MONTHLY AVERAGE DAILY INSOLATION AND OPTIMUM TILT ANGLE OF SOLAR COLLECTOR IN P.D.R. YEMEN

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Received December 20, 1991

Abstract

The knowledge of the available solar irradiation is valuable for the design and assessment of solar energy conversion systems. For P.D.R. Yemen, an accurate prediction of solar irradiation is needed for designing solar systems to be used in the country. In this study an attempt has been made to generate theoretically the daily average insolation on horizontal and tilted surfaces. The optimum tilt angle of the surface is also determined. Different locations covering a wide range of latitude of the country have been studied. A simple mathematical model is used which is mainly based on the latitude, altitude and climatic conditions of the place. A computer program is developed so that the optimum tilt angle of the surface can be determined yearly and at a particular period (seasonally) at different places in the Republic.

Keywords: solar energy, insolation, flat-plate collector, optimum slope of flat-plate collector.

Introduction

The worldwide shortage of petroleum emphasizes the need for alternate energy sources, which are both inexpensive and clean. Among possible alternate energy sources, the most pollution-free limitless source is solar energy. Thus the use of solar energy for space heating, cooling and domestic hot water supply has received serious attention in recent years.

Flat-plate solar collectors are the most common device for intercepting solar heat at low temperature and have been extensively used for water and air heating.

However, under most circumstances the flat-plate collector is a fixed installation which has no tracking device to follow the sun. In order to maximize the absorbed energy, the collector plane should always be perpendicular to the sun rays. This cannot be achieved with a fixed installation, because the declination of the sun varies over the year. Hence one uses some value for the collector tilt angle which results in a reasonably high yearly or (part of the year) average of collected energy.

Extensive work has been done for determining the total solar insolation on a flat plate surface at any orientation and their optimum tilt [1–9] where different ways are discussed.

This paper presents firstly a survey of the monthly average daily insolation on tilted collectors for various locations in P.D.R.Y. The insolation calculation is based on clear sky radiation, HOTTLE [10], due to sparsity and shortage of actual sky radiation data for most of the locations.

A computer program has been developed to evaluate the monthly average daily insolation on a tilted surface facing south (i. e. azimuth angle is zero) at various tilt angles, where certain locations covering a wide range of latitude in P.D.R.Y. are considered in *Fig. 1.* By examining the density of insolation at different tilt angles, the optimum tilt of the collector is determined.

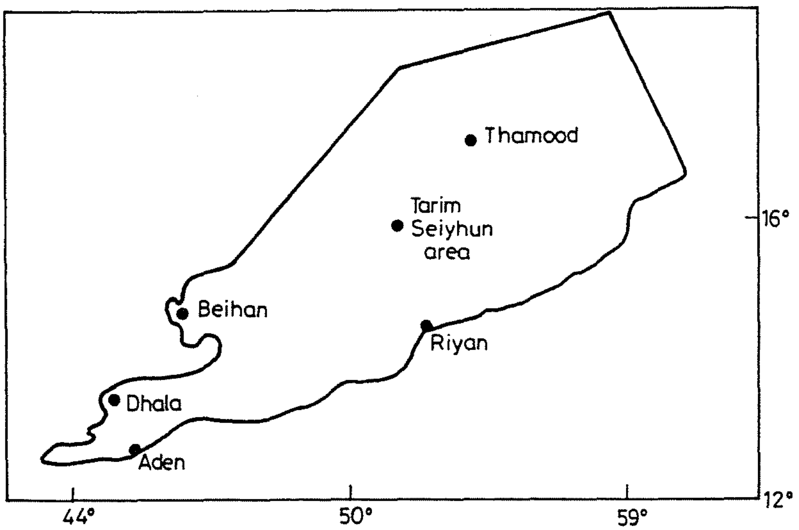

Fig. 1. Locations considered in P.D.R. Yemen

Analysis

Due to shortage of actual sky radiation data, a simple model is suggested for calculating the insolation on tilted surface.

HOTTLE [10] presented a convenient method for estimating the beam radiation under clear sky conditions in terms of zenith angle (Θz), latitude,

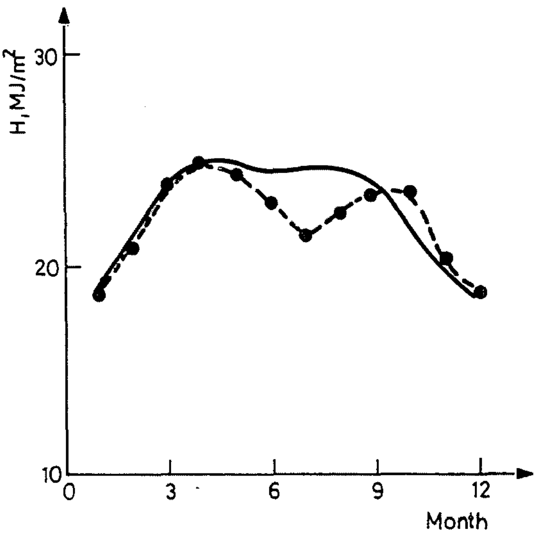

Fig. 2. Measured (— · —) and calculated (—) values of monthly global daily irradiance on a horizontal plane in Aden

and altitude of the site. The beam radiation (Ib) on the horizontal is indicated by:

$$Ib = Isc\left[1 + 0.033 \cdot \cos\left(360 \cdot \frac{N}{365}\right)\right] \cdot \left[a_0 + a_1 \cdot \exp\frac{-k}{\cos(\Theta z)}\right] \cdot \cos(\Theta z)\,, \tag{1}$$

where Isc is the solar constant (1353 W/m^2),
N is the day of the year,
Θz is the zenith angle which can be found from the following equation:

$$\cos(\Theta z) = \cos(\phi) \cdot \cos(\delta) \cdot \cos(w) + \sin(\phi) \cdot \sin(\delta)\,, \tag{1.1}$$

$$\delta = 23.45 \cdot \sin\left[360 \cdot \frac{284 + N}{365}\right]\,, \tag{1.2}$$

where ϕ, δ and w are the latitude, declination and hour angles in deg. respectively.

The coefficients are:

$$a_0 = r_0 \cdot a_0' \qquad \text{with} \qquad a_0' = 0.4237 - 0.00821(6 - A)^2\,, \tag{1.3}$$

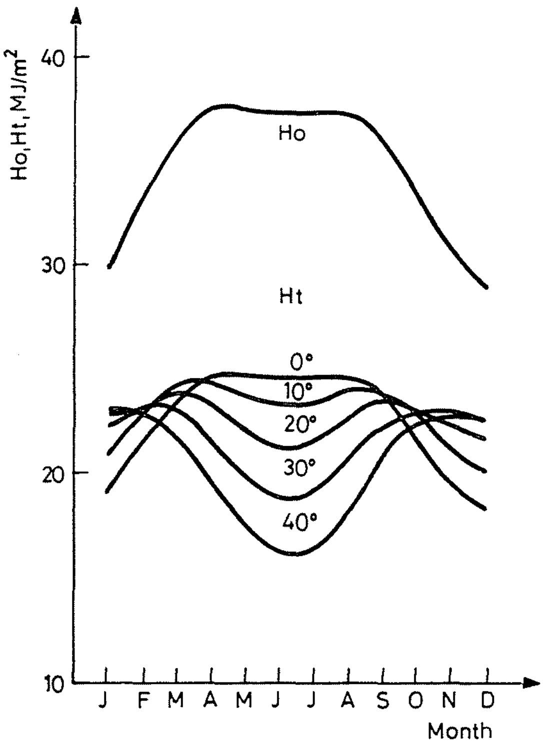

Fig. 3. Extraterrestrial radiation and monthly average daily radiation for different tilts in Aden

$$a_1 = r_1 \cdot a_1' \qquad \text{with} \qquad a_1' = 0.5055 + 0.001(6 - A)^2\,, \tag{1.4}$$

$$k = r_k \cdot k' \qquad \text{with} \qquad k' = 0.2711 + 0.01858(2.5 - A)^2\,, \tag{1.5}$$

where A is the elevation of the site above sea level in km, r_0, r_1, and r_k are the climate correction factors. Their values for tropical climate are 0.95, 0.98 and 1.02, respectively.

The clear sky diffuse irradiance (Id) on the horizontal surface can be estimated from a relation due to LIU and JORDAN [11]

$$Id = \left\{0.0271 \cdot \left[Isc\left(1 + 0.033 \cdot \cos\left(360 \cdot \frac{N}{365}\right)\right)\right] - 0.2939 \cdot Ib\right\} \cdot \cos(\Theta z)\,. \tag{2}$$

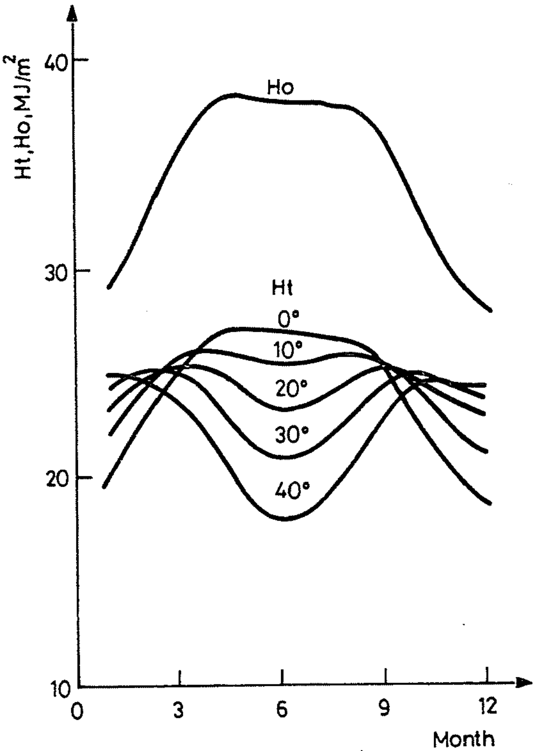

Fig. 4. Extraterrestrial radiation and monthly average daily radiation for different tilts in Tarim and Seyhun

Therefore the clear sky radiation (Ic) on a horizontal surface at any instant of time is:

$$Ic = Ib + Id. \tag{3}$$

The insolation on a horizontal surface at a required interval of time can be calculated by *Eq.* (3). While the global daily clear sky radiation (H), and the daily diffuse radiation (Hd) can be obtained by a numerical integration in steps of 15 minutes from sunrise to sunset that is:

$$H = \int_{w_{st}}^{w_{ss}} Ic \, \mathrm{d}w, \tag{4}$$

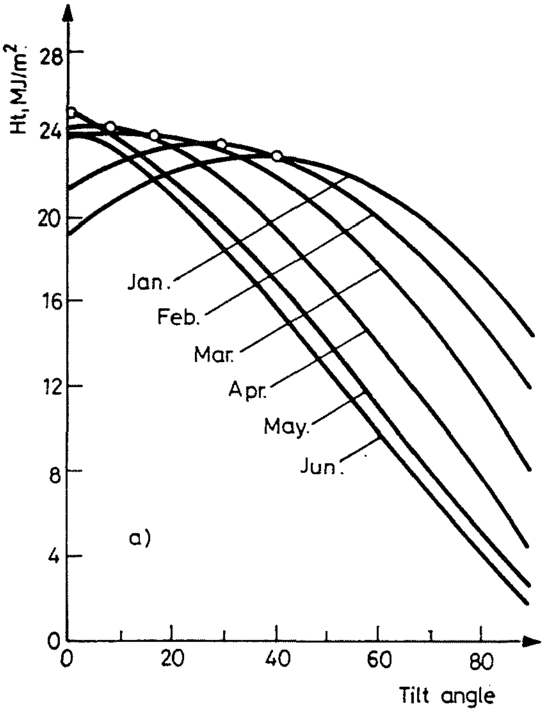

Fig. 5a. Monthly average daily insolation versus collector tilt from Jan. to Jun. in Aden

$$Hd = \int_{w_{sr}}^{w_{ss}} Id\,\mathrm{d}w\,, \tag{5}$$

where w_{sr} and w_{ss} are the sunrise and sunset hour angles in (deg).

The 15, or 16 of each month is mostly used for estimating the monthly average daily values (H), however, recommended days for which the extraterrestrial radiation is nearly the same as the monthly average [12]. These recommended days shown in *Table 1* are used in calculation of the daily insolation (H) and monthly average extraterrestrial radiation (Ho) in this paper.

The solar radiation incident upon a tilted surface (Ht) is composed of direct beam radiation, the sky diffuse radiation, and that part of radiation reflected from the ground.

$$Ht = H\left(1 - \frac{Hd}{H}\right) \cdot Rb + Hd\frac{1+\cos(\beta)}{2} + H \cdot Rg\frac{1-\cos(\beta)}{2}\,, \tag{6}$$

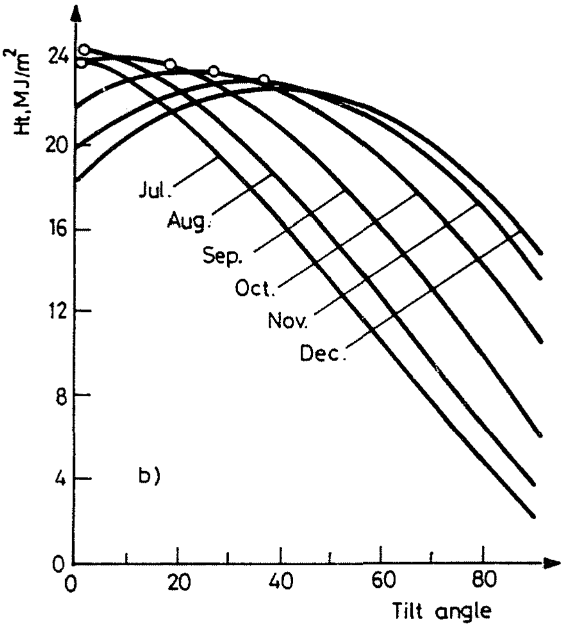

Fig. 5b. Monthly average daily insolation versus collector tilt from Jul. to Dec. in Aden

$$Rb = \frac{\cos(\phi-\beta)\cdot\cos(\delta)\sin(w_s') + \frac{\pi}{180}\cdot w_s'\cdot\sin(\phi-\beta)\cdot\sin(\delta)}{\cos(\delta)\cdot\sin(w_{ss}) + \frac{\pi}{180}\cdot w_{ss}\cdot\sin(\phi)\cdot\sin(\delta)}, \qquad (7)$$

$$w_s' = \min\left[w_{ss}, \arccos\left(\tan(\phi-\beta)\cdot\tan(\delta)\right)\right], \qquad (8)$$

$$w_{ss} = \arccos\left(\tan(\phi)\cdot\tan(\delta)\right), \qquad (9)$$

where Rb is the monthly mean tilt factor (dimensionless)
β is the collector tilt angle (deg.).
Rg = 0.2 is the ground reflectance (dimensionless).

Results and Discussion

The aim of this paper is to find the optimum slope of a flat-plate collector at different locations in P.D.R.Y. The insolation calculation is based on a simple model developed by HOTTLE [10].

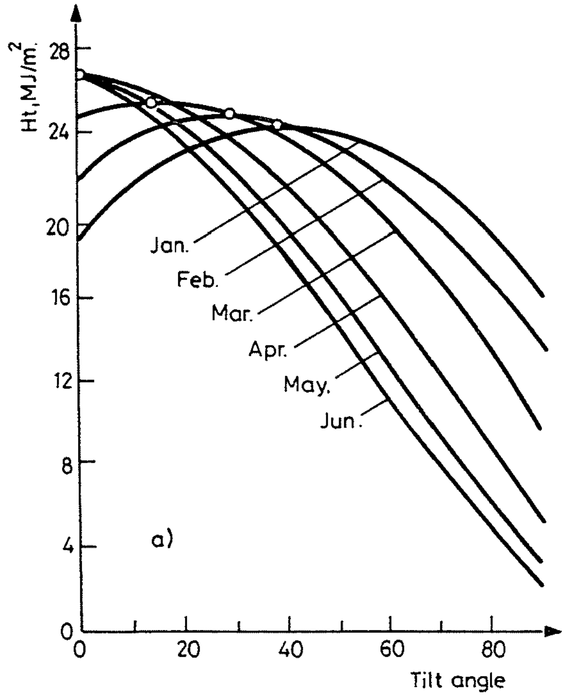

Fig. 6a. Monthly average daily insolation versus collector tilt from Jan. to Jun. Tarim and Seyhun area

Fig. 2 shows the measured and calculated values of the daily global radiation on a horizontal plane for Aden [13] (the only place where several back years data are available). The deviation between the measured and calculated values seems to be small, in other words, this unrecognizable deviation (less than 10% error) supports our suggestion of applying this model for the estimation of Ht for different locations in P.D.R.Y. where no actual data is available. In addition, its accuracy is fairly sufficient for the purpose of specifying the optimum tilt.

The monthly average daily insolation data at various tilt ($\beta = 0$ to 40 deg.) for a number of locations *Tables 2–7, Figs 3,* and *4* showing the clear effect of the tilt angle (β) on the values of insolation Ht (coastal and inland locations) are considered as an example.

Monthly average daily insolation (Ht) versus collector tilt angle are shown in *Figs. 5,* and *6,* the optimum tilt angle is marked with an x to show the optimum tilt at each month.

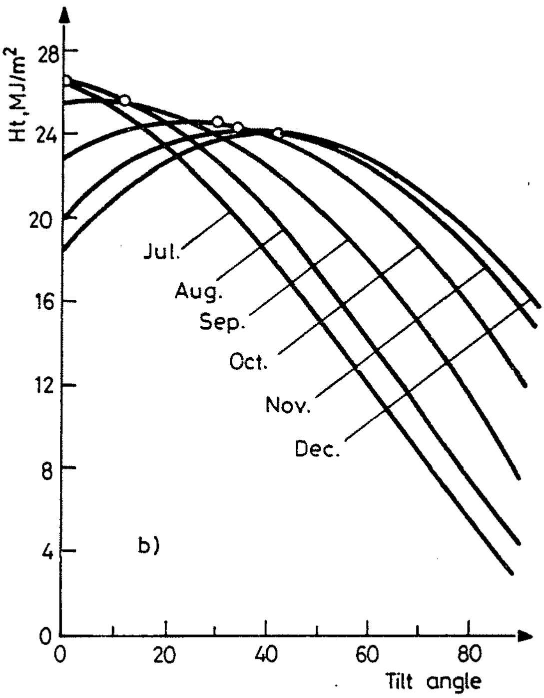

Fig. 6b. Monthly average daily insolation versus collector tilt from Jul. to Dec. Tarim and Seyhun area

When a collector is a fixed installation, usually be concerned with a tilt angle to intercept maximum solar energy for a year or at a particular part of the the year, rather than the optimum tilt angle for each month.

Here, in this paper the optimum tilt (β) is studied on the basis of two arguments, one according to the maximum energy received yearly, and due to this it is found that the maximum energy is received when the slope is 10 deg. from the horizontal facing south for all the locations (see *Tables 2–7* and *Figs. 9–10*).

In the other case the optimum tilt is found on the basis of the seasonal variation of the insolation where the monthly average daily insolation is calculated at different tilt angles 0, 10, 20, 30 and 40 deg. (*Figs. 7–10*). It can be seen that for winter heating, the optimum tilt is 30 deg. from the horizontal, in the sense that it collects maximum insolation at that period (Oct. – Mar.), whereas a horizontal collector is the optimum (i. e. $\beta = 0$) for summer cooling (Apr. – Sep.).

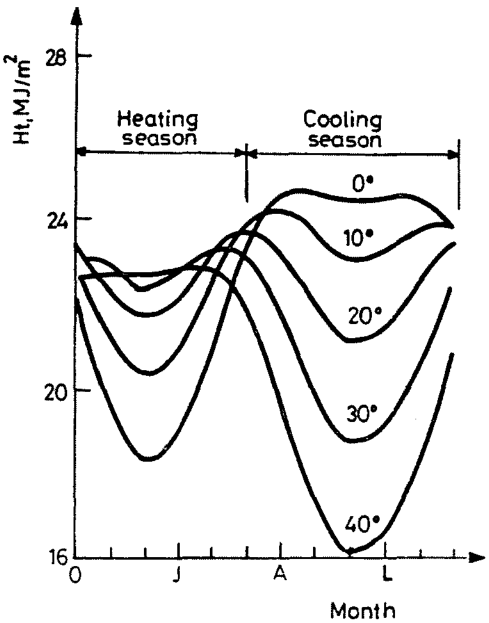

Fig. 7. Seasonal variation of insolation for different tilt angles in Aden

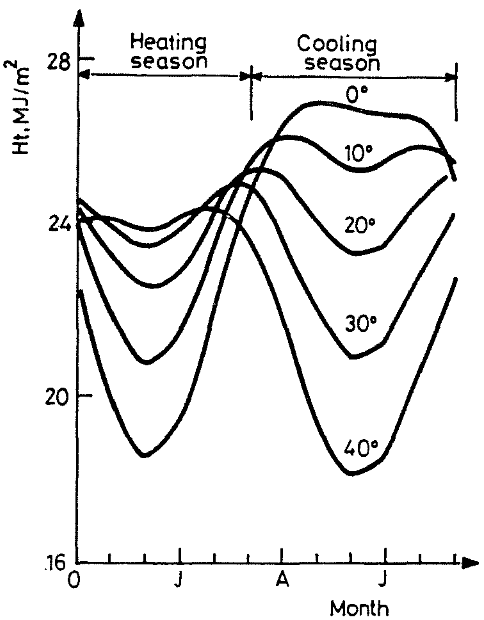

Fig. 8. Seasonal variation of insolation for different tilt angles in Tarim and Seyhun area

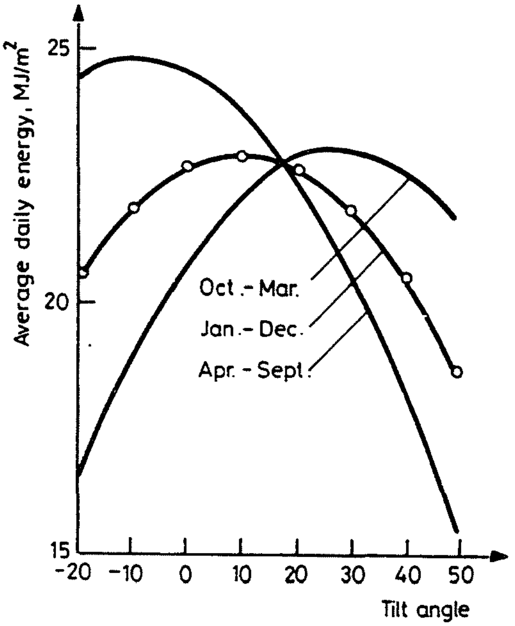

Fig. 9. Mean average daily energy intercepted at particular periods in Aden

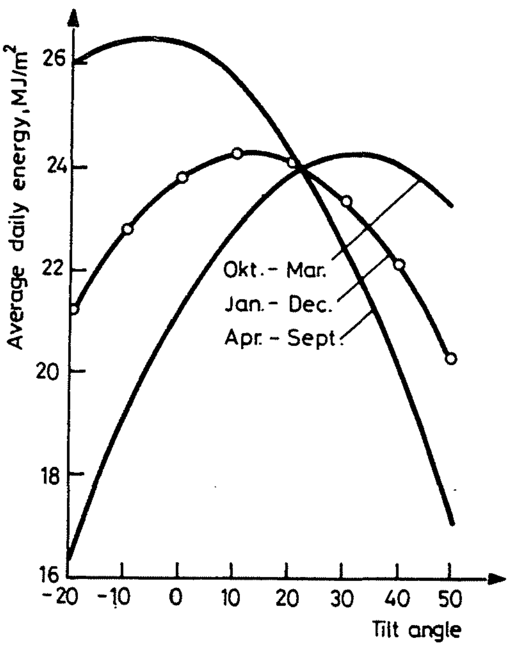

Fig. 10. Mean average daily energy intercepted at particular periods in Tarim and Seyhun area

Table 1
Recommended days for calculating the monthly average daily radiation

Month	Day of the year	Date
Jan.	17	17 Jan.
Feb.	47	16 Feb.
Mar.	75	16 Mar.
Apr.	105	15 Apr.
May	135	15 May
Jun.	162	11 Jun.
Jul.	198	17 Jul.
Aug.	228	16 Aug.
Sep.	258	15 Sep.
Oct.	288	15 Oct.
Nov.	318	14 Nov.
Dec.	344	10 Dec.

Table 2
Monthly average daily insolation (MJ/m^2 day)

Aden (12 50 N°)											
		$\beta = 0$		$\beta = 10$		$\beta = 20$		$\beta = 30$		$\beta = 40$	
Month	*Ho*	*Rb*	*H*	*Rb*	*Ht*	*Rb*	*Ht*	*Rb*	*Ht*	*Rb*	*Ht*
Jan.	30.31	1	19.2	1.14	20.9	1.25	22.2	1.32	22.9	1.35	23.0
Feb.	33.02	1	21.2	1.10	20.6	1.16	23.4	1.19	23.5	1.18	23.1
Mar.	36.03	1	23.6	1.04	24.1	1.04	24.0	1.02	23.3	0.96	22.0
Apr.	37.65	1	24.8	0.98	24.4	0.93	23.4	0.85	21.8	0.75	19.7
May	37.76	1	24.9	0.93	23.7	0.84	22.1	0.73	19.9	0.60	17.3
Jun.	37.43	1	24.5	0.91	23.1	0.81	21.2	0.68	18.8	0.54	16.1
Jul.	37.42	1	24.6	0.92	23.3	0.82	21.5	0.70	19.2	0.56	16.6
Aug.	37.46	1	24.7	0.96	24.0	0.89	22.7	0.80	20.9	0.68	18.6
Sep.	36.43	1	23.9	1.01	24.1	0.99	23.7	0.95	22.6	0.87	20.9
Oct.	33.90	1	21.9	1.07	23.1	1.12	23.5	1.12	23.4	1.10	22.6
Nov.	30.91	1	19.6	1.13	21.3	1.23	22.5	1.29	22.9	1.31	22.9
Dec.	29.34	1	18.4	1.16	20.3	1.28	21.7	1.37	22.5	1.41	22.7
Annual Total (MJ/m^2 year)			271.7		275.2		271.8		261.9		245.7

Table 3
Monthly average daily insolation (MJ/m^2 day)

Tarim & Seyhun Area (15 59 N°)											
		$\beta = 0$		$\beta = 10$		$\beta = 20$		$\beta = 30$		$\beta = 40$	
Month	*Ho*	*Rb*	*H*	*Rb*	*Ht*	*Rb*	*Ht*	*Rb*	*Ht*	*Rb*	*Ht*
Jan.	28.07	1	19.6	1.16	21.7	1.28	23.3	1.36	24.2	1.40	24.4
Feb.	32.05	1	22.0	1.11	23.6	1.18	24.6	1.22	24.9	1.22	24.7
Mar.	35.52	1	24.8	1.04	25.5	1.06	25.6	1.04	25.0	0.99	23.7
Apr.	37.67	1	26.5	0.99	26.2	0.94	25.3	0.87	23.7	0.78	21.5
May	38.22	1	26.9	0.94	25.8	0.86	24.1	0.75	21.8	0.63	19.1
Jun.	38.08	1	26.7	0.92	25.3	0.82	23.3	0.70	20.7	0.56	17.8
Jul.	37.99	1	26.7	0.93	25.4	0.84	23.5	0.72	21.1	0.59	18.3
Aug.	37.67	1	26.5	0.97	25.9	0.91	24.6	0.82	22.7	0.71	20.3
Sep.	36.14	1	25.3	1.02	25.7	1.01	25.3	0.97	24.3	0,90	22.7
Oct.	33.10	1	22.9	1.08	24.2	1.14	24.9	1.15	24.9	1.13	24.2
Nov.	29.75	1	20.2	1.14	22.2	1.25	23.5	1.32	24.3	1.35	24.4
Dec.	28.03	1	18.8	1.17	21.0	1.31	22.7	1.41	23.8	1.46	24.2
Annual Total (MJ/m^2 year)			287.2		292.5		290.7		281.4		265.4

Table 4
Monthly average daily insolation (MJ/m^2 day)

Riyan (14 39 N°)											
		$\beta = 0$		$\beta = 10$		$\beta = 20$		$\beta = 30$		$\beta = 40$	
Month	*Ho*	*Rb*	*H*	*Rb*	*Ht*	*Rb*	*Ht*	*Rb*	*Ht*	*Rb*	*Ht*
Jan.	29.4	1	18.5	1.15	20.4	1.27	21.7	1.35	22.5	1.39	22.7
Feb.	32.3	1	20.7	1.11	22.2	1.18	23.1	1.21	22.3	1.21	22.9
Mar.	35.7	1	23.4	1.04	23.9	1.05	23.4	1.03	23.4	0.98	22.2
Apr.	37.7	1	24.9	0.98	24.6	0.94	24.9	0.87	22.1	0.77	20.1
May	38.1	1	25.2	0.94	24.1	0.86	25.2	0.75	20.4	0.62	17.9
Jun.	37.9	1	25.0	0.92	23.6	0.82	21.7	0.69	19.4	0.56	16.7
Jul.	37.8	1	24.9	0.93	23.7	0.83	21.9	0.72	19.8	0.58	17.1
Aug.	37.6	1	24.9	0.97	24.3	0.90	23.0	0.81	21.3	0.70	19.0
Sep.	36.2	1	23.9	1.02	24.1	1.01	23.7	0.96	22.8	0,89	21.2
Oct.	33.3	1	21.6	1.08	22.7	1.13	23.3	1.14	23.2	1.12	22.6
Nov.	30.1	1	19.1	1.14	20.8	1.24	22.0	1.31	22.6	1.34	22.7
Dec.	28.4	1	17.8	1.17	19.3	1.30	21.2	1.40	22.1	1.45	22.4
Annual Total (MJ/m^2 year)			269.8		274.1		271.8		262.8		247.4

Table 5
Monthly average daily insolation (MJ/m² day)

	Thamood (17 22 N°)										
	$\beta = 0$			$\beta = 10$		$\beta = 20$		$\beta = 30$		$\beta = 40$	
Month	*Ho*	*Rb*	*H*	*Rb*	*Ht*	*Rb*	*Ht*	*Rb*	*Ht*	*Rb*	*Ht*
Jan.	28.04	1	18.6	1.17	20.8	1.30	22.4	1.39	23.4	1.44	23.7
Feb.	31.23	1	21.2	1.12	22.8	1.20	23.9	1.25	24.4	1.26	24.2
Mar.	35.07	1	24.2	1.05	25.1	1.07	25.2	1.06	24.7	1.01	23.6
Apr.	37.64	1	26.3	0.99	26.1	0.96	25.3	0.89	23.7	0.80	21.7
May	38.54	1	26.9	0.95	25.9	0.87	24.3	0.77	22.1	0.65	19.5
Jun.	38.57	1	26.8	0.93	25.5	0.83	23.6	0.72	21.2	0.58	18.3
Jul.	38.40	1	26.7	0.94	25.6	0.85	23.8	0.74	21.5	0.61	18.8
Aug.	37.79	1	26.4	0.97	25.9	0.92	24.7	0.84	22.9	0.73	20.6
Sep.	35.85	1	24.9	1.03	25.3	1.02	25.1	0.99	24.2	0.92	22.7
Oct.	32.42	1	22.2	1.09	23.5	1.15	24.3	1.18	24.4	1.16	23.9
Nov.	28.78	1	19.3	1.15	21.3	1.27	22.7	1.35	23.6	1.39	23.8
Dec.	26.95	1	17.8	1.18	20.0	1.33	21.8	1.44	22.9	1.51	23.4
Annual Total (MJ/m² year)			281.4		287.7		286.9		278.9		264.1

Table 6
Monthly average daily insolation (MJ/m² day)

	Dhala (13 44 N°)										
	$\beta = 0$			$\beta = 10$		$\beta = 20$		$\beta = 30$		$\beta = 40$	
Month	*Ho*	*Rb*	*H*	*Rb*	*Ht*	*Rb*	*Ht*	*Rb*	*Ht*	*Rb*	*Ht*
Jan.	29.87	1	21.3	1.15	23.5	1.26	25.1	1.34	26.0	1.37	26.2
Feb.	32.68	1	23.6	1.10	25.3	1.17	26.3	1.20	26.5	1.20	26.1
Mar.	35.86	1	26.3	1.04	26.9	1.05	26.9	1.02	26.2	0.97	24.9
Apr.	37.66	1	27.7	0.98	27.3	0.93	26.2	0.86	24.5	0.76	22.1
May	37.93	1	27.9	0.94	26.7	0.83	24.7	0.74	22.3	0.61	19.4
Jun.	37.67	1	27.7	0.92	26.0	0.81	23.8	0.69	21.1	0.55	18.0
Jul.	37.63	1	27.7	0.93	26.2	0.83	24.1	0.71	21.6	0.57	18.5
Aug.	37.54	1	27.7	0.96	26.8	0.90	25.5	0.81	23.4	0.69	20.9
Sep.	36.33	1	26.7	1.02	26.9	1.01	26.5	0.96	25.4	0.88	23.6
Oct.	33.63	1	24.5	1.08	25.7	1.12	26.3	1.13	26.3	1.11	25.5
Nov.	30.50	1	21.9	1.13	23.9	1.24	25.3	1.30	26.0	1.32	26.1
Dec.	28.87	1	20.5	1.16	22.8	1.29	24.6	1.38	25.7	1.43	26.0
Annual Total (MJ/m² year)			303.4		308.3		305.4		294.9		277.2

Table 7
Monthly average daily insolation (MJ/m^2 day)

	Beihan (14 47 N°)										
		$\beta = 0$		$\beta = 10$		$\beta = 20$		$\beta = 30$		$\beta = 40$	
Month	*Ho*	*Rb*	*H*	*Rb*	*Ht*	*Rb*	*Ht*	*Rb*	*Ht*	*Rb*	*Ht*
Jan.	29.35	1	20.6	1.15	22.8	1.27	24.4	1.35	25.3	1.39	25.5
Feb.	32.27	1	22.9	1.11	24.6	1.18	25.6	1.21	25.6	1.21	25.6
Mar.	35.64	1	25.7	1.04	26.4	1.05	26.5	1.03	25.8	0.98	24.5
Apr.	37.67	1	27.4	0.98	27.0	0.94	26.0	0.87	24.3	0.77	22.0
May	38.12	1	27.7	0.94	26.5	0.86	24.7	0.75	22.3	0.62	19.5
Jun.	37.94	1	27.5	0.92	25.9	0.82	23.8	0.70	21.2	0.56	18.1
Jul.	37.87	1	27.4	0.93	26.1	0.83	24.1	0.72	21.6	0.59	18.6
Aug.	37.63	1	27.3	0.97	26.6	0.90	25.3	0.82	23.3	0.70	20.8
Sep.	36.21	1	26.2	1.02	26.5	1.01	26.1	0.97	25.1	0.89	23.4
Oct.	33.29	1	23.8	1.08	25.8	1.15	25.2	1.13	25.8	1.13	25.1
Nov.	30.01	1	21.2	1.14	23.2	1.25	24.6	1.31	25.4	1.34	25.5
Dec.	28.33	1	19.7	1.17	22.1	1.30	23.8	1.40	24.9	1.45	25.3
Annual Total (MJ/m^2 year)			297.6		302.9		300.7		290.9		274.1

Conclusion

The model can be used with fairly good accuracy, to serve the purpose of specifying the optimum slope of a number of locations in P.D.R.Y. where no actual data is available.

The yearly maximum energy is received when the collector tilt is 10 deg. for all locations considered.

The optimum tilt for considering seasonal basis is 30 deg. for heating season (Oct. – Mar.), and mostly 0 deg. for cooling season (Apr. – Sep.).

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