STUDY OF AN ASYMMETRICALLY ARRANGED INDUCTION HEATING EQUIPMENT

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Summary

An asymmetrically arranged induction heating equipment has been studied. The paper discusses the effects of the asymmetry: it calculates the magnetic field distribution, the leakage reactance, the resistances, the efficiency, the surface heat flux and the transient temperature distribution in an appropriately chosen model. The consequences of the asymmetry are as follows: The maximal value of the magnetic field can be 5 times greater then the mean value. The leakage inductance decreases by 40%. The resistance of the conductors increases by 100%. The efficiency decreases by 2%. The current consumption increases by 20%. The first harmonic of the temperature rise can reach 200 °C.

Introduction

A research work on the waste recovering of braided steel wire rings built in the rubber tyres of trucks has been carried on at the Department of Electrotechnics, at the Technical University of Budapest. The results promise a considerable save of valuable material.

For this purpose, it is obvious to adopt a medium-frequency induction heating process, since it is necessary to have only a thin surface layer of the steel wire ring heated in order to preserve the favourable mechanical properties of the wire and to recover the rings with minimal energy consumption.

One of the results of the measurements carried out at frequencies of 2 and 8 kHz by the Department of Electrotechnics [6], [7] was that under seemingly similar circumstances the form of the temperature rise in the steel insert had remarkable scattering. This phenomenon was attributed to the following. On the one hand there is a considerable screening effect caused by the steel cord layer surrounding the wire ring, on the other hand the wire ring is not placed symmetrically between the copper inductor plates.

This paper discusses the effects of the above mentioned asymmetry: it calculates the magnetic field distribution, the leakage reactance, the resistances, the efficiency, the surface heat flux and the transient temperature distribution in an appropriately chosen model.

The model used for the discussion

The axonometric view of the inductor insert system drawn out of proportion for the sake of illustration is shown in Fig. 1.

A medium-frequency current is carried by the copper inductor plates. The alternating magnetic field produced by this current induces eddy-currents in the steel insert. Therefore, the inductor insert system operates like a transformer short-circuited in the secondary circuit. The primary coil is the copper plate, the secondary one is the steel insert.

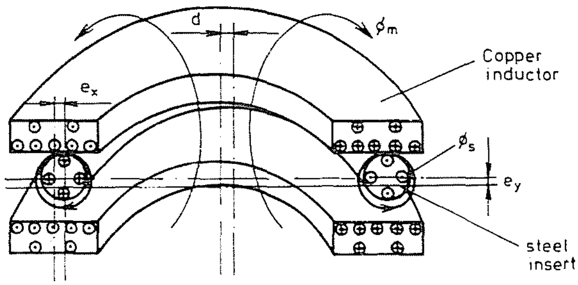

Fig. 1

Because of the skin effect, the current is carried by only a thin surface layer of the conductors thus the current density can be modelled by surface currents distributed along the surface of the conductors. In knowledge of the surface current distribution, the magnetic field, the leakage reactance, the resistances, the surface heat flux and the transient temperature distribution can be calculated.

The surface current distribution can not be analytically expressed. The exact distribution can be determined by numerical methods, for example by the finite elements, the integral equation or the variational method. However, for the design and construction of the induction heating equipment first a qualitative knowledge of the current distribution is imperative. This is why an infinite long arrangement shown in Fig. 2 was selected for calculating approximately the surface current distribution. It makes possible to determine the current distribution analytically, thus the effect of the asymmetry can be expressed by a mathematical relation.

The current carried by the inner surface of the copper inductor is equal to the current I_2 of the steel insert. The main field flux Φ_m is produced by the current flowing on the outer surface of the inductor. The leakage flux Φ_l is excited by the insert current. The primary current I_1 is the sum of the magnetizing current I_m and the secondary current I_2:

$$I_1 = I_m + I_2 \tag{1}$$

In accordance with this physical picture, the transformer equivalent circuit is shown in Fig. 3. R_1, R_m, R_2 are the resistances of the inner and outer surface of the inductor and the steel insert, respectively. Comparing Fig. 3

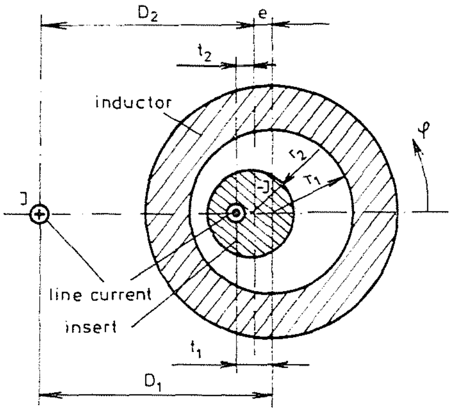

Fig. 2

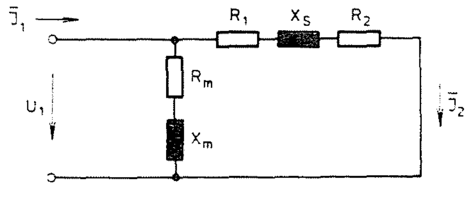

Fig. 3

with the usual equivalent circuit of transformers, there is a remarkable difference, because the heat produced in the primary coil is taken into account by the aid of two resistors. This is the consequence of the fact that two currents independent of each other are carried by the copper inductor.

According to Fig. 1, the center of the steel insert is placed at a "vertical" distance e_y from the symmetrical plane of the copper inductor. The "horizontal" distance e_x is owing to the eccentricity d between the axes of the steel wire ring and the copper inductor. The value of the distance e_x changes from $-d$ to d along the perimeter of the inductor—insert system.

In a given perimetrical angle the eccentricity e shown in Fig. 2 can be calculated as

$$e=\sqrt{e_x^2+e_y^2} \tag{2}$$

The change of the eccentricity e will be neglected, and it will be replaced by a mean value.

Because of the restricted length of the paper the details of the deduction are not elaborated. They can be found in [5].

Magnetic field between the conductors

In order to calculate the magnetic field between the conductors, we adopt the assumption that the skin depth δ is zero both in the copper inductor and in the steel insert. It is a good approximation since the skin depth at medium frequencies is much less than the width of the inductor.

This assumption means that there is no magnetic field in the conductors. Since the normal component of the flux density B_n is continuous, it equals zero on the surface of the conductors, thus the surface of the conductors is a magnetic line of force.

According to Fig. 2 the surface currents of the conductors are replaced by line currents so that the excited magnetic field should be the same as the one produced by the surface currents. Since the surface of the conductors is a magnetic line of force, the location of the line currents corresponding to prescibed circular magnetic lines of force can be found by the aid of the theory of images [8], [9], [10].

Using the theory the following equations are valid Fig. 2:

$$t_1 D_1 = r_1^2 \tag{3a}$$

$$t_2 D_2 = r_2^2 \tag{3b}$$

$$t_1 = t_2 + e \tag{3c}$$

$$D_1 = D_2 + e \tag{3d}$$

Solving the set of equations (3):

$$t_1=(1+K-\sqrt{K^2-\left(\frac{r_2}{e}\right)^2})e \tag{4a}$$

$$t_2=(K-\sqrt{K^2-\left(\frac{r_2}{e}\right)^2})e \tag{4b}$$

where

$$K=\frac{r_1^2-r_2^2-e^2}{2e^2} \tag{4c}$$

It can be seen from (4), that if the eccentricity e vanishes, t_1 and t_2 also tend towards zero. This is in accordance with the physical concept that a surface current distributed evenly along the surface of a cylinder can be replaced by a line current placed in the axis of the cylinder.

Having obtained the coordinates of the line currents, the magnetic field strength on the surface of the conductors can be formulated as follows*:

$$H_{f0}(\varphi)=H_{SO}\frac{1-\gamma^2}{1+2\gamma\cos\varphi+\gamma^2}=H_{SO}\left[1+\sum_{k=1}^{\infty}(-\gamma)^k\cos k\varphi\right] \tag{5a}$$

where H_{SO} is the magnetic field produced when the eccentricity e equals zero:

$$H_{SO}=\frac{I_2}{2\pi r_0} \tag{5b}$$

and

$$\gamma=\frac{t_0}{r_0} \tag{5c}$$

On the surface of the steel insert $t_0=t_2$ and $r_0=r_2$, on that of the copper inductor $t_0=t_1$ and $r_0=r_1$. I_2 is the current of the steel insert.

The magnetic field distribution on the surface of the conductors based on eq. (5) is shown in Fig. 4.

The leakage flux Φ_l can be determined by evaluating a surface integral of the flux density on a surface streching from one conductor to the other. Dividing the leakage flux Φ_l by the exciting current I_2 the leakage inductance can be obtained:

$$L_l=\frac{\mu_0 l}{2\pi}\ln\frac{r_1-e-t_2}{r_2-\frac{t_2}{r_2}(r_1-e)} \tag{7}$$

where l is the length of the inductor, μ_0 is the permeability of vacuum.

* In eq. (5a) the Fourier series of the magnetic field strength was calculated by the aid of the following formula [4]:

$$\int_0^{\pi}\frac{\cos\alpha n x\,\mathrm{d}x}{1-2\cos\alpha x+\alpha^2}=\pi\frac{\alpha^n}{1-\alpha^2};\ n\leq 0;\ |\alpha|<1 \tag{6}$$

If the eccentricity e vanishes, (7) takes the following form:

$$L_l = \frac{\mu_0 l}{2\pi} \ln \frac{r_1}{r_2} \tag{8}$$

(8) is the wellknown expression of the inductance of a coaxial cable.

According to Fig. 1 and eq. (2), the eccentricity e and thus the leakage inductance per unit length change along the inductor, so multiplying by l in order to obtain the leakage inductance L_l is not correct.

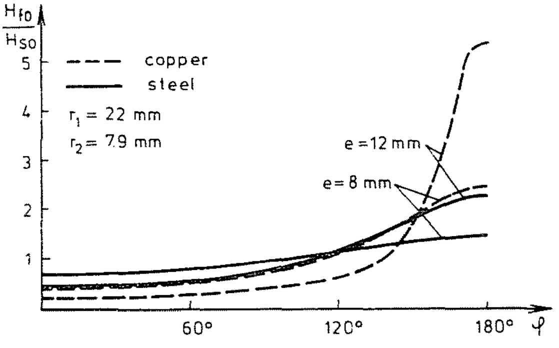

Fig. 4

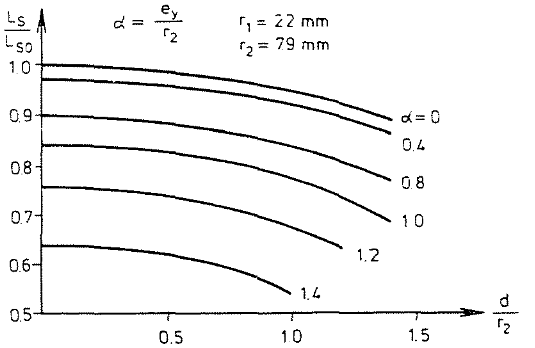

Fig. 5

Instead of multiplying we integrated L_l/l, the leakage inductance of unity length along the inductor. Of course, there is a contradiction in this calculation, since calculating the magnetic field and the leakage flux of unity length we adopted the assumption that the length of the inductor is infinite and at integrating it is considered to be infinitesimal, but if the diameter of the inductor is large enough this calculation can provide a good approximation.

The calculated inductance is shown in Fig. 5. Comparing the effects of the horizontal eccentricity d and the vertical distance e_y (Fig. 1) it can be seen that the effect of the horizontal eccentricity is relatively much less, so in the following we deal with the effect of the vertical distance e_y only.

Magnetic field in the conductors

It is obvious that calculating the magnetic field in the conductors the assumption that the skin depth δ is equal to zero cannot be upheld, but the fact that the skin depth δ is small as compared with the dimensions of the conductors can be utilized. Therefore the Cartesian co-ordinate system shown in Fig. 6 is used in the calculation instead of the polar one.

The magnetic field H and current density J are periodic functions of the variable X. The length of a period is $2r_0$ where $r_0=r_1$ in the copper inductor and $r_0=r_2$ in the steel insert. It is assumed that the magnetic field H at $r=0$ is equal to the field calculated by the assumption of $\delta=0$ i.e. it can be expressed by (5).

Employing Faraday's law

$$\operatorname{rot} \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}, \tag{9}$$

Amper's law

$$\operatorname{rot} \bar{H} = \bar{J} = \sigma \bar{E} \tag{10}$$

and using the sourcelessness of the current density J:

$$\operatorname{div} \bar{J} = 0, \tag{11}$$

the wellknown diffusion equation

$$\Delta \bar{E} = \mu\sigma \frac{\partial \bar{E}}{\partial t} \tag{12}$$

can be obtained where Δ is the Laplace operator μ is the permeability, σ is the conductance of the conductors, t is time, $\bar{E}$ is the electric field intensity.

The partial differential equation (12) is to be solved under the boundary conditions (5). The solution can be found by the wellknown method of separation of variables [2], [3], [8], [9], [10]. The solution is the following

$$\bar{E} = \bar{e}_z \sum_{k=0}^{\infty} E_k e^{-\frac{r}{\delta_k}} \cos\left(k \frac{X}{r_0}\right) \cos(\omega t + \alpha_k - \beta_k r) \tag{13a}$$

where
$$E_0 = \frac{I_2}{\sqrt{2}\,\pi r_0 \delta_0 \sigma} \tag{13b}$$

$$E_k = 2\cdot\sqrt{2}\,(-\gamma)^k E_0 \frac{\delta_0}{\delta_k} \cos\alpha_k \quad k=1,2,3 \tag{13c}$$

$$\delta_0 = \sqrt{\frac{2}{\omega\mu\sigma}} \text{ is the skin depth} \tag{13d}$$

$$\beta_0 = \frac{1}{\delta_0} \text{ is the wave number} \tag{13e}$$

$\bar{e}_z$ is the z direct unit vector;

δ_k and β_k are the skin depth and the wave number at the k^{th} harmonic, which can be expressed by the following formulae:

$$\frac{\delta_k}{\beta_k} = \delta_0^2 \tag{13f}$$

$$\frac{1}{\delta_k^2} - \beta_k^2 = \frac{k^2}{r_0^2} \tag{13g}$$

Solving (13f) and (13g)

$$\frac{\delta_k}{\delta_0} = \beta_k \delta_0 = -\frac{k^2\delta_0^2}{2r_0^2} + \sqrt{\frac{k^4\delta_0^4}{4r_0^4} + 1} \tag{13h}$$

α_k is the phase angle at the k^{th} harmonic:

$$\alpha_0 = 45° \tag{13i}$$

$$\operatorname{ctg}\alpha_k = \delta_k \beta_k \quad k=1,2,3 \tag{13j}$$

It can be seen from (13) that the higher is the order number of the harmonic the less are the skin depth and the wave number, and the larger is the phase angle. If the order number tends to infinity, $\delta=0$, $\beta=0$ and $\alpha=90°$.

If the eccentricity e vanishes, which means that $\gamma=0$, it can be seen from (13c) that all harmonics will vanish, so the expression valid at the symmetrical case can be obtained.

Surface heat flux

Knowing the distribution of the current density J the Joule heat J^2/σ can be determined. Since the skin depth δ is small we adopt the assumption that the total heat is produced by a thin layer of depth δ_0 on the surface of the conductors, i.e. the volume heat flux is substituted by a surface heat flux.

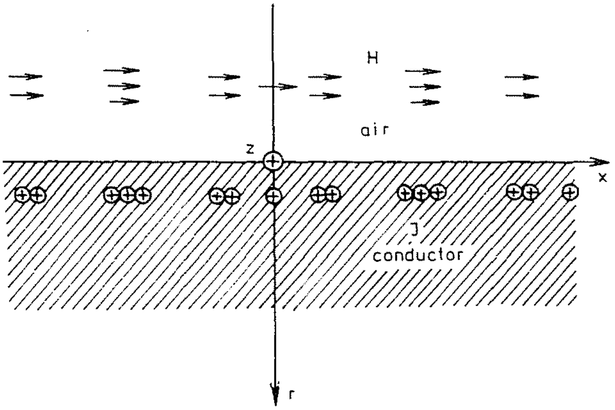

Fig. 6

The mean value of the surface heat flux q is the following: (See Fig. 6):

$$q(x)=\int_{r=0}^{\infty}\frac{1}{T}\int_{t=0}^{T}\sigma\bar{E}^2\,\mathrm{d}t\,\mathrm{d}r \tag{14}$$

where T is the period. Performing the operations in (14) the following result can be obtained:

$$q(x)=q_0+\sum_{k=1}^{\infty}q_k\cos\left(k\frac{x}{r_0}\right) \tag{15a}$$

where

$$q_0=\frac{1}{2}Q_{00}+\sum_{n=0}^{\infty}\frac{1}{2}Q_{nn} \tag{15b}$$

$$q_k=\sum_{n=k}^{\infty}Q_{n,n-k}+\frac{1}{2}\sum_{n=0}^{\infty}Q_{n,k-n} \tag{15c}$$

$$Q_{mn}=\frac{\sigma E_m' E_0''}{2}\,\frac{\cos\alpha_m\cos\alpha_n\cos(\alpha_m-\alpha_n+\alpha_{mn})}{\sqrt{\left(\frac{1}{\delta_m}+\frac{1}{\delta_n}\right)^2+(\beta_m-\beta_n)^2}} \tag{15d}$$

$$E_0'=\frac{1}{2}E_0'' \tag{15e}$$

$$E_n'=E_n''\quad n=1,2,3\ \ldots \tag{15f}$$

$$E_n''=\frac{2I_2}{\pi r_0\delta_0\sigma}\,\frac{\delta_0}{\delta_n}(-\gamma)^n\quad n=0,1,2\ \ldots \tag{15g}$$

$$\sin \alpha_{mn} = \frac{\beta_m - \beta_n}{\sqrt{\left(\frac{1}{\delta_m} + \frac{1}{\delta_n}\right)^2 + (\beta_m - \beta_n)^2}} \qquad m, n = 0, 1, 2 \ldots \tag{15h}$$

The relative value of the amplitude of the first harmonic is shown in Fig. 7.

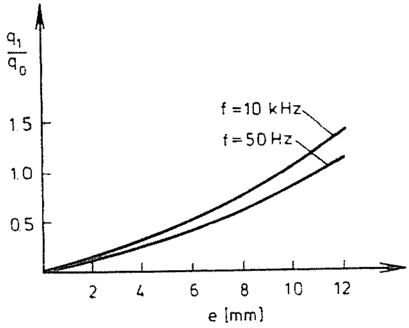

Fig. 7

Resistance

In the knowledge of the surface heat flux the whole heat produced in the conductors can be expressed:

$$P = l \int_{x=0}^{2r_0\pi} q(x)\,\mathrm{d}x = 2r_0\pi l q_0 \tag{16}$$

On the other hand

$$P = \frac{1}{2} R I_2^2 \tag{17}$$

(in (17) I_2 denotes peak value)
Comparing (16) and (17)

$$R = \frac{4\pi r_0 l q_0}{I_2^2} \tag{18}$$

Substituting (15b) into (18)

$$R = R_0 \left[1 + 4 \sum_{k=1}^{\infty} \gamma^{2k} \frac{\frac{\delta_0}{\delta_k}}{1 + \left(\frac{\delta_0}{\delta_k}\right)^4} \right] \tag{19a}$$

where

$$R_0 = \frac{l}{2\pi r_0 \delta_0 \sigma} \tag{19b}$$

is the wellknown expression of the resistance of an infinite half space.

It can be seen that (19a) tends to R_0 if the eccentricity e vanishes. The resistances of the copper inductor and the steel insert are shown in Fig. 8.

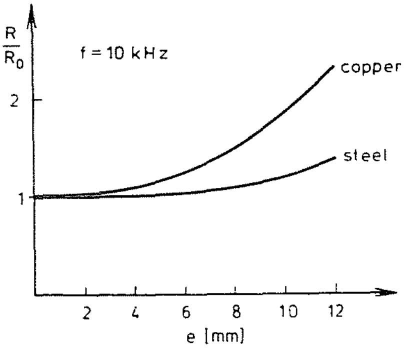

Fig. 8

Efficiency and current ratio

By the aid of the equivalent circuit shown in Fig. 3 the efficiency of the inductor can be determined:

$$\eta = \frac{P_2}{P_1 + P_2 + P_m} = \frac{1}{1 + \dfrac{R_1}{R_2} + \dfrac{R_m}{R_2} \dfrac{(R_1 + R_2)^2 + X_s^2}{R_m^2 + X_m^2}} \tag{20}$$

Since the third term in the denominator is much smaller than the second one, the change of the efficiency is determined by the ratio R_1/R_2. According to Fig. 8 this ratio increases, so the efficiency decreases if the eccentricity e increases.

According to Fig. 3 the ratio of the primary and secondary currents is expressed as

$$\frac{I_2}{I_1} = \sqrt{\frac{X_m^2 + R_m^2}{(X_s + X_m)^2 + (R_1 + R_2 + R_m)^2}} \tag{21}$$

The change of the efficiency and the ratio of the primary and secondary currents are shown in Fig. 9.

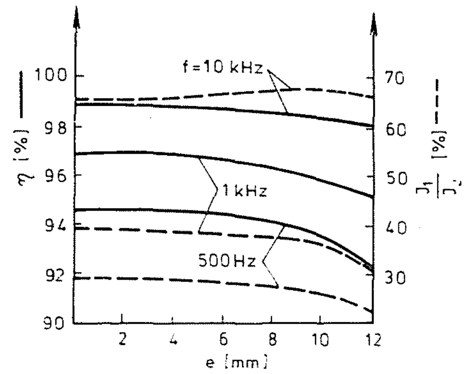

Fig. 9

Transient temperature distribution

Since the surface heat flux is not evenly distributed along the perimeter of the steel insert, its temperature also varies. The temperature rise on the surface of the steel insert is sought in the following form:

$$\vartheta(x, t) = \vartheta_0(t) + \sum_{k=1}^{\infty} \vartheta_k(t) \cos\left(k \frac{x}{r_0}\right) \tag{22}$$

$\vartheta_0(t)$ is the time function of the average temperature rise of the surface of the steel insert. We aim at determining the temperature rise $\vartheta_k(t)$ as a function of $\vartheta_0(t)$.

In order to calculate the transient temperature rise, the wellknown diffusion equation (23) is to be solved [1], [2], [3]:

$$\lambda \Delta \vartheta = \rho c \frac{\partial \vartheta}{\partial t} - p \tag{23}$$

where λ is the thermal conductivity c is the specific heat, ρ is the density and p is the heat flux produced in a unit volume. Since the total heat is produced in a thin layer of depth δ_0, p is expressed in (24) taking (15a) into account:

$$p(x) = \frac{q(x)}{\delta_0} = p_0 + \sum_{k=1}^{\infty} p_k \cos(kx/r_0) \tag{24a}$$

where

$$p_k = q_k/\delta_0 \quad k = 0, 1, 2 \ldots \tag{24b}$$

The time function of the heat flux p is supposed to be zero before and after the heating of the steel insert and constant during it.

The diffusion equation (24a) takes the following form in the surface layer of depth δ_0:

$$\frac{\partial \Theta}{\partial \tau} = \left[1 + \sum_{k=1}^{\infty} \frac{q_k}{q_0} \cos(k\varphi)\right] = \frac{\partial^2 \Theta}{\partial \varphi^2} \tag{25a}$$

where the dimensionless temperature Θ and the dimensionless time τ are introduced:

$$\Theta = \frac{\delta_0}{q_0} \frac{\lambda}{r_2^2} \vartheta \tag{25b}$$

$$\tau = \frac{\lambda}{\rho c r_2^2} t \tag{25c}$$

$$\varphi = \frac{x}{r_0} \tag{25d}$$

According to (22), the dimensionless temperature Θ is sought in a Fourier series form:

$$\Theta = \Theta_0 + \sum_{k=1}^{\infty} \Theta_k \cos(k\varphi) \tag{26}$$

Since the diffusion equation (25a) is linear, the principle of superposition can be used:

$$\frac{\partial \Theta_0}{\partial \tau} - 1 = \frac{\partial^2 \Theta_0}{\partial \varphi^2} \tag{27a}$$

$$\frac{\partial \Theta_k}{\partial \tau} - \frac{q_k}{q_0} \cos k\varphi = \frac{\partial^2 \Theta_k}{\partial \varphi^2} \quad k = 1, 2, 3 \ldots \tag{27b}$$

Since Θ_0 does not depend on the angle φ, the solution of (27a) under the initial condition $\Theta_0(\tau = 0) = 0$ takes the following form:

$$\Theta_0 = \tau \tag{28}$$

Let us seek ϑ_k by the aid of the separation of variables:

$$\vartheta_k = T(\tau) \cos(k\varphi) \tag{29}$$

Substituting (29) into (27b)

$$\frac{\mathrm{d}\,T}{\mathrm{d}\,\tau} + k^2 T = \frac{q_k}{q_0} \tag{30}$$

Solving (30) under the initial condition $T(\tau=0)=0$:

$$T(\tau)=\frac{1}{k^2}\frac{q_k}{q_0}(1-e^{-k^2\tau}) \tag{31}$$

Comparing (26), (28), (29) and (31), the transient temperature rise Θ is the following:

$$\Theta=\tau+\sum_{k=1}^{\infty}\frac{1}{k^2}\frac{q_k}{q_0}(1-e^{-k^2\tau})\cos(k\varphi) \tag{32}$$

According to (25b) and (32), the amplitude of the k^{th} harmonic of the temperature rise can be expressed as:

$$\vartheta_k=\vartheta_0\frac{1}{k^2}\frac{q_k}{q_0}\frac{1-e^{-k^2\tau}}{\tau} \tag{33}$$

ϑ_0 is the temperature at the end of the heating process. It can be seen from (33) that the shorter the time of the heating process, the larger are the harmonics of

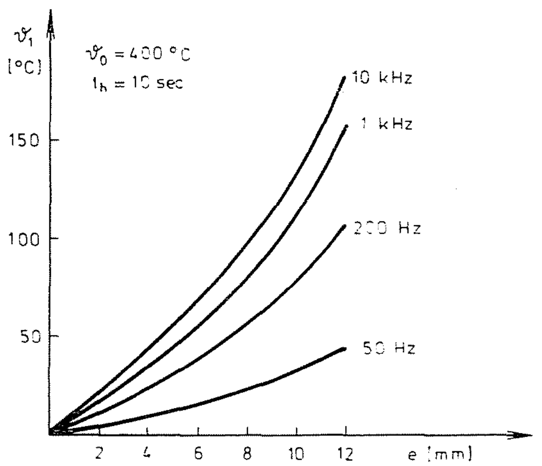

Fig. 10

the temperature rise. The ratio of ϑ_k/ϑ_0 tends to q_k/q_0 if the dimensionless heating time τ vanishes, which expresses the physical concept that there is no thermal conductivity.

The first harmonic ϑ_1 of the temperature rise is plotted against the eccentricity e in Fig. 10 in the case when the heating time is 10 seconds and ϑ_0 is 400 °C. It is remarkable that the value of ϑ_1 can reach as high a value as 200 °C.

Conclusions

An asymmetrically arranged induction heating equipment has been studied. The consequences of the asymmetry are as follows:

1. The magnetic field on the surface of the conductors is not evenly distributed. Its maximal value can be 5 times greater then the mean value on the surface of the copper inductor (See Fig. 4).
2. The leakage inductance L_l decreases. Its value can be 60% that of the symmetrical case. The effect of vertical asymmetry is much greater (See Fig. 5).
3. The resistance of the conductors increases. The value of the resistance R_1 can be greater than 200% of that of the symmetrical one (See Fig. 8).
4. The efficiency decreases. The loss is 2% at a frequency of 1 kHz (See Fig. 9).
5. The current consumption increases by 20% (See Fig. 9).
6. The temperature rise of the steel insert is not evenly distributed. The first harmonic of the temperature rise can reach 200 °C (See Fig. 10).

References

1. BEJI SZABÓ, D.: Induction heating (In Hungarian) Müszaki Könyvkiadó, Budapest, 1965.
2. BIRÓ, O.—GYIMESI, M.: Calculation of a thermal transient process at the decoupling of shrinking clutch. (In Hungarian) Elektrotechnika, *75*, No, 9–10. 1982.
3. FRANK, PH.—MISES, R.: Die Differenzial- und Integralgleichungen der Mechanik und Physik. Dover (New York) 1961.
4. GRADSTEIN, I. S.—RIZHIK, I. M.: Integralov, Summ, Ryadov I Proizvedieny (In Russian) Gosudarstvennoe Izdatelstvo Tablici Fiziko-Matematitseskoy Literatury, Moskow, 1963.
5. GYIMESI, M.: Study of an inductor-insert system arranged asymmetrically. (In Hungarian) TUB Department of Electrotechnics, 1982.
6. KANYO, P.—NAGY, I.—TÓTH, F.: Theoretical and experimental studies on the waste recovering of braided steel wire rings by induction heating (In Hungarian) TUB Department of Electrotechnics, 1981.
7. KANYO, P.—NAGY, I.—TÓTH, F.: Theoretical and experimental studies of induction heating process for the waste recovering of braided steel wire rings. (In Hungarian) TUB Department of Electrotechnics. 1982.
8. KÜPFMÜLLER, K.: Einführung in die theoretische Elektrotechnik (In German), Springer-Verlag, Berlin, Göttingen, Heidelberg, 1949.
9. NEUMANN, L. R.—KALANTAROV, P. L.: Teoretitseskye Osnovi Elektrotechniki (In Russian) Gosenergoizdat, 1959.
10. STRATTON, J. A.: Electromagnetic Theory, Mc. Graw-Hill Book Company, Inc. New York, London, Toronto, 1941.

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