

DYNAMICS AND RELIABILITY OF A MAN-MACHINE SYSTEM

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Abstract

This study deals with the dynamics and reliability of a man-machine system with the technical part containing a hydraulic servomechanism. A new method has led to some interesting results of use for both analysis and synthesis of such system. Furthermore, with a convenient modification these results may be generalized for studying other structural systems.

Investigation of the system reliability is based on concept of multistate system analysis.

Introduction

Dynamics and reliability of a man-machine system will be studied where the technical part consists of a hydraulic servosystem and a mass-spring-damper load. Such a system operation may be vehicle driving, crane manipulation and so on.

This problem is of interest for several engineering activities. Some peculiarities will be picked out felt to be of importance in system operation. Such is, e.g., the influence of the change of hydraulic fluid compressibility due to air bubbles arising in the fluid usual in tropical and humid environment. Other changes of oil properties are irrelevant and may be disregarded.

Dynamics of technical part will be investigated for its transient behavior and stability as well as the limit cycle occurrence of the servosystem affected by hysteresis backlash.

The subsequent problem is the modelling of the man-machine system. Several models have been suggested by different authors for the man, i.e. the operator of them that seeming the most convenient for our case has been chosen.

The last problem studied is that of the reliability of the complex system, considered as a multistate system. So theory of multistate system analysis will be relied on. At this intermediate stage of investigations, here the essential concepts will be given, likely to help solving the outlined problem.

The hydraulic servosystem in the technical part

Influence of the compressibility change of the hydraulic fluid

In tropical and humid environment, bubbles have often been observed in the fluid. The presence of air bubbles affects of course fluid properties, hence, also the hydraulic servosystem. Among them the compressibility change of the hydraulic fluid is determinant for the system dynamics. A fluid containing air bubbles less than one millimeter in diameter may be considered as a one-phase, so-called Newtonian fluid. Fed to the system by a high pressure pump, the bubbles are compressed with two effects: first, air temperature in the bubbles increases increasing also oil temperature, hence reducing the viscosity and, also the damping; second, as a function of the air-oil volume ratio, the modulus of compressibility of the working fluid, i.e. the air-oil mixture, is very strongly changed.

Oil temperature and viscosity variation may be stated to be very small and negligible, in conformity with relationship [1]:

$$\Delta T = \frac{24}{427} \frac{n}{n-1} p_a v_a \left[\left(\frac{p_0}{p_a} + 1 \right)^{\frac{n-1}{n}} - 1 \right] \frac{c_1 \gamma_1 v}{c_0 \gamma_0 (1-v)} \div \left(1 + \frac{c_1 \gamma_1 v}{c_0 \gamma_0 (1-v)} \right)$$

where

ΔT = temperature difference;

p_a = atmospheric pressure;

v_a = surrounding air specific volume;

p_0 = operating pressure in the system;

n = polytropical specific heat power;

c_0, c_1 = specific heats of oil, and air, respectively;

γ_0, γ_1 = specific weight of oil, and air, respectively;

v = volume ratio of air bubbles in oil.

Under normal operating conditions, the temperature increase is seen not to exceed 0.05 °C.

Some authors report of the so-called Diesel effect, where air in the bubbles is mixed with the oil vapour and under high compression this mixture burns like the burning process in the diesel engine, so the temperature may increase. But, by a simple calculation can show that in this case the oil temperature does not increase noticeably after all. For instance for a 10% air to oil ratio by volume and an air surplus ratio $\alpha = 1.25$, at atmospheric pressure the temperature increase is but 0.11 °C. Therefore at a dynamic equilibrium of

temperature in the system there is no considerable variation. Thus in the following this fact may be disregarded.

Now, let us consider the influence of the compressibility on the change of the modulus $E = 1/\beta$ of the hydraulic fluid. The isothermal state variation between β and v : [1]

$$\beta = \frac{1 + E_0 \frac{p_a}{p_0^2} \frac{v}{1-v}}{1 + \frac{p_a}{p_0} \frac{v}{1-v}} \beta_0$$

where β is the compressibility coefficient of oil with bubbles and E_0 is the volumetric modulus of pure i.e. bubble-free oil. Obviously for $v=0$, $\beta = \beta_0$ (subscript 0 refers to pure oil).

Substituting this relationship into the corresponding coefficients of the characteristic equation of our servosystem:

$$K(s) = C_2(\beta)s^3 + C_1(\beta)s^2 + s + K_0(\beta) = 0$$

or:

$$K(s) = C_2(v)s^3 + C_1(v)s^2 + s + K_0(v) = 0$$

where s is the Laplace transform variable. Using the well-known Routh-Hurwitz stability criterion, the stability condition results as a function of v :

$$av^3 + bv^2 + cv + d > 0$$

where a , b , c and d are constants. For a given system, and given operation parameters this condition leads to the permissible value of v , granting stability to the servosystem.

It is interesting to mention here that also β can be of help in stating analitically the variation law of the gain, of the damping ratio as well as of the time constant. Namely with increasing v , also the gain and the time constant increases, but, in turn, the damping ratio decreases [1].

Behavior of the servosystem considering the hysteresis backlash and the hydraulic fluid compressibility

The problem will be investigated by means of a convenient digital simulation based on a system model involving the equation system:

$$1) \quad T^2 \ddot{x}_s + 2\zeta T \dot{x}_s + \dot{x}_s + K(a-h) = 0$$

$$2) \quad T^2 \ddot{x}_s + 2\zeta T \dot{x}_s + \dot{x}_s + K(x_s + 1) = 0$$

$$3) \quad T^2 \ddot{x}_s + 2\zeta T \dot{x}_s + \dot{x}_s + K(-A+h) = 0$$

$$4) \quad T^2 \ddot{x}_s + 2\zeta T \dot{x}_s + \dot{x}_s + K(x_s - h) = 0$$

where

T = system time constant;

ζ = damping ratio;

K = gain;

x_a = system input signal in our case $x_a = \delta(t)$, the Dirac impuls;

x_s = system output signal or signal to be controlled;

A = timely amplitude of input signal of the hysteresis backlash nonlinearity.

According to the computer program flowchart shown in Fig. 1 many different phase trajectories in plane and space of the state variables are

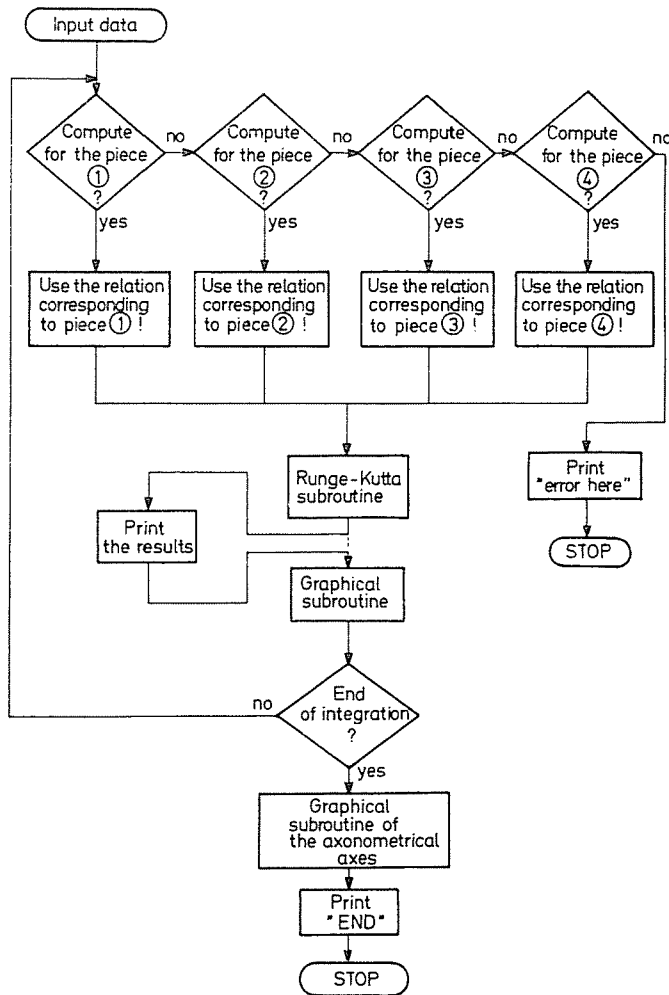


Fig. 1

obtained. Furthermore, diagrams of displacement, speed and acceleration may be plotted as time functions $x(t)$, $y(t)$ and $z(t)$ respectively [2].

Some concrete cases have led to the following conclusions:

— If the hydraulic fluid compressibility is negligible the backlash causes a certain limit cycle. This fact is in total agreement with results of some others methods, for instance, the isocline method [3]. It is very interesting that the limit cycle amplitude and the backlash magnitude are in linear relation (see [2]).

— The fluid compressibility omits the limit cycle occurrence and the system behaves as a conditionally stable one.

— The hysteresis backlash improves the stability of the considered system, although to the detriment of controlling accuracy. But in practice, this situation is not always disadvantageous, because for example in a roughing operation at a copying shaper, it is needless to maintain a magnitude smaller than a few micrometers.

— If the backlash-free system is stable, then no backlash can induce instability. Besides, in third-order system i.e. when the fluid compressibility is considered the backlash, within certain limits, brakes the feedback effect, moreover it keeps the feedback amplitude of the initial output signal not greater than the backlash magnitude h .

— Considering the ratio by volume of air oil mixture as a system operation parameter, between the initial magnitude of the controlled signal $x_{s0} = x_0$ and the backlash h there exists a linear relation which divides the first quarter of plane (x_0, h) into two regions, namely one for the stability, and the other for the instability (see [2]).

— Despite of the outlined linearity the system retains an essentially nonlinear feature. Namely the system is only stable for signal of small magnitude.

Dynamics of our man-machine system

A new method will be suggested to approach the problem of our man-machine system, easy to use in engineering practice for the analysis and synthesis of these systems [4].

After the construction of the general model, a simplified one will be started from gradually proceeding to more complicated one.

System model

The model of a man-machine system will be constructed where the human operator manipulates a machine by means of a hydraulic servomechanism. Fig. 2a-b shows the block-diagram of this complex system.

Great many continuous and sampled-data models have been suggested for the human transfer function. We shall use a continuous one [5]:

$$H(s) = K_e e^{-T_H s} \frac{1 + T_e s}{(1 + T_N s)(1 + T_I s)}$$

where

- K_e = operator's gain;
- T_H = dead time at operator;
- T_e = operator's lead time constant;
- T_I = operator's lag time or time constant;
- T_N = neuro-muscular time constant.

As a first approach, the last two time constants are assumed to be zero. By the way, dead time is of great importance in system stability and it causes

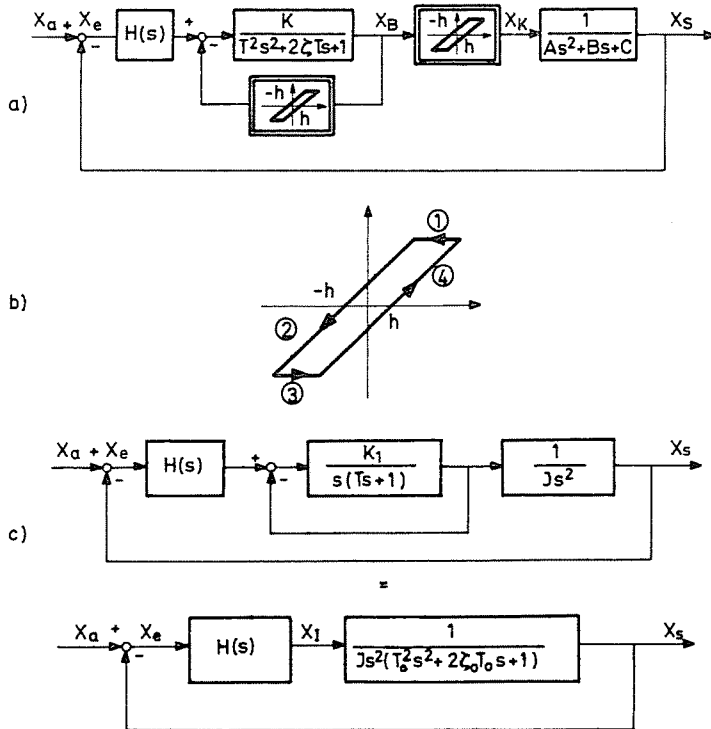


Fig. 2

serious mathematical problems in system synthesis. This fact induced us to simplify the model.

First, the compressibility of the hydraulic fluid and the hysteresis backlash of the servomechanism will be disregarded. Second, the controlled object will be assumed to be spring-damper free. In this way the system becomes linear and simpler as shown by its block-diagram (see Fig. 2c).

After some conversions and simplifications in the block-diagram and of the calculation steps, the retarded differential difference equation (RDDE) becomes:

$$x^{IV}(t) + a_1 \ddot{x}(t) + a_0 \ddot{x}(t) + b_1 \dot{x}(t-1) + b_0 x(t-1) = 0$$

where

$$a_1 = 2\zeta_0 \frac{T_H}{T_0}$$

$$a_0 = \left(\frac{T_H}{T_0}\right)^2$$

$$b_1 = \frac{K_e}{J} \frac{T_H^3 T_e}{T_0}$$

$$b_0 = \frac{K_e}{J} \frac{T_H^4}{T_0^2}$$

taking: $x_e = x$, and under the condition that the type number of the input signal $x_a(t)$ is not greater than 2.

Mathematically, the asymptotic stability of the trivial solution $x(t) = 0$ is intended to be investigated after Lyapunov; namely the system equilibrium or system stability!

There are several methods in the special literature to investigate the stability of RDDEs. But in the actual case, the most effective solution method is based on a recently developed theorem

Theorem

Let us consider the characteristic function of the mentioned RDDE in the form

$$D(\lambda) = \lambda^4 + a_1 \lambda^3 + a_0 \lambda^2 + b_1 \lambda e^{-\lambda} + b_0 e^{-\lambda}.$$

Let the function $M(y)$ and $S(y)$ be defined as

$$M(y) = \operatorname{Re} D(iy) = y^4 - a_0 y^2 + b_1 y \sin y + b_0 \cos y$$

$$S(y) = \operatorname{Im} D(iy) = -a_1 y^3 + b_1 y \sin y - b_0 \sin y$$

where $y \in R^+$, $i = \sqrt{-1}$.

Trivial solution $x(t)=0$ of our RDDE is asymptotically stable iff

$$S(y_k) \neq 0, \quad k=1, 2, \dots, m \quad \text{and}$$

$$\sum_{k=1}^m (-1)^k \text{sign } S y_k = 2$$

where $y_1 \geq y_2 \geq \dots \geq y_m \geq 0$ are real zeros of $M(y)$.

Here we shall omit the proof of this theorem, and refer instead to [4]. Also, the physical fact will be involved that $b_0 > 0$ and $b_1 > b_0$ are necessary conditions of stability. Indeed, they practically are always fulfilled in the case of a similar control system.

From these conditions, after a number of calculation steps, some stability charts in different parameter planes of interest may be derived, e.g. in planes (a_0, b_1) , (T_H, T_e) , (T_0, J) , and (T_0, ζ_0) . The shaded domains in both stability charts refer to the stability region (see Figs 3 through 6).

These stability charts suit both system analysis and system synthesis in a certain sense.

Some controlling examples point to the accuracy of these charts (Fig. 7).

It is interesting to see that the complex man-machine system may be stable in the case of a negative damper! Besides, the Nyquist plot has an

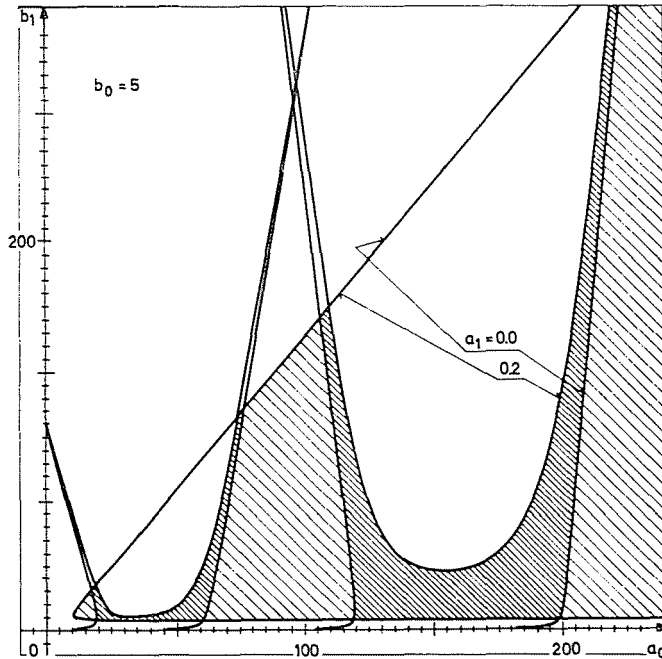


Fig. 3

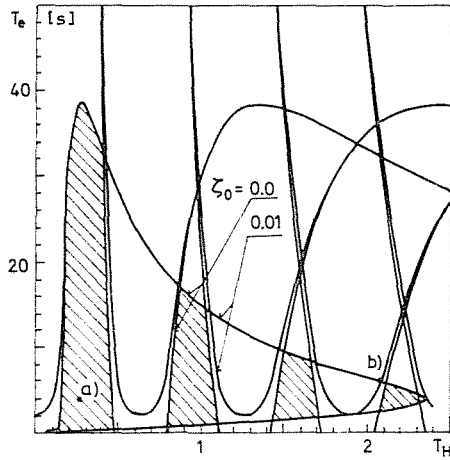


Fig. 4

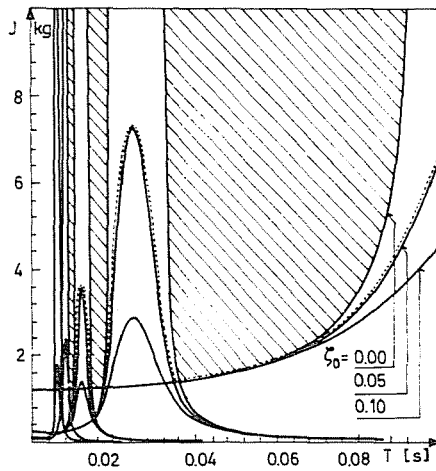


Fig. 5

asymptote at zero damping ratio (Figs 8a, b, c) and in this case the system may be stable depending on how much it follows the course of the Nyquist plot. This problem will be discussed latter. Via a number of concrete examples and stability charts, several interesting and important conclusions may drawn about this man-machine system.

Before hand, let us extend our model to the case where the load, i.e. the object to be controlled, is of a mass-spring-damper character and between the servomechanism and the load there is a hysteresis backlash, in correspondence

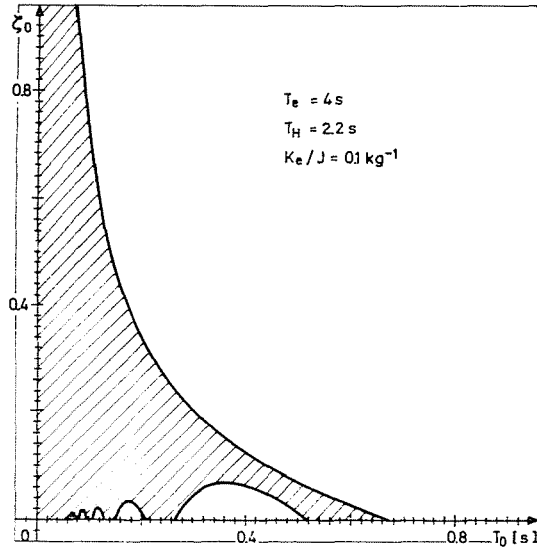


Fig. 6

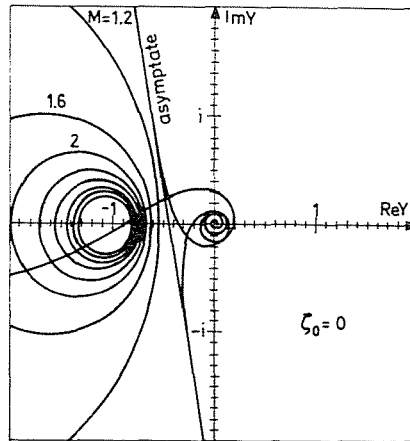


Fig. 7

with engineering practice. Its block-diagram is seen in Fig. 2a, except that there is no backlash.

In this case, different stability charts result, for instance in plane (T_H, T_e) and (T_1, ζ_1) (see Fig. 9). Now the Nyquist plot is also modified. Namely, there are two asymptotes when both damping ratios are zero (Fig. 10) [6].

As concerns the hysteresis backlash between the servomechanism and the load the describing function method has to be used as in the customary case.

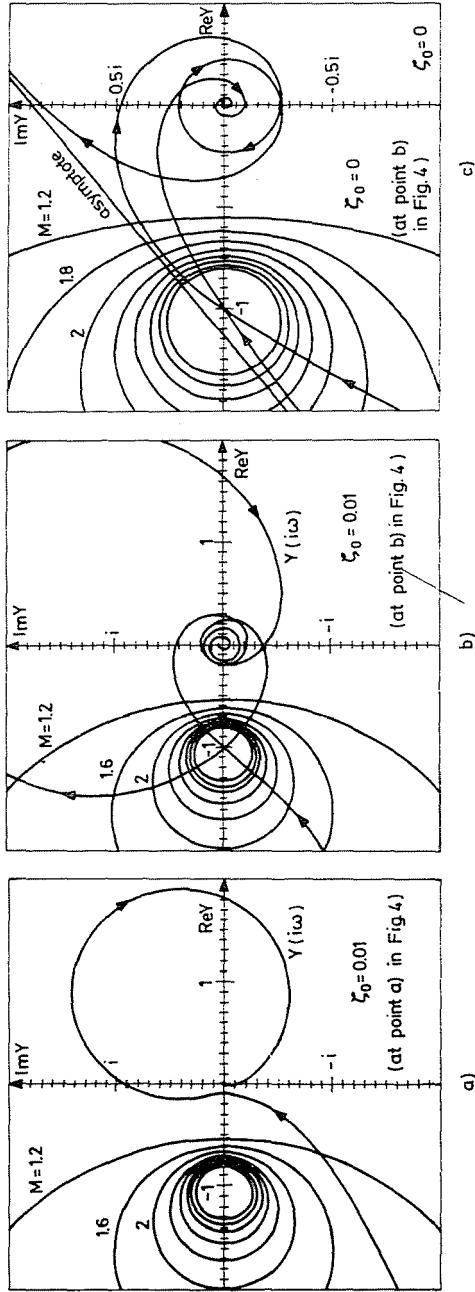


Fig. 8

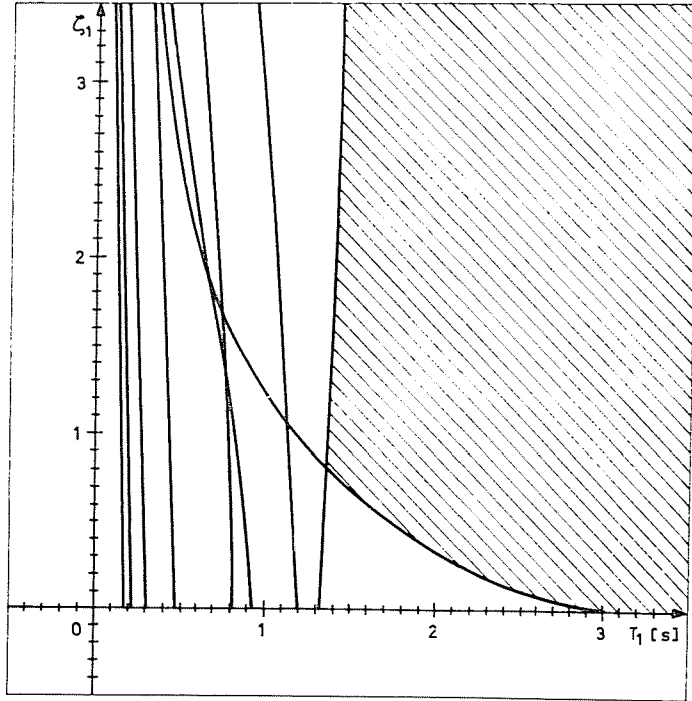
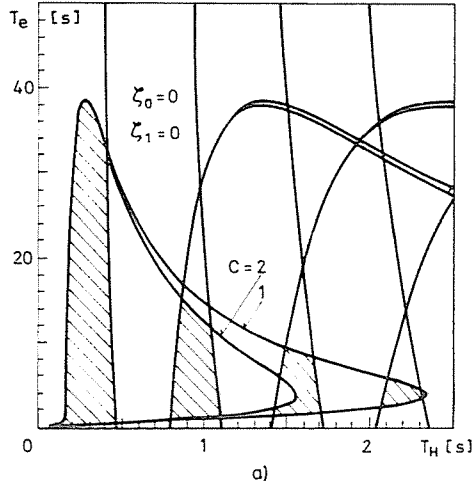
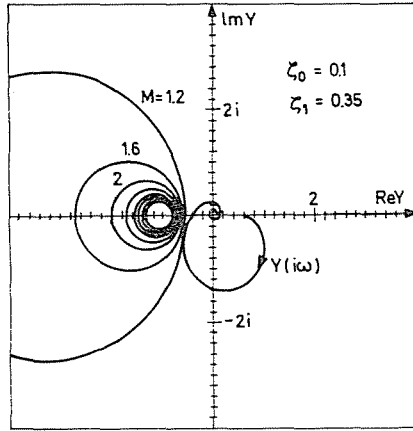
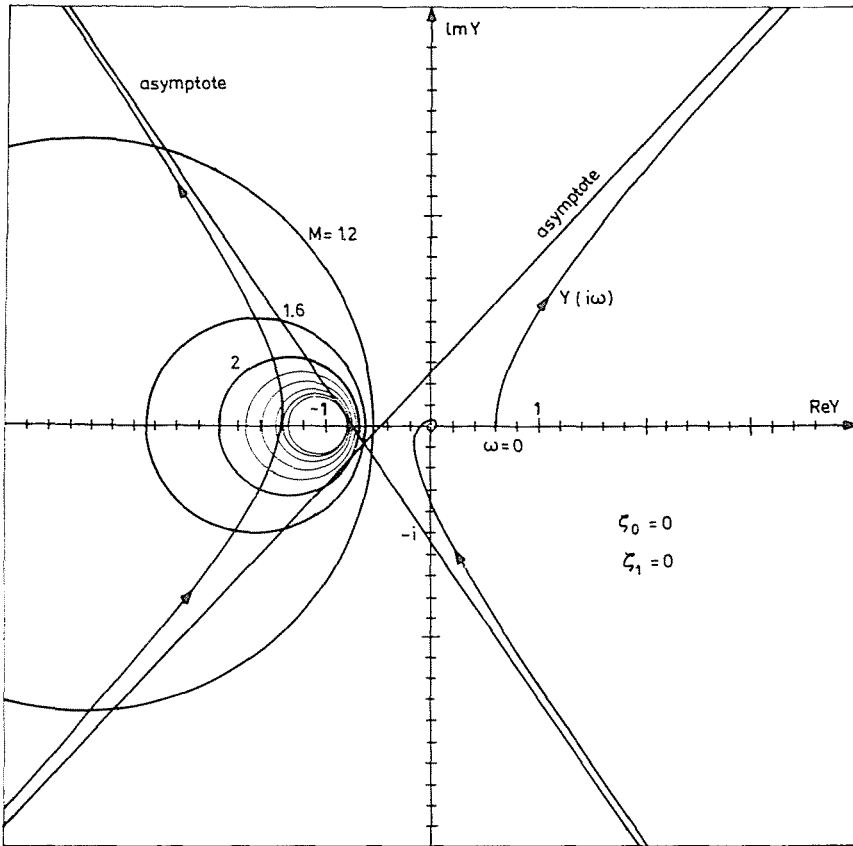


Fig. 9



a)



b)

Fig. 10

Now, let us discuss some important conclusions:

1) A human operator of adequate may cause the system to become stable for a zero or negative damping ratio. In this way the human presence improves the stability of the complex system.

2) The Nyquist stability criterion is henceforward valid, but in a more general sense. We may draft some rules for the easy use of this criterion.

3) In conformity with the hysteresis backlash between the servomechanism and the load—whether the load is of a spring-damper-free character or not—the system may involve one or more limit cycles of different characters, namely, convergent or divergent. In some cases the hysteresis backlash improves the stability situation of this system [2].

Reliability of man-machine system

Basic concept

From the aspect of reliability, it seems more exact to consider the man-machine system as a multistate system, inducing to apply the theory of multistate system analysis. From the multitude of relevant procedures and algorithms, preference is given the fault tree representation, with minimal cutset (or pathset). Of course, the graph theory or discrete function theory recourse to if necessary [7], [8], [9], [10], [11].

For the sake of simplicity it may be supposed that
 — the failure of the component are s -independent;
 — as concerns reliability, our system is always s -coherent.

Now, let us define the concept of coherence:

Let c_i be a component of a system $f(\mathbf{X})$; c_i being a s -coherent of the system means:

- 1) c_i play a role in the system;
- 2) any state change of c_i influences the system;
- 3) the direction of this state change is consistent with that of the state change of the system.

These conditions of the s -coherent system also imply that if any component is not s -coherent the system itself is not s -coherent.

So it is right to suppose that this system is s -coherent, because failure of any component does not improve the system reliability.

This theory lends itself to investigate a system where the components have different numbers of states, and not only the probability of system failure or of system operability can be predicted but probability that system is at any performance level of interest, and so on.

Let us mention here that in multistate system analysis the lattice theory is rather efficient.

After all, the multistate system theory is the extension of the binary-state theory. Therefore the well known laws of the binary-state system may be used at a suitable logical analogy. Namely if in the multistate system

\mathbf{X} = vector of component states (X_1, X_2, \dots, X_n);

$\Phi(\mathbf{X})$ = state of the system;

X_i = state of component c_i ; $i = 1, 2, \dots, n$;

N_i = best state of component c_i or maximal state level of component c_i .

M = best state (max. state level) of the system; and so on, then for the binary-state system:

$\mathbf{X} = (X_1, X_2, \dots, X_n)$; $X_i \in \{0, 1\}$ = failed or functioning, $i = 1, 2, \dots, n$

$P_i = \Pr(X_i = 1)$;

$\Phi(\mathbf{X}) \in \{0, 1\}$.

Of course, we may write for the multistate system:

$X_i \in \{0, 1, \dots, N_i\}$;

$P_{ij} = \Pr\{X_{ij} = 1\}$ where j refers to the state level j of component c_i .

Hence for the binary-state system $j = 2$

$$\Phi(\mathbf{X}) = \sum_{k=1}^2 \Phi^k(\mathbf{X})$$

but for the multistate system

$$\Phi(\mathbf{X}) = \sum_{k=1}^M \Phi^k(\mathbf{X}), \quad k = \text{system state level}$$

where

$$\Phi^k(\mathbf{X}) = \begin{cases} 0 & \text{if } \bar{\Phi}(\mathbf{X}) < k \\ 1 & \text{if } \bar{\Phi}(\mathbf{X}) \geq k \end{cases}$$

As to the human reliability several works have been published in the last ten years [7], [12].

In general, human error may result from any of the following cases:

- the operator pursues a wrong goal;
- the required goal is not met because the operator acted wrongly;
- the operator fails to act in the moment of need.

As to the levels of human error, different numbers of finite states are found depending on the actual case, and sometimes this problem may be individual one.

It is important to note that the operator may be considered as an "element" with a continuous reliability model. This is a case of a mix system model which may raise a lot of interesting problems not to be concerned with here.

Illustrative examples for the use of the multistate system theory

Let us consider some simple cases involving technical devices, and see how to model a multistate system with multistate components using binary variables.

Example 1

Consider an oil supply system consisting of two identically performing pumps. Corresponding to the requirements of the system, there are three levels, namely

- level 0 = system fails;
- level 1 = system performance is acceptable;
- level 2 = system functions perfectly or normally.

The system performs normally if one pump is at full power (level 2 of element) and the other pump is at half power (level 1 of element).

The system operates acceptably if one pump is at full power and the other fails (level 0 of element), or both pumps are at half power.

The system fails if one pump is at half power and the other fails, or both fail.

The conditions above yield the state function of the system:

$$\Phi(2, 2) = \Phi(2, 1) = \Phi(1, 2) = 2$$

$$\Phi(0, 2) = \Phi(2, 0) = \Phi(1, 1) = 1$$

$$\Phi(0, 1) = \Phi(1, 0) = \Phi(0, 0) = 0$$

or as logical tabulated function:

X_2	X_1	0	1	2
0	0	0	0	1
1	0	0	1	2
2	1	1	2	2

and as the state function of level k :

$$\Phi^k(\mathbf{X}) = \bar{\Phi}^k(X_{1k}, X_{2k}), \quad k = 1, 2,$$

namely

$$\Phi^1(\mathbf{X}) = \sup(X_{12}, X_{22}, X_{11}X_{21}) = \max(X_{12}, X_{22}, X_{11}X_{21})$$

$$\Phi^2(\mathbf{X}) = \sup(X_{11}X_{22}, X_{11}X_{21}) = \max(X_{11}X_{22}, X_{12}X_{21}).$$

Remember that in the domain of discrete functions the supremum is equivalent to the maximum.

From these state functions the flowcharts in Fig. 11 may be constructed.

After the flowchart the reliability at level k may be determined without special difficulties. Namely, the system reliability at level 1 is

$$R = E\{\Phi^1(\mathbf{X})\} = P_{12}EE\{\Phi^1(\mathbf{X})X_{12} = 1\} + (1 - P_{12})E\{\Phi^1(\mathbf{X})/X_{12} = 0\}$$

$$= P_{12} + (1 - P_{12})P_{22} + (1 - P_{12})(1 - P_{22}) \times (P_{11} - P_{12})(P_{21} - P_{22})$$

(where $E\{X_{11}/X_{12} = 0\} = \Pr\{X_1 = 1\} = \Pr\{X_1 \geq 1\} - \Pr\{X_1 \geq 2\} - P_{11} - P_{12}$).

And at level 2:

$$R = E\{\Phi^2(\mathbf{X})\} = P_{11}P_{22}[1 + P_{12}(P_{21} - P_{11}P_{22})].$$

From the flowchart of $\Phi^k(\mathbf{X})$ the fault tree is easy to construct. For example, from $\Phi^1(\mathbf{X})$ and $\Phi^2(\mathbf{X})$ of the example above the fault trees are those in Fig. 12.

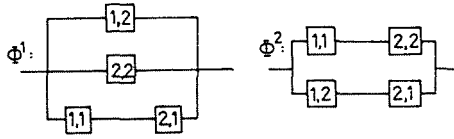


Fig. 11

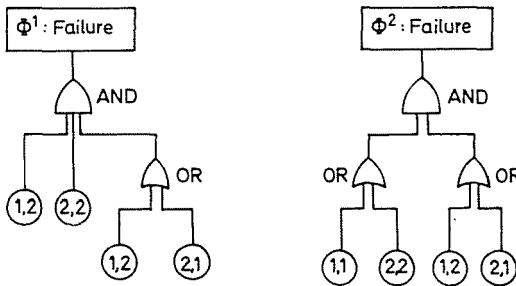


Fig. 12

Example 2

Consider an oil supply as shown in Fig. 13. From the given system and with the given data number of component levels and of logical functions, noted in Fig. 14 the corresponding event tree can be constructed. The given logical functions are

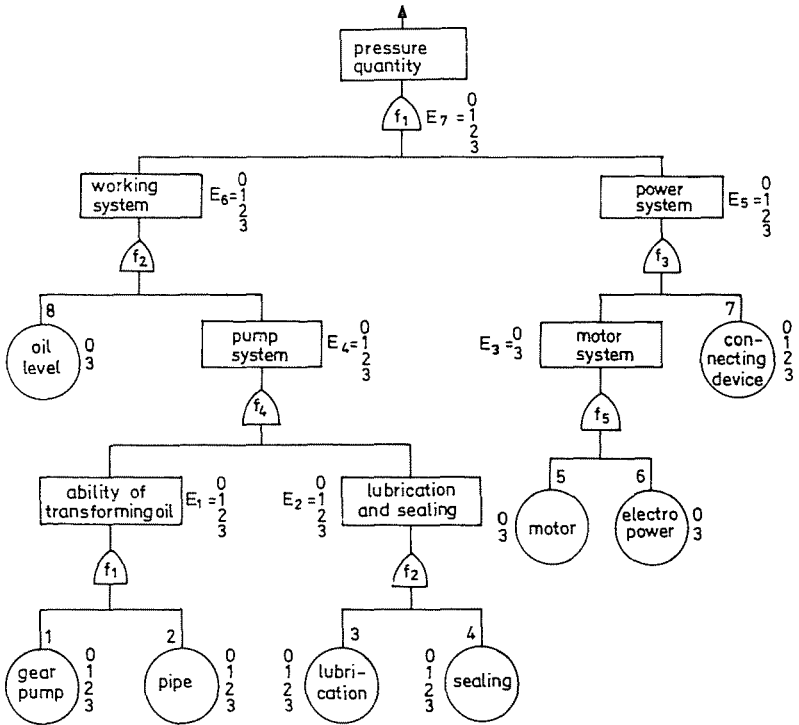
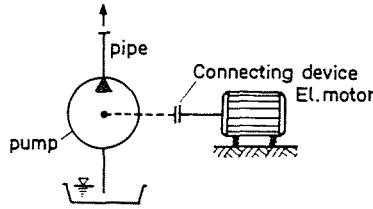


Fig. 14

$f_1: X_2 \backslash X_1$	0	1	2	3
0	0	0	0	0
1	0	0	1	1
2	0	1	1	2
3	0	1	2	3

$f_2: X_2 \backslash X_1$	0	1	2	3
0	0	0	0	0
3	0	1	2	3

$f_3: X_2 \backslash X_1$	0	3
0	0	0
1	0	1
2	0	2
3	0	3

$f_4: X_2 \backslash X_1$	0	1	2	3
0	0	0	0	0
1	0	1	1	1
2	0	1	1	2
3	0	1	2	3

$f_5: X_2 \backslash X_1$	0	3
0	0	0
3	0	3

The level probability vectors of components are

$$\mathbf{p}_1 = \mathbf{p}_2 = \mathbf{p}_3 = \mathbf{p}_4 = \mathbf{p}_7 = (0.01, 0.1, 0.2, 0.69)$$

$$\mathbf{p}_5 = \mathbf{p}_6 = \mathbf{p}_8 = (0.01, 0, 0, 0.99).$$

The same procedure has led to the result:

$$R = (0.121121, 0.43604, 0.296081, 0.151758).$$

That is, probability of the system insufficiency is 0.121121 ;
 probability of the pressure level 1 is 0.43604 ;
 that of the pressure level 2 is 0.296081 ;
 that of the maximal or highest pressure level
 is 0.151758 .

The probability of a high-level pressure seems to be relatively small, in spite of that the components have a great probability at high level!

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