# THE RELIABILITY MODEL OF A SELECTED CLASS OF THE HIERARCHICAL SYSTEMS

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Received june 30, 1988

**Abstract**

The paper analyses digital systems a with three-level hierarchical structure. It has taken into account reliability and functional aspects of the system. A reliability-functional model of the proposed digital system has been presented.

## 1. Introduction

The aim of our considerations is the reliability functional analysis of digital systems with a hierarchical structure. A structure example of the system is presented in figure 1. Each element of the structure is capable of processing some local task and transfering the results to the elements on higher and lower levels in the hierarchy.

In the reference literature available to date one can hardly find any works dealing simultaneously with both reliability and functional aspects of digital systems. In [5] a reliability-functional model of digital systems has been presented, considering, in particular single processing systems. The problem of assessing the properties of reliability of digital systems with a hierarchical structure with the use of Markov processes is discussed in [1, 2, 3]. In [4] the problems of reliability of digital systems with simple and complex form are dicussed, too.

The present paper aims at a reliability-functional analysis of digital system with a hieararchical structure, using the semi-Markov processes.

Contrary to Markov's processes, the semi-Markov processes allow us to reveal the complexity of operational rules of digital systems and to consider the strategies of information and hardware renewal used in case of some inefficiency.

Advantages of the semi-Markov processes result from the possibilities of introducing an arbitrary time distribution for a system in a single state as well as an arbitary probability of transfers between states.

## 2. Assumptions

It is assumed that the digital system with a hierarchical structure (Fig. 1) realizes the definite set of tasks. The task is a finite sequence of the packets of instructions the realization of which takes some time for the system to operate. After the task to be processed has been introduced the system divides it into packets and ascribes to the latter appropriate levels of processing. Each task fed into the system is split into phases related to given levels of the system structure. It is assumed that no communication between the elements of the same level of the system structure takes place. The subsequent task is introduced into the system through a device located on level no. 1 as soon as the preceding task introduced by this device has been finished. The subsystem operation realizing any task is defined by functional configuration (Fig. 2).

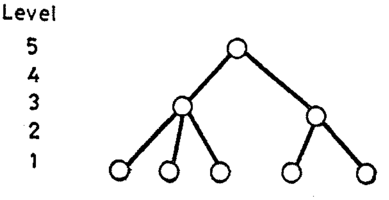


*Fig. 1.* An example of the system structure

*Fig. 2.* A functional configuration

On the basis of the definition of failure, a reliability structure of functional configuration is defined. The reliability structure defines the ways of cooperation between devices of the case of failures.

Any device of each of the system structure levels may fail. If the device necessary to realize the task fails, the task is destroyed. It is revived after renewal of the damaged device.

At the time when realizing $i$-th phase of the task, the set of the $i$-th phase hardware supplies $H^{(i)}$, and its set of operational system procedures $SO^{(i)}$ are generated: Moreover for the $i$-th phase, the set of input task elements $Z_{in}^{(i)}$ and the set of output task elements $Z_{out}^{(i)}$ are distinguished.

Thus, realization of the $i$-th phase of task is based on the set:

$$\{Z_{in}^{(i)}, H^{(i)}, SO^{(i)}, Z_{out}^{(i)}\};\ i \in I,\ \text{card}\ I = 2(M-1)$$

where: $M$ — number of the system structure levels,
card $I$ — number of executed phases of the task.

The sets $Z_{in}^{(i)}$ and $Z_{out}^{(i)}$ are defined as follows:

$$Z_{in}^{(i)} = \{D^{(i)}, P^{(i)}, K_A^{(i)}, V^{(i)}, W^{(i-1)}, K_R^{(i-1)}\}$$

$$Z_{out}^{(i)} = \{D^{(i+1)}, P^{(i+1)}, K_A^{(i+1)}, V^{(i+1)}, W^{(i)}, K_R^{(i)}\}$$

where: $D^{(i)}$ — set of input data for the $i$-th and subsequent phases,
$P^{(i)}$ — set of procedures for the $i$-th phase (program realizing this phase of a the task),
$K_A^{(i)}$ — set of forecast chronicles (of the a'priori assumed initial and final moments and mean times of the task phase duration) of the $i$-th phase,
$K_R^{(i)}$ — set of real chronicles of the $i$-th phase and of the phases which have been completed,
$W^{(i)}$ — set of results of the $i$-th phase and the preceding phases,
$V^{(i)}$ — component of the vector $\mathbf{V}$ initiating the $i$-th phase realization.

The above vector $\mathbf{V}$ allows for systems characteristics. For the input task for the $i$-th phase, the component $V^{(i)}$ selects the sets of hardware supplies ($H^{(i)}$, $SO^{(i)}$), subsets of data $D^{(i)}$, procedures $P^{(i)}$, forecast chronicles $K_A^{(i)}$, as well as the results of previous processing $W^{(i-1)}$, which are to be realized in the $i$-th phase.

The notion of an operator $\$$ subordinating (for the $i$-th phase of the task) the set $Z_{out}^{(i)}$ to the set $Z_{in}^{(i)}$ is introduced by means of hardware-software reserves $H^{(i)}$ and $SO^{(i)}$, i.e. subordinates the set of results $W^{(i)}$ to the data $D^{(i)}$ and to the procedures $P^{(i)}$ and completes the chronicle $K_R^{(i)}$; that is:

$$S^{(i)} = \$\{Z_{in}^{(i)}, H^{(i)}, SO^{(i)}, Z_{out}^{(i)}\}$$

with

$$((\forall Z_{in}^{(i)} \in Z_{in})(\exists S^{(i)} \subset S)) \Leftrightarrow (Z_{in}^{(i)}\, S^{(i)}\, Z_{out}^{(i)})$$

$$Z_{(i)} = \bigcup_{i=1}^{\text{card } I} Z_{in}^{(i)}$$

$$S = \bigcup_{i=1}^{\text{card } I} S^{(i)}.$$

The above magnitude $S^{(i)}$ is otherwise called a functional configuration of the system for the $i$-th phase of the task.

It should be noted that in order to maintain continuous realization of particular phases of a task, the following equality must be fulfilled $Z_{in}^{(i+1)} = Z_{out}^{(i)}$.

The system failure state, $A_i$, during realization of the $i$-th phase takes place if and only if, in state $S_i$, the task $Z_{in}^{(i)}$ is being realized $W_R^{(i)}$, where there are differences from the assumed ones $W_A^{(i)}$

$$A_i \Leftrightarrow (S_i \wedge (W_A^{(i)} \neq W_R^{(i)}))$$

Let $e_i^{(k)}$ stand for the $k$-th inefficiency (error) in the $i$-th phase, renewable from the level of operational systems. The following takes place:

$$e_i^{(k)} \in E_i,$$

where: $E_i = \{e_i^{(1)}, e_i^{(2)}, \ldots, e_i^{(k)}, \ldots\}$ — detectable-by the diagnostic system — set of inefficiencies renewable during realization in the $i$-th phase of the task $k = \text{card } E_i$.

If the detected failure requires only informational renewal, then the loss time $t$ in the $i$-th phase is equal to the sum of the time needed to eliminate failure ($t_I$) and the duration $\bar{t}$ of the part of the task destroyed in case of failure:

$$(A_i \Rightarrow e_i^{(k)}) \Rightarrow (t = t_I + \bar{t})$$

If the detected failure $A_i$ requires renewal of hardware, then the loss time $t$ in the $i$-th phase is equal the sum of the time $t_I$ of all the attempts the system undertakes to eliminate the failure and the time $t_u$ of renewal of hardware, and the sum of the time $\bar{t}$ of the part of task destroyed by the failure:

$$(A_i \Rightarrow f_i^{(k)}) \Rightarrow (t = t_I + t_u + \bar{t}); \; f_i^{(k)} \in F_i$$

where: $f_i^{(k)}$ — $k$-th inefficiency of hardware supplies during the $i$-th phase realization,
$F_i$ — set of inefficiencies of hardware supplies during the $i$-th phase realization.

## 3. Functional model

Figure 3 presents the graph of states corresponding to the realization of successive phases of the task in an ideal case.

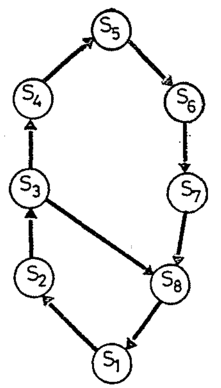


*Fig. 3.* Functional model of digital system

A task introduced into the device of the 1st level is initially processed (state of processing, $S_1$), then transmitted to the device of level no. 3 (state of transmission, $S_2$), and processed in the device of level no. 3 (state of processing $S_3$). Next, it is transmitted to the central device (state of transmission $S_4$) where it is processed (state of processing $S_5$), and then transmitted back to the device of level no. 3 (state of transmission $S_6$) processed in this device (state of processing $S_7$), and finally transmitted to the device of the first level (state of transmission, $S_8$). Additionally, we have considered the possibilities of the task transfers from level 3 to 1 without level 5.

Presentation of the task realization process in the form of a sequence of phases enables easy consideration of optional hierarchical structures with many levels.

## 4. Reliability-functional model

Each phase of the task is represented by 3 states: 1) state of correct realization of the $i$-th phase: $S_i$; 2) state of technical renewal of the device of the $i$-th level in the system structure: $U_i$; 3) state of informational renewal on the $i$-th level: $I_i$. The states in which both reliability and functional aspects have been taken into account are called reliability functional states.

Figure 4 presents the graph for transfers between particular reliability-functional states.

If the $i$-th phase of the task is performed correctly, then with probability $p_{i,\ i+1}$ the system will start realization of the $i+1$ phase, i.e. state $S_{i+1}$. Should, however, some incorrectness appear when realizing the $i$-th phase, this will be discovered with a probability equal to 1, and the system-with a probability $p_{i\ I}$ — will move to state $I_i$ of informational renewal. $N$ attempts at removing the inefficiency will be undertaken in state $I_i$. If the failure is caused by an error, then information renewal will take place, and the system — with a probability $p_{I.\ i}$ — will return to the phase of task realization. If, however, the attempts at removing the inefficiency should fail i.e., a device of the $i$-th phase has failed during the realization of the task, then the system — with probability $p_{i,u}$ will move to state $U_i$ of technical renewal, and after accomplishing it will return to the state of task realization with a probability $p_{u,i}$.

## 5. The throughput of the system

On the basis of the graph in Fig. 4 we obtain the matrix of **P** — probabilities of transfers between particular reliability-functional states of dimensions $n \times n$, where $n$ — number of states (in the graph of Fig. 4 the number of states $n=24$).

It is assumed that the $i$-th phase of the task is realized during time $\tau_i$.

As a reliability measure of the analysed digital system a throughput of the system $V$ has been proposed. It is defined in the following way:

$$V = \frac{E(N(t))}{E(N_0(t))}$$

where: $E(N(t))$ — expected number of tasks realized in an unreliable real system during fixed time $t$,

$E(N_0(t))$ — expected number of tasks realized in an ideal system in fixed time $t$.

It is assumed that the distributions of duration times of all the tasks are identical and the system works in stationary conditions.

Let $\pi_{si}$ $(\pi'_{si})$ be the stationary probability of state $S_i$ in the case of a real (ideal) system and $\pi_{Ii}$ $(\pi'_{ui}$ (be stationary probability of the state $I_i$ $(U_i)$ in the case of a real (unreliable) system.

Apart from this, the following denotations are introduced: $\tau_{si}$ ($\tau'_{si}$) mean time of the real (ideal) realization in the $S_i$ state in the real (ideal) system and $\tau_{Ii}$ ($\tau_{ui}$) — mean time wasted for informational (hardware) renewal in the $S_i$ state.

Throughput of the system $V$, with $t \to \infty$ is expressed by relation:

$$V = \frac{\Pi_{S1} - \Pi_{I1}}{\Pi'_{s1}} \cdot \frac{\sum\limits_{i=1}^{2} \tau'_{S1} \cdot \Pi'_{S1} + \sum\limits_{i=3}^{7} \frac{k+1}{2} \cdot \tau'_{si} \cdot \Pi'_{si} + \tau'_{S8} \cdot \Pi'_{S8}}{\sum\limits_{i=1}^{2} \Theta_i + \sum\limits_{i=3}^{7} \frac{k+1}{2} \cdot \Theta_i + \Theta_8}$$

where: $\Theta_i = \tau_{si} \cdot \Pi_{si} + \tau_{Ii} \cdot \Pi_{Ii} = \tau_{ui} \cdot \Pi_{ui}$; $i = \overline{1, 8}$

$k$ — number of terminals on the first level (Fig. 1).

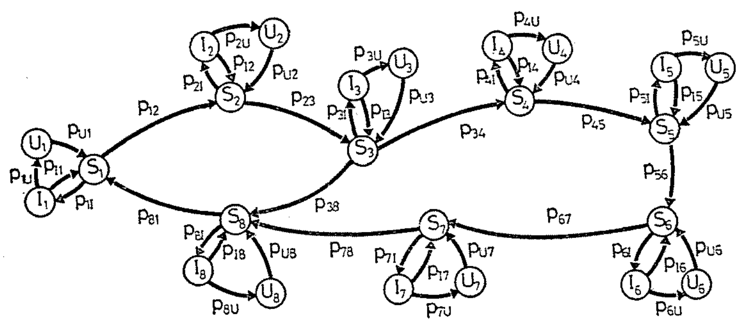

*Fig. 4.* Graph of the states of digital system.

## 6. Conclusion

The reliability-functional model of digital system of 5-level hierarchical structure has been presented. The model is intended for functional configuration realizing a single task. Due to the limited space of this paper, system model which would perform multiple tasks have not been discussed.

On the assumption that the reliability-functional model forms the basis of a further reliability analysis and in particular serves to determine the assumed reliability measure of the system, a way has been suggested to determine the assumed measure: the relative system throughput.

It should be noted that the presented model is a general one for hierarchical digital systems. It can be expanded taking into account some more detailed characteristics.

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