being proportional to the differences in intensive characteristics. The intensity characteristic itself can be defined for a definite point of the space. Should the distribution of the intensity characteristic be inhomogeneous (that is, there should be differences in the intensity characteristics), extensive currents will start flowing as a result of these differences, in a direction in which these differences are eliminated. Inhomogeneity is a source of a driving force; the system tries to move towards equilibrium.

Designating the general extensive quantity, its current will be

$$\Phi = \frac{d\Psi}{d\tau}.$$

According to the phenomenological Ohm's law the relationship between current and driving force can be expressed as

$$\Phi = \frac{1}{R} X$$

where X = difference in intensity characteristic as the source of driving force
R = overall resistance in the way of the current.

As mentioned above, a feature of the system is that extensive currents are brought about by the differences in intensity characteristics, which in effort to eliminate inhomogeneities within the system result in a combination of an extensive and an intensive characteristic into a coherent pair. (Principal effects in the Onsager's non-equilibrium thermodynamics.) E.g. the temperature difference brings about heat flux, while the electric voltage difference charging current etc.

The product of the coherent extensive-intensive pairs results in energy:

$$E = \Psi X,$$

or, in the case of the more frequently used, energy current (power)

$$\Phi_E = \Phi \cdot X.$$

Definite relationships are prevailing between the coherent extensive-intensive pairs. These relationships are illustrated in Fig. 2, taking into consideration also the general notion of the inductance (L) and the capacitance

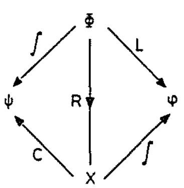

Fig. 2

(C) in addition to resistance (R). The relationships that can be written in accordance with the Figure are, as follows:

$$X = R\Phi$$

$$\varphi = L\Phi$$

$$\Psi = CX$$

$$\Psi(C)=\Psi(0)+\int_0^\tau \Phi(\tau)d\tau$$

$$\varphi(\tau)=\varphi(0)+\int_0^\tau X(\tau)d\tau$$

Considering that the product of the coherent extensive-intensive pairs is obtained in energy dimension, the energy current (or power) can be defined, as follows:

$$\Phi_E=\Phi X.$$

Accordingly, the capacitive and inductive energy storage, respectively, as well as the dissipation energy are obtained, as follows:

$$E_C(\tau)=\int_0^\tau X\Phi d\tau=\int_0^\tau X d\Psi,$$

$$E_L(\tau)=\int_0^\tau \Phi X d\tau=\int_0^\tau \Phi d\varphi,$$

and

$$E_R(\tau)=\int_0^\tau \Phi X d\tau.$$

The dynamic behaviour of physically feasible systems is described by the general vector differential equation (Fig. 3).

$$F(U, V, \dot{U}, \dot{V}, \ldots)=0$$

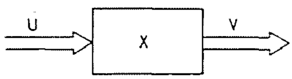

Fig. 3

$$U=\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} \quad \text{input signal vector}$$

$$V=\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \quad \text{output signal vector}$$

The inputs of a complex technological process (system) are mass and energy currents. The total input vector (U) of the system includes usually more independent mass and energy current vectors which acts at different input points of the system. Within this, each input current is set up of more components which may be the currents of extensive characteristics, or intensity characteristics alike. Ordinates $u_1, u_2, \ldots u_m$ of the total input vector shall thus be defined accordingly.

Output vector V, however, cannot be defined so unambiguously. In analyzing a system it is not only the actual output characteristics observed at the physical boundary of the system which are of interest; what we want to know is rather the instantaneous *state* of the system.

A system that can be considered to be a lumped parameter system can be decomposed into simple elementary units on the basis of analytical engineering considerations, the decomposition resulting in a network model consisting of simple resistances and simple storage elements. The dynamic state (motion) of the system is determined by the instantaneous value of the input signals and by the instantaneous filling state in the storage elements of the network. The *response* of the system to definite inputs is described by *state variables* characteristic of the latters, hence these state variables are components $v_1, v_2, \ldots v_n$ of the output vector V.

The state variables are intensity characteristics having a unambiguous bearing on the state of filling of the storage elements in the system, or occasionally other characteristics proportional to them (e.g. height of liquid level in a tank expressed in meters, proportional to the bottom pressure).

Since the instantaneous state of filling in the storage elements preserves the history of the system, the state variables carry information on the past. The state of systems consisting of more storage elements can be described by independent state variables of a number identical with the number of storage elements.

In general, the state variable is a motion characteristic signal of the system, the value of which at given instant shall be known for the determination of the next time characteristics, if the system characteristics and inputs are known. To define the state variable as an intensity characteristic, it seems most reasonable to take the following physical formula as a basis: if an extensive current entering an arbitrary confined area within the space differs from the current leaving this area, the intensity characteristic describing the state of filling of the system will change, the extent of change being inversely proportional to the capacity of the system:

$$\frac{dv}{dt}=\frac{1}{C}(\Phi_b-\Phi_k).$$

The above relationship expresses the non-stationary conservation law applying to extensive quantities.

In order to distinguish the state variables of the system from the arbitrarily selected signals considered to be output signals, the vector of the state variables will be designated X in the following. Should all the state variables be considered the output of the system and so followed, it will be found that vector X is identical with output vector V, however, vector V contains, in most cases, only some preferred components of vector X, or other variables having a definite bearing on that components (Fig. 3).

In the most general nonlinear case, the dynamic state function with one variable for an elementary unit having one storage element can be written in the state variable form, as follows:

$$X(t)=f[X(t), u(t), t].$$

In the functional connection, the role of time (t, as a variable) has been specially emphasized. This means that the relations described by the equation depend not only on the time functions of the state variables and inputs but also on the time-dependent parameters of the system.

Let us consider the state variables to be the scalar ordinates of the state vector, and the input signals to be those of the input vector. Treating these vectors as a column matrix, we obtain the general nonlinear state equation of the system:

$$X(t)=F[X(t), U(t), t].$$

In the linear variable coefficient case (allowing system characteristics to be time-dependent), the vector differential equation can be written as

$$X(t)=A(t)X(t)+B(t)U(t)$$

where A = system matrix of size $n\times n$, always quadratic
B = input matrix of size $n\times m$.

The corresponding Functional Block Diagram of the above equations can be seen in Fig. 4.

According to what has been said above, the system variables considered to be output signals are not necessarily identical with the state variables.

Therefore, the following formula applies in general to the output vector:

$$V(t) = G[X(t), U(t),]$$

In the linear case:

$$V(t) = C(t)X(t) + DU(t).$$

D transmits the direct effect of input signals on output signals and it is often left out of consideration in actual investigations provided the state variables can be selected so favourably that the output signals depend on the

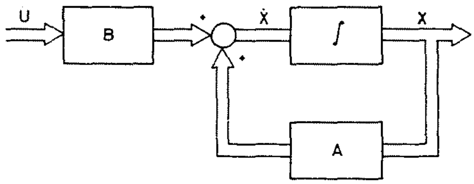

Fig. 4

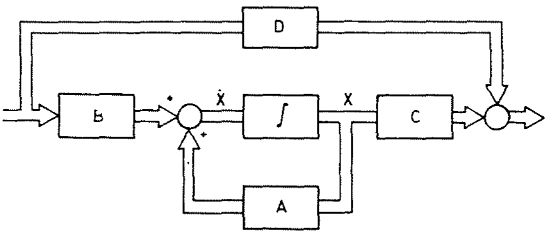

Fig. 5

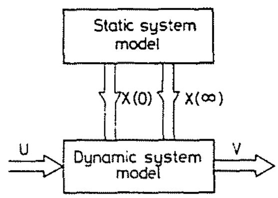

Fig. 6

inputs only through these variables. The Functional Block Diagram corresponding to the latter equation is given in Fig. 5.

No systems analysis in the space of states can be made without a decomposition of the system by an engineering approach as this is a priori demanded when a dynamic mathematical model is set up. However, after the decomposition has been accomplished, the state equations of the system components can be simply written on the basis of analysis of the phenomenon, making use of the knowledge in natural sciences. Then, the system model resulting from synthesis can be produced methodically. No systems analysis can be made without the production of a static mathematical model. State vector X and/or response V, associated with given input signal U, depend(s) on the history of the system. In solving the system of differential equations, this follows from the initial conditions. Assuming that the system has been in steady state when input U is connected to it, the initial conditions can be determined from the static model of the system. In case of a stable system, a new steady belongs to a new input U after the transients have disappeared thus the values of state variable X, associated with t, are again given by the static model (Fig. 6). Obviously, the classic method of linear differential equation with one variable, as well as the method of frequency and transfer functions are simpler cases of investigations in the space of states. It is a different matter that, apart from quite simple cases, mathematical system models produced in the space of states can not be treated by classic methods without the aid of a computer.

Literature

1. SZABÓ, I.: Systems theory of machines and processes. (Gépek és folyamatok rendszertana). Tankönyvkiadó, 1978
2. IMRE, L.—SZABÓ, I.: Transport processes. (Transzportfolyamatok). Tankönyvkiadó, 1970
3. SZABÓ, I.: Systems and control engineering. (Rendszer és irányítástechnika). Tankönyvkiadó (under publication)

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