GENERALIZATION OF THE UP-TO DATE THEORY OF PLASTICITY FOR THE DESCRIPTION OF CREEP STRAIN

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Abstract

The papers [4,11] contain the description of the synthesis of slip conception and flow theory in accordance with the ideas of Koiter [7]. The present work contains comments to the mentioned [4,11] up-to-date theory of plasticity and its generalization for the description of creep strain.

Keywords: slip and flow theory, plasticity, creep strain.

Transformation of Tresca Surface

The Batdorf–Budiansky theory is greatly based on the Tresca condition of plasticity

$$\tau_{\max} = \tau_s, \tag{1}$$

that is, the maximum of shear stress is equal to the yield shear stress.

The new method of construction of plasticity surface is discussed below. For this purpose the plane which passes through the given point is considered. We define the plane by unit normal $\vec{n}$. The unit direction $\vec{l}$ is studied in the stated plane.

We mark components of the mentioned vectors in some Descartes coordinate system $oxyz$ in 3 - dimensional space in which the body is loaded by n_x, n_y, n_z and l_x, l_y, l_z, respectively.

The conditions of orthonormalization are

$$lx^2 + ly^2 + lz^2 = 1; \quad n_x^2 + n_y^2 + n_z^2 = 1; \tag{2}$$

$$l_x n_x + l_y n_y + l_z n_z = 0$$

As the 6 values $l_x, \dots, n_z$ are connected by relations (2), these values could be expressed through α, β and ω. It could be two spherical coordinates α

and β of vector $\vec{n}$ and angle (ω). The last expresses direction of axis $\vec{l}$ in the given plane [9,10].

The component of tangential load in system $\vec{n}$, $\vec{l}$ is denoted by τ_{nl}

$$\tau_{nl} = \sigma_{ij} l_i n_j. \tag{3}$$

The summation is taken with respect to i, j; $i, j = x, y, z$. Let's take into consideration the 5-dimensional space of Ilyushin [6], with Descartes coordinate system. The components of load vector S_k are on the axes of the stated system. These components are

$$\begin{aligned} &S_1 = \frac{3}{2}(S_{zz} + S_{xx}); \quad S_2 = \frac{\sqrt{3}}{2}(S_{zz} - S_{xx}); \\ &S_3 = \sqrt{3} S_{xz}; \quad S_4 = \sqrt{3} S_{xy}; \quad S_5 = \sqrt{3} S_{yz}, \end{aligned} \tag{4}$$

where $S_{xx}, \dots, S_{zz}$ - components of deviator of tensor σ_{ij}. According to these components we rewrite formula (3) as

$$\begin{aligned} &\tau_{nl} = (l_x n_x + l_z n_z) S_1 + \frac{1}{\sqrt{3}}(-l_x n_x + l_z n_z) S_2 + \\ &\frac{1}{\sqrt{3}}(l_x n_z + l_z n_x) S_3 + \frac{1}{\sqrt{3}}(l_x n_y + l_y n_x) S_y + \frac{1}{\sqrt{3}}(l_y n_z + l_z n_y) S_5. \end{aligned} \tag{5}$$

Due to the fact that τ_{nl} is the smooth function of angles α, β and ω, the correlations (1) are equivalent to the equations

$$\frac{\partial \tau_{nl}}{\partial \alpha} = \frac{\partial \tau_{nl}}{\partial \beta} = \frac{\partial \tau_{nl}}{\partial \omega} = 0; \quad \tau_{nl} = \tau_S. \tag{6}$$

In terms of (5) the last correlations from (6) could be transformed to the form

$$\begin{aligned} &\sqrt{3}(l_x n_x + l_z n_z) S_1 + (-l_x n_x + l_z n_z) S_2 + (l_x n_z + l_z n_x) S_3 + \\ &+ (l_x n_y + l_y n_z) S_y + (l_y n_z + l_z n_y) S_5 = \sqrt{3} \tau_S. \end{aligned} \tag{7}$$

Under fixed given values of directing cosines $l_x, \dots, n_z$ the *Eq*(7) is the equation of a plane in space of stresses.

For the definition of its orientation we turn to (6). Relations (6) we treat as a system of equations for finding $S_1, \dots, S_5$. Usually, under the known stress, we look for orientation of $\vec{l}$, $\vec{n}$ system, in which the tangential stress is maximal. Now we are solving the reverse task: $l_x, \dots, n_z$ are given, and we are looking for such a stress condition, when in the given system $\vec{l}$, $\vec{n}$ the tangential stress is maximal.

Solution of the named system satisfies the *Eq.* (1), as the first three equalities from (6) formulate condition of maximal τ_{nl}, and the last equality under $\tau_{nl} = \tau_{\max}$ is in agreement with (1). That solution is in agreement also with the plane *Eq.* (7), as it is evaluated of (7). Thus, the plane (7) and surface (1) have the common point — the plane (7) is tangent to surface (1).

Altering the defining cosines $l_x, \ldots, n_z$, we obtain various planes (7), they all are tangent to (1). Thus, the Tresca surface of plasticity could be treated as passing over the set of tangential planes (7).

We could have doubts about the mentioned idea because of the following: in 5-dimensional space, the set of all planes is depending on four parameters (less by one than the dimension of space). Planes (7) depend only on the three angles α, β and ω. We notice that solution of the equation system (6) is ambiguous as we have four *Eq.* (6) with five $(S_1, \ldots, S_5)$ unknowns. It means that plane (7) is tangent to surface (1) at the same line (not at point), which substitutes the fourth parameter. As an example we can consider the cylinder or cone, where the tangent plane is tangent to them at all their surface.

Further we assume [4,7,11] that plane (7) could move parallel with itself. For a stressed body the vector of stresses moves on these planes, to which it is tangent. We assume that this movement will lead to plastic strains, normal to plane and dependent on value of its displacement. Macrostrains are equal to sum of elementary translations of set of planes. Such an assumption results in the well-known formula of Batdorf–Budiansky for plastic strain components [4,7,11].

Named results drive to the simplification of Budiansky formula: it is necessary to change the condition of plasticity (1). We shall discuss this question in detail.

Let us consider 3-dimensional subspace S_1, S_2, S_3 of 5-dimensional space of stresses $S_1, S_2, \ldots, S_5$. In this subspace the track of tangent plane will be described by the equation

$$\begin{aligned}&\sqrt{3}(l_x n_x + l_z n_z)S_1 + (-l_x n_x + l_z n_z)S_2 + \\ &+ (l_x n_z + l_z n_x)S_3 = \sqrt{3}\tau_S.\end{aligned} \tag{8}$$

Based on formula (1) it is easy to prove that normalizing multiplier (d) for plane (7) is equal to one and for its track (8) we obtain

$$d = \sqrt{1 - l_y^2 - n_y^2 + 4l_y^2 n_y^2}, \tag{9}$$

with

$$0 \leq d \leq 1. \tag{10}$$

Let's write *Eq.* (8) through the characteristics of the considered 3-dimensional subspace. Normed equation of a plane is presented in a form

$$m_1S_1 + m_2S_2 + m_3S_3 = h_0, \tag{11}$$

where h_0 – distance between the origin of coordinates and plane (11); m_1, m_2 and m_3 — directing cosines of normal $\vec{m}$ to it. They could be

$$m_1 = \cos\hat{\alpha}\cos\hat{\beta}, \quad m_2 = \sin\hat{\alpha}\cos\hat{\beta}, \quad m_3 = \sin\hat{\beta}. \tag{12}$$

The angles $\hat{\alpha}$, $\hat{\beta}$ are shown in *Fig. 1.* The planes (8) and (11) are identical, that's why

$$m_1 = \frac{\sqrt{3}(l_xn_x + l_zn_z)}{d}, \quad m_2 = \frac{-l_xn_x + l_zn_z}{d}, \tag{13}$$

$$m_3 = \frac{l_xn_z + l_zn_x}{d}.$$

Moreover

$$h_0 = \frac{\sqrt{3}\tau_S}{d}. \tag{14}$$

From that and inequalities (10) it follows that distance h_0 could take the value

$$\sqrt{3}\tau_s \leq h_0 \leq \infty. \tag{15}$$

Formulas (13) and (14) with the correlations (2) establish connection between parameters $l_x, \ldots, n_z$ (or α, β, ω), and m_1, m_2, m_3, h_0 (or $\hat{a}$, $\hat{\beta}$, h_0). Certain values $m_1, m_2, \ldots, m_5$ are in agreement with certain values of $l_x, \ldots, n_z$, and value of distance h_0 from interval (15). It means that optional plane of the S_1, S_2, S_3 subspace with given distance h_0 is tangent to a Tresca surface of plasticity in a 5-dimensional space of stresses $S_1, \ldots, S_5$.

Further instead of distance h_0 we introduce a new value — λ –the angle between normal $\vec{N}$ to plane (7) and normal $\vec{m}$ to track of a plane (8) or (11). We obtain

$$\cos\lambda = \vec{N} \cdot \vec{m}. \tag{16}$$

where dot between the vectors means their scalar multiplication. From formulae (7), (8) and (16) we obtain $\cos\lambda = d$, which means that the desired correlation on the basis of formula (14) is presented as

$$h_0 \cos\lambda = \sqrt{3}\tau_S. \tag{17}$$

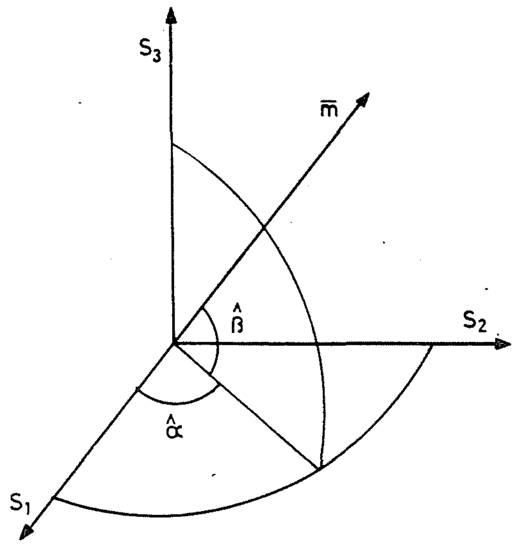

Fig. 1.

Fig. 2 illustrates the *Eq.* (17). *Fig. 1* presents part of plasticity surface, 2 — plane which is tangent to it, 3 — the track of named plane; $OA = \sqrt{3}\tau_S$, $OB = h_0$, O — the origin of coordinates.

As the connection exists between parameters $S_1, \ldots, S_5$ and $\hat{\alpha}$, $\hat{\beta}$, $\hat{\lambda}$, the plane *Eq.* (7) could be presented through $\hat{\alpha}$, $\hat{\beta}$, $\hat{\lambda}$ or m_1, m_2, m_3, λ. On the basis of formulae (2), (9), (12), (13) and (17) we have [4]

$$\begin{aligned} &S_1 m_1 \cos\alpha + S_2 m_2 \cos\beta + S_3 m_3 \cos\alpha \pm \\ &\pm S_4\sqrt{\frac{\sin^2\alpha}{2} + x} \pm S_5\sqrt{\frac{\sin^2\alpha}{2} - x} = \sqrt{3}\tau_S, \end{aligned} \tag{18}$$

where $x = x(\hat{\alpha}, \hat{\beta}, \hat{\lambda})$. In [4] formula for $x(\hat{\alpha}, \hat{\beta}, \hat{\lambda})$ is presented; we do not need it and do not draw it.

Further we have the main simplifying assumption. The Tresca surface, which bends around the set of planes (18), is replaced by a new surface, which bends around the next set of planes:

$$\begin{aligned} &S_1 m_1 \cos\alpha + S_2 m_2 \cos\alpha + S_3 m_3 \cos\alpha \pm \\ &\pm S_4\sqrt{\frac{\sin^2\alpha}{2} + x_0(\alpha)} \pm S_5\sqrt{\frac{\sin^2\alpha}{2} - x_0(\alpha)} = \sqrt{3}\tau_S. \end{aligned} \tag{19}$$

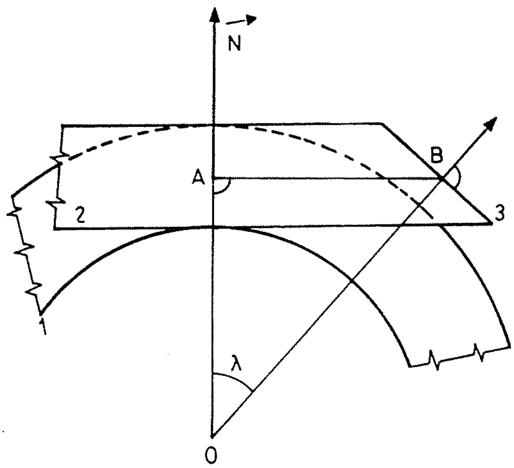

Fig. 2.

The difference between *Eqs* (18) and (19) is that the function of three variables $x(\hat{\alpha}, \hat{\beta}, \lambda)$ in (19) is replaced by function of one variable $x_0(\lambda)$.

Formula $x_0(\lambda)$ is used in [4,11] but now it is unnecessary. Replacement of x by x_0 substantially simplifies the formula for components of plastic strain. Moreover, it permits to present in explicit form the plasticity surface in 3-dimensional subspace $S_1S_2S_3$. In fact, conditions (6) are replaced by

$$\frac{\partial\phi}{\partial\hat{\alpha}} = 0; \quad \frac{\partial\phi}{\partial\hat{\beta}} = 0; \quad \frac{\partial\phi}{\partial\lambda} = 0, \tag{20}$$

where ϕ is the left part of formula (19). The first two *Eqs* (20) cause

$$\text{tg } \hat{\alpha} = \frac{S_2}{S_1}; \quad \text{tg } \hat{\beta} = \frac{S_3}{S_1 \cos\hat{\alpha} + S_2 \sin\hat{\alpha}}, \tag{21}$$

and from the third *Eq* (20) under conditions $S_4 = S_5 = 0$ we obtain

$$S_1^2 + S_2^2 + S_3^2 = 3\tau_S^2. \tag{22}$$

Thus, there is such a surface enveloped by the set of planes (19) that in subspace S_1, S_2, S_3 it coincides with the Huber–Mises plasticity surface. Notice that transformed plasticity condition in a 5-dimensional space does not coincide either with criterion of Tresca or with criterion of Huber–Mises.

Components of Plastic Strain

Let's consider the case when components of tangential stress τ_{xy} and τ_{yz} are equal to zero, i.e. in accordance with formula (4) we obtain $S_4 = S_5 = 0$.

Consider the stress vector

$$\vec{S} = S_i \cdot \vec{e}_i, \tag{23}$$

where e_i–unit vectors, axes — directed. As mentioned, plastic strain is caused by movement of tangent planes (19) of 5-dimensional space, and those planes to which vector (23) is tangent. It is located in subspace S_1, S_2, S_3 ($S_4 = S_5 = 0$), thus tangents takes place to tracks of planes (11) in given subspace. The tracks of planes (11) not only bend around sphere (23), but fill all the space $S_1S_2S_3$ outside the sphere. The set of planes, which are tangent to named sphere in the range of angles $\hat{\alpha}$ and $\hat{\alpha} + d\hat{\alpha}$, $\hat{\beta}$ and $\hat{\beta} + d\hat{\beta}$, could be specified by solid angle equal to $\cos\hat{\beta}d\hat{\alpha}d\hat{\beta}$.

The set of planes, which are parallel to the named one and the distance from which to the origin of coordinates is in interval h_0, $h_0 + dh_0$, is characterized by elementary 'volume'

$$dv = \cos\beta d\alpha d\beta q(h_0)dh_0, \tag{24}$$

or

$$dv = \cos\beta d\alpha d\beta d\lambda\ . \tag{25}$$

It is easy to obtain from formula (17) that

$$q(h_0) = \frac{\sqrt{3}\tau_S}{h_0\sqrt{h_0^2 - 3\tau_s^2}}, \tag{26}$$

Further we shall use Eq (25).

This displacement or planes with 'volume' dv leads to elementary plastic strain $d\bar{\varepsilon}$, directed normally $\overline{N}$ to them and dependent on volume of these displacement, i.e.

$$d\vec{\varepsilon} = \vec{N}F(H_S)dv, \tag{27}$$

where F–characteristic function of material, H_S–the distance from plane to the origin of coordinates. As the initial distance is equal to $(\sqrt{3}\tau_S)$ for all planes H_S characterizes the named displacement.

The components of general plastic strain, caused by displacement of planes with different normals $\vec{N}$, could be stated as follows;

$$\varepsilon_K = \iiint\limits_{\Omega\lambda} N_K F(H_S) \cos\beta d\hat{\alpha} d\hat{\beta} d\hat{\lambda}, \quad (K = 1, 2, 3), \tag{28}$$

where ε_K–components of plastic strain vector; which are connected with components of strain tensor by correlation, similar to (4); N_K–projections of normal $\vec{N}$ to coordinate axes, which under (17) take form

$$N_K = m_K \cos\lambda \quad (K = 1, 2, 3). \tag{29}$$

Besides formulae (28) we have in condition of plasticity (19) $\varepsilon_4 = \varepsilon_5 = 0$ under S_4 and S_5 (because of varying sign of directing cosines).

Formulae (28) and (29) are the sought-for correlations for components of plastic strain. They were derived in [4,11] by another, more rigorous method. We use here a deduction of the named formulae, which is easier to grasp than that in [4,11].

Further we shall give some more explanation to *Eq* (28). The distances from the origin of coordinates to the tangent plane ($\sqrt{3}\tau_S$) in 5-dimensional space and to its track (h_0) in subspace S_1, S_2, S_3 are connected by correlation (17). As planes could only move parallel with themselves, formula (17) is valid for distances H_S and h_S from origin of coordinates to the named planes after their displacement, i.e.

$$h_S \cos\lambda = H_S. \tag{30}$$

As the distance to the movable plane track is defined by stress vector $\vec{S}$,

$$h_S = \vec{S} \cdot \vec{m},$$

we obtain from formulae (30) and (31)

$$H_S = (S_1 m_1 + S_2 m_2 + S_3 m_3) \cos\lambda. \tag{32}$$

It is necessary to substitute obtained value H_S in formulae for components of plastic strain (28) and complete integration for λ.

For defining the boundaries of variation with λ we note that in the planes under plastic strain, which are tangent to Huber–Mises surface, begins displacement the earliest. For them $\lambda = 0$. In growth of magnitude of the stress vector, new planes begin to participate in the displacement.

Angle λ for these planes is obtained from (17). It is necessary to make distance h_0 equal to $\overline{S}$, $\overline{m}$ in (17), i.e.

$$0 \le \lambda \le \lambda_1, \quad \cos\lambda_1 = \frac{\sqrt{3}\tau_S}{\vec{S}\cdot\vec{m}}. \tag{33}$$

For definition of domain Ω of variety of angles $\hat{\alpha}$ and $\hat{\beta}$ in formula (28), we again note that in sphere (22) we obtain $\lambda = \lambda_1 = 0$. Hence and from formula (33) we get the equation of the named domain boundary at sphere (22) in a form

$$S_1 m_1 + S_2 m_2 + S_3 m_3 = \sqrt{3}\tau_S. \tag{34}$$

Under the given components of stress vector $S_1 S_2 S_3$ formulae (12), (28), (29), (32), (33) fully define components of the plastic strain vector ε_1, ε_2, ε_3.

Note the following. Plastic strain is conditioned by displacement of planes in a 5-dimensional space. In spite of this, the stated formulae for ε_K include only characteristics of their track's displacements in a 3-dimensional subspace. In particular, on the basis of formula (34), it is easy to analyse the transformation of the domain of a variety of angles $\hat{\alpha}$ and $\hat{\beta}$ at sphere (22).

Further I.F. Andrusik has demonstrated [4] that from formula (28) follows (under $S_4 = S_5 = 0$) the existence of the universal connection between intensities of tangential stresses and shear strains. As known, such a connection is peculiar to many materials. Thus the problems, which were moved out in [5] against the slip conception on the basis that function F is not universal, does not cover formula (28).

Formula (28) also results in the following: Under torsion of preliminary tensioned thin-walled tube the overeall module is defined by Cicala formula [3] (as in terms of slip conception).

Steady-state Creep

The formula (28) defines the plastic strain of materials, which increase their strength at a process of loading, i.e. increase their resistance to strain. Hardening is characterized by plane's displacement — their removal from the origin of coordinates.

Let's turn to experimental data. If test piece is unloaded, the return of mechanical features to their previous values takes place. In particular, the strain strength of a material decreases. The mentioned return also happens in a loaded test piece and creep is the result of a strain strength decrease. This idea is wellknown and generally accepted in physics of metals and in other allied branches of science [8,10].

To get formulae for creep strain at the basis of mentioned idea, we rewrite the basic *Eq* (28) in the other form, we introduce the notion of intensity (φ) or irreversible strain (in direction of normal $\vec{N}$):

$$\varphi = \frac{|d\vec{\varepsilon}|}{dv}, \tag{35}$$

where 'volume' dv is defined by formula (25). Thus, on the basis of correlations (28) and (29) we obtain

$$\varepsilon_K = \iiint\limits_{\Omega\lambda} \varphi N_K dv, \tag{36}$$

where

$$\varphi = F(H_S). \tag{37}$$

We write formula for components of velocity of irreversible strain

$$\dot{\varepsilon}_k = \iiint\limits_{\Omega\lambda} \dot{\varphi} N_K dv, \tag{38}$$

where dot over letter means derivative with respect to time.

In accordance with *Eqs* (32) and (37) we have

$$\dot{\varphi} = \frac{dF}{dH_S} \dot{\vec{S}} \cdot \vec{m} \cos\lambda. \tag{39}$$

And $\dot{\varphi} > 0$, if

$$\dot{\vec{S}} \cdot \vec{m} > 0, \quad \vec{S} \cdot \vec{m} = h_S = (\vec{S} \cdot \vec{m})_{\max}, \tag{40}$$

where $(\vec{S}\cdot\vec{m})_{\max}$ means the maximal volume of scalar product $\vec{S}\cdot\vec{m}$ over the whole history of loading. If at least one of conditions (40) is violated, $\dot{\varphi} = 0$. In correlations (36) and (38) the limits of integration could be different. In *Eq* (38) Ω and λ mean the set of correspondent angles under which $\dot{\varphi} > 0$.

In terms of theory of plasticity, the displacement of planes could be realized only under the variation of intensity φ. Distance from plane H_S and intensity φ are one-to-one correspondent by means of formula (37). In accordance with the mentioned idea, the planes could return in the direction to the origin of coordinates without change of φ. Thus, in terms of this idea, it is necessary to abandon correlations (37) and (39) and introduce a new function ψ, which depends on position of a plane in a 5-dimensional space, i.e.

$$\psi = F(H_S). \tag{41}$$

Let's call ψ intensity of strengthening (in direction of normal $\vec{N}$). This is determined by formula (41), as ψ depends on displacement of a plane, and the last condition defines strengthening of the material.

Further we specify the connection between ψ, φ by the next differential dependence

$$d\psi = d\varphi - K(\tau_i, T_0)\psi dt, \tag{42}$$

where $d\psi$ and $d\varphi$ — variations of functions ψ and φ over time dt, K — the characteristic function of a material. It depends on intensity of tangent stresses τ_i and homological temperature T_0, i.e.

$$T_0 = \frac{T}{T_{ml}} \tag{43}$$

and T — temperature at a given moment, T_{ml} — temperature of melting.

The first component of the right part of the formula (42) means that strengthening depends on unelastic strain, the second one indicates that recovery takes place, and strengthening and loosening of material take place simultaneously.

Correlations, analogous to (42) were used [1,10] for a long time for description of some interior parameters.

Formulae (38), (41) and (42) define the components of irreversible strain rate.

Realization of *Eq* (31) is a condition for production of residual strain ε in the direction of normal $\vec{N}$ in a 5-dimensional space. This equality means that stress vector $\vec{S}$ is tangent to track of a plane (with normal $\vec{N}$ in 3-dimensional space $S_1S_2S_3$. Under this condition we obtain from formulae (30), (31) and (41)

$$\psi = F(\vec{S} \cdot \vec{m} \cos\lambda), \tag{44}$$

and we have from *Eqs* (42) and (44)

$$\dot{\varphi} = \frac{dF}{dH_S} \cdot \dot{\vec{S}} \cdot \vec{m} \cdot \cos\lambda + K(\tau_i, T_0) F(\vec{S} \cdot \vec{m} \cos\lambda). \tag{45}$$

In these directions, where $\overline{S} \cdot \overline{m} < h_s$, we have $\dot{\varphi} = 0$ and from (42) we obtain

$$\dot{\psi} = -K(\tau_i, T_0)\psi. \tag{46}$$

Eq. (46) describes strengthening recovery decrease of function ψ, and formula (41) demonstrates reset of plane to coordinates origin. Formulae (44) and (45) are invalid under the condition $\overline{S} \cdot \overline{m} < h_S$.

Let's consider two extreme cases. The first — the loading is so quick that creep doesn't manage to realize. When $dt = 0$, that is equivalent to the case when we gain $K = 0$, formula (45) agrees with equality (38), derived for the plastic body.

In another extreme case — let temperature and stress be fixed and remain constant ($\dot{s} = 0$) after rise of loading. Thus, we gain from (45)

$$\dot{\varphi} = K(\tau_i, T_0)F(\vec{S} \cdot \vec{m} \cdot \cos\lambda). \tag{47}$$

As the right part of the last formula does not depend on time, we obtain

$$\varphi = \varphi_0 + K(\tau_i, T_0)F(\vec{S} \cdot \vec{m}\cos\lambda)t, \tag{48}$$

where — φ_0 is φ in the moment ($t = 0$) in agreement fixed stress. Formulae (36) and (48) describe strain of stationary creep.

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