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APPROXIMATE CALCULATION OF PRESSURE DROP IN BENDS BUILT IN PNEUMATIC CONVEYING PIPES IN THE CASE OF HIGH DENSITY CONVEYING

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Abstract

In this paper a computing method is described for the approximate determination of pressure drop in bends of different plain built into pneumatic pipes in the case of high density conveying.

Particles moving in the bend decelerate because of the force acting on them. The conveying gas flow accelerates the decelerated particles in the connecting straight pipe section following the bend to a limit velocity belonging to the straight section if it is long enough. Pressure drop in the bend is approximately equal to the pressure drop necessary for accelerating the material again.

Maximum velocity change occurs in a bend of vertical plane starting from vertical position, therefore this bend has the greatest pressure drop, too.

Keywords: pressure drop of bends, material velocity in pipes, pneumatic conveying.

The following simplifying conditions are done by stating the mathematical-physical model for the determination of pressure drops in the bends:

a) The driving force originating from air flow and acting on the particles moving in the bend will not be considered.
b) The friction force acting on the particles moving in the bend are calculated from the Coulomb friction.
c) The pressure drop in the bend will be approximated by taking into consideration paragraphs a. and b., with a value originating from the fact that particles decelerated in the bend will be accelerated by the conveying gas flow in the connecting straight line following the bend, to a velocity belonging to this vertically or horizontally positioned pipe section.

Due to the reducing velocity of the decelerating particles, the concentration of the material will change in the bend. The value of the concentration is smaller at the beginning of the bend than at the end of it.

The following types of differently positioned bends built in conveying pipes will be discussed:

1. Bend of vertical plane, starting from horizontal.
2. Bend of vertical plane, starting from vertical.
3. Bend of horizontal plane.

Approximate Determination of Pressure Drop in a Horizontally Starting Vertical Plane Bend

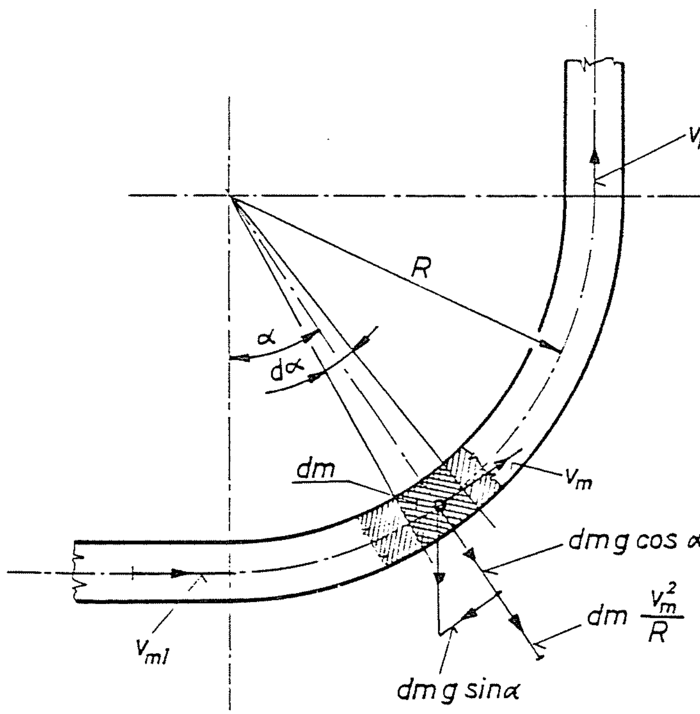

Fig. 1. Vertical plane, horizontally starting bend

Friction force acting on the mass element moving in an elementary section cut from the bend at an arbitrary angle 'α' can be calculated by the aid of 'μ' friction coefficient as follows:

$$dF_F = \left(dm\, g \cos\alpha + dm \frac{v_m^2}{R} \right) \mu. \tag{1}$$

Force component opposite to the direction of motion:

$$dF = dm\, g \sin\alpha. \tag{2}$$

Thus the force braking motion, i. e. the frictional force:

$$dF_R = dF_F + dF = dm \left[\left(g\cos\alpha + \frac{v_m^2}{R} \right) \mu + g \sin\alpha \right]. \tag{3}$$

Equation for the decelerating mass element:

$$dm \frac{dv_m}{dt} = -dF_R. \tag{4}$$

In *Eq.* (4) the negative sign indicates deceleration. From *Eqs.* (3) and (4);

$$\frac{dv_m}{dt} = -g\sin\alpha - \left(g\cos\alpha + \frac{v_m^2}{R} \right) \mu. \tag{5}$$

The left side of *Eq.* (5) can be written as follows:

$$v_m = \frac{Rd\alpha}{dt}. \tag{6}$$

After some transformations, the following differential equation is obtained for describing the decelerating motion of the particle in the bend:

$$\frac{dv_m}{d\alpha} v_m + \mu v_m^2 = -gR\sin\alpha - \mu g R \cos\alpha. \tag{7}$$

The dimensionless form of *Eq.* (6):

$$\pi_1 \frac{dv_m^*}{d\alpha} v_m^* \, \pi_1 \, \mu v_m^{*2} = -\sin\alpha - \mu\cos\alpha. \tag{8}$$

Solving the dimensionless differential *Eq.* (8) at the initial conditions

$$\alpha = 0; \qquad v_m = v_{m1}$$

the following equation is obtained:

$$v_m^* = \frac{v_m}{v_{m1}} = \left[(1-\pi_2)e^{-2\mu\alpha} - \pi_3\sin\alpha + \pi_2\cos\alpha \right]^{1/2}. \tag{9}$$

By the aid of *Eq.* (9) the velocity of the particle moving in the bend taken as a function of the angle can be calculated from point to point. Thus, for example, the 'v_{m2}' velocity of the particle coming out from a bend of $\alpha = 90°$ can be determined from its 'v_{m1}' entering velocity.

In the straight pipe section following the bend, and being in this case vertical, the material from the decreased 'v_{m2}' velocity accelerates to the 'v_{mv}' velocity belonging to the vertical conveying pipe if this section is long enough after the bend.

The pressure drop of the bend being actually equal to the pressure drop that is needed for the reaccelerating of the material in the vertical pipe section:

$$\Delta p_B = \frac{\dot{m}_m}{A}(v_{mv} - v_{m2}). \tag{10}$$

As particles, due to the force acting on them, are decelerating in the bend, the concentration of material along the bend is changing as a function of angle. The value of the concentration:

$$C = \frac{\dot{m}_m}{v_m A} = \frac{\dot{m}_m}{v_{m1} v_m^* A}. \tag{11}$$

Approximate Determination of Pressure Drop on a Vertically Starting Vertical Plane Bend

The friction force acting on the elemental material being in an elemental section in the bend of *Fig. 2* cut out at an arbitrary angle 'α':

$$dF_F = \left(dm \frac{v_m^2}{R} - dm\, g \sin\alpha \right) \mu. \tag{12}$$

The component opposite to the direction of motion:

$$dF = dm\, g \cos\alpha. \tag{13}$$

The decelerating force is the sum of the above two equations:

$$dF_R = dF_F + dF = \left(dm \frac{v_m^2}{R} - dm\, g \sin\alpha \right) \mu + dm\, g \cos\alpha. \tag{14}$$

Equation for the decelerating mass element:

$$dm \frac{dv_m}{dt} = -dF_R. \tag{15}$$

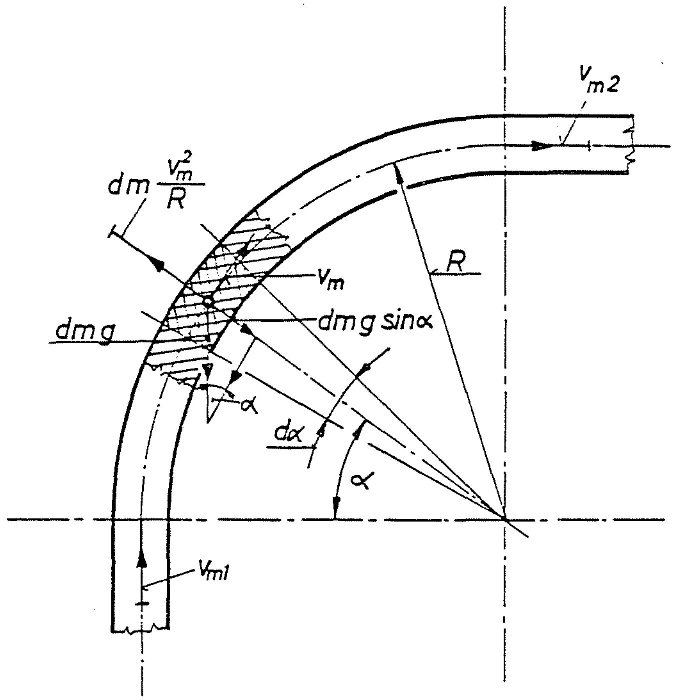

Fig. 2. Vertical plane, vertically starting bend

Using *Eq.* (6), *Eq.* (15) can be written as follows:

$$\frac{dv_m}{}d\alpha v_m + \mu v_m^2 = -gR\cos\alpha + \mu gR\sin\alpha. \tag{16}$$

Dimensionless form of *Eq.* (16):

$$\pi_1 \frac{dv_m^*}{d\alpha} v_m^* + \mu\,\pi_1\, v_m^{*2} = -\cos\alpha + \mu\sin\alpha. \tag{17}$$

Eq. (17) differs from *Eq.* (8) only in the right side, thus their solutions are similar, too.

The solution of the differential equation (17) at the initial conditions

$$\alpha = 0; \qquad v_m = v_{m1}$$

is as follows:

$$v_m^* = \left[(1+\pi_3)e^{-2\mu\alpha} - \pi_2\sin\alpha - \pi_3\cos\alpha\right]^{1/2}. \tag{18}$$

With *Eq.* (8) the velocity of the particle decelerating in the bend can be calculated as a function of the angle.

In *Eq.* (12) we supposed that

$$\frac{v_m^2}{R} > g \sin \alpha \tag{19}$$

along the whole bend.

It can occur at angle $\alpha = \alpha_o$ that $\frac{v_{mo}^2}{R} = g \sin \alpha_o$. In case when $\alpha > \alpha_o$

$$\frac{v_m^2}{R} < g \sin \alpha \tag{20}$$

inequality is valid.

In case of *Eq.* (20) friction force is:

$$dF_f = \left(dm \, g \sin \alpha - dm \, \frac{v_m^2}{R} \right) \tag{21}$$

Using *Eq.* (21), dimensionless form of the equation describing the motion in bend:

$$\pi_1 \frac{dv_m^*}{d\alpha} v_m^* - \mu \pi_1 v_m^{*2} = - \cos \alpha - \mu \sin \alpha. \tag{22}$$

Eq. (22) differs from *Eq.* (17) only in sign.

The solution of the differential equation at the initial conditions

$$\alpha = \alpha_o; \qquad v_m = v_{mo}$$

is as follows:

$$v_m^* = \left[e^{2\mu(\alpha - \alpha_o)} (v_{mo}^{*2} + \pi_2 \sin \alpha_o - \pi_3 \cos \alpha_o - -\pi_2 \sin \alpha + \pi_3 \cos \alpha \right]^{1/2}. \tag{23}$$

In the horizontal straight pipe section following the bend, the material velocity of which reduced from 'v_{m2}' in the bend with central angle $\alpha = 90^\circ$ accelerates to the velocity 'v_{mh}' dominating in the connection straight horizontal section.

Pressure drop in the bend can be calculated similarly to *Eq.* (10):

$$\Delta p_B = \frac{\dot{m}_m}{A} (v_{mh} - v_{m2}). \tag{24}$$

The value of concentration:

$$C = \frac{\dot{m}_m}{v_m A} = \frac{\dot{m}_m}{v_{m1} v_m^* A}. \tag{25}$$

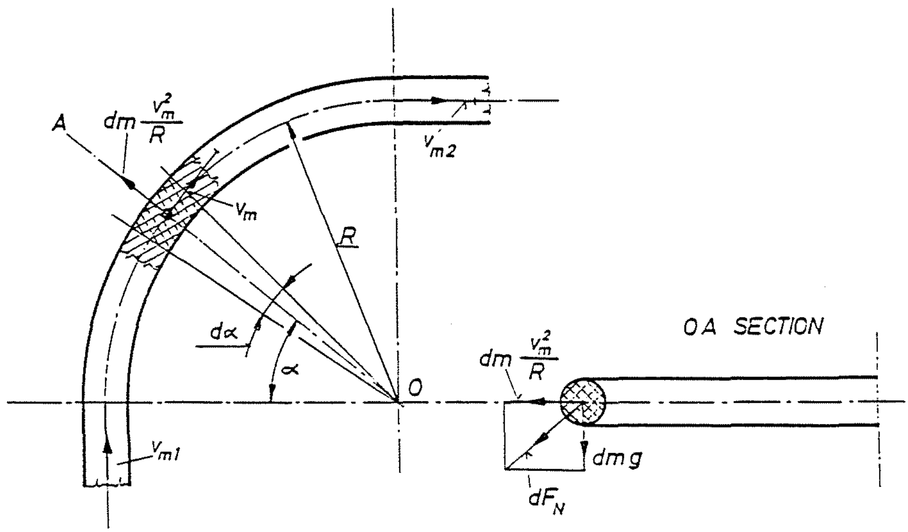

Fig. 3. Horizontal plane bend

Approximate Determination of Pressure Drop in a Horizontal Plane Bend

The friction force acting on the mass element moving in the elemental section cut from the bend of *Fig. 3* at angle 'α':

$$dF_F = \mu\, dF_N = \mu \left[\left(dm\, \frac{v_m^2}{R} \right)^2 + (dm\, g)^2 \right]^{1/2} . \tag{26}$$

Equation of the motion of particles decelerating in the bend:

$$dm\, \frac{dv_m}{dt} = -dF_F. \tag{27}$$

Negative sign here also means deceleration. Using

$$v_m = R\, \frac{d\alpha}{dt} \tag{28}$$

is obtained that:

$$\frac{dv_m}{d\alpha}\, v_m = -\mu R \left(\frac{v_m^4}{R^2} + g^2 \right)^{1/2} . \tag{29}$$

Making *Eq.* (24) dimensionless:

$$\pi_1 \frac{dv_m^*}{d\alpha} v_m^* = -\mu \left(\pi_1^2 v_m^{*4} + 1\right)^{1/2}. \tag{30}$$

The solution of this equation at the initial conditions

$$\alpha = \alpha_o; \qquad v_m = v_{mo}$$

is as follows:

$$v_m^* = \left[\frac{\pi_4^2 - e^{4\mu\alpha}}{2\pi_1\pi_4 e^{2\mu\alpha}}\right]^{1/2}. \tag{31}$$

Pressure drop along the bend is again equal to that needed for reaccelerating to the 'v_{mh}' velocity, which is dominant in the horizontal straight section which follows the bend. This velocity is equal to the entering velocity, because identical horizontal pipe sections are connected to the bend at both ends, i. e. $v_{mh} = v_{m1}$.
Pressure drop in the bend:

$$\Delta p_B = \frac{\dot{m}_m}{A}(v_{mh} - v_{m2}). \tag{32}$$

The value of concentration:

$$C = \frac{\dot{m}_m}{v_m A} = \frac{\dot{m}_m}{v_{m1} v_m^* A}. \tag{33}$$

Comparison of Bends of Different Types

Using the relations given in this paper, the effect of altering parameters being in these relations on the change of the individual physical values has been determined.

During the investigation the following data were kept constant:

Pipe diameter	$D = 50$ mm
Material velocity at bend begin	$v_{m1} = 10$ m/s
Material mass flow	$\dot{m}_m = 10$ t/h
Bend radius	$R = 1$m

In *Fig. 4* the velocity of particles moving in the bend can be seen as a function of central angle in the cases of differently positioned bends.

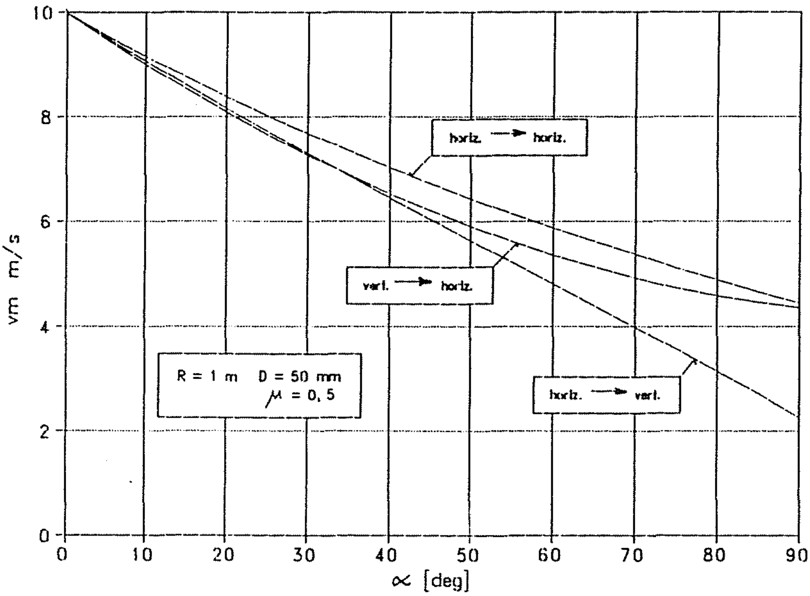

Fig. 4. Material flow velocity change as a function of angle.
Parameter: bend position

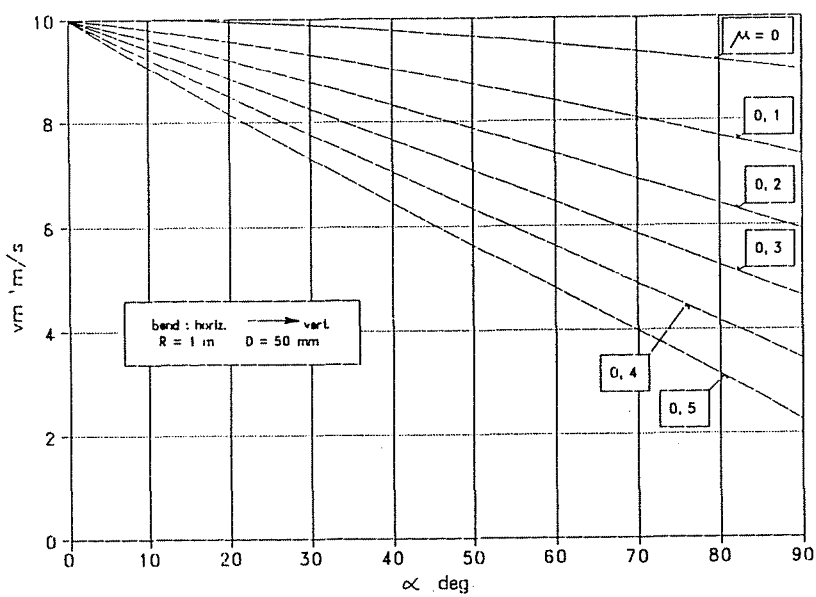

Fig. 5. Material flow velocity change as a function of angle in the case of bend turning from horizontal to vertical
Parameter: friction coefficient

The 'v_{m2}' velocity of the material leaving the bend is minimal at bends going from horizontal to vertical (i. e. the braking of the material is maximal here), while it is the greatest in the case of horizontal plane bend.

In *Fig. 5* the change of material velocity can be seen in the case of vertical plane horizontally starting bend. The 'μ' friction coefficient is the parameter.

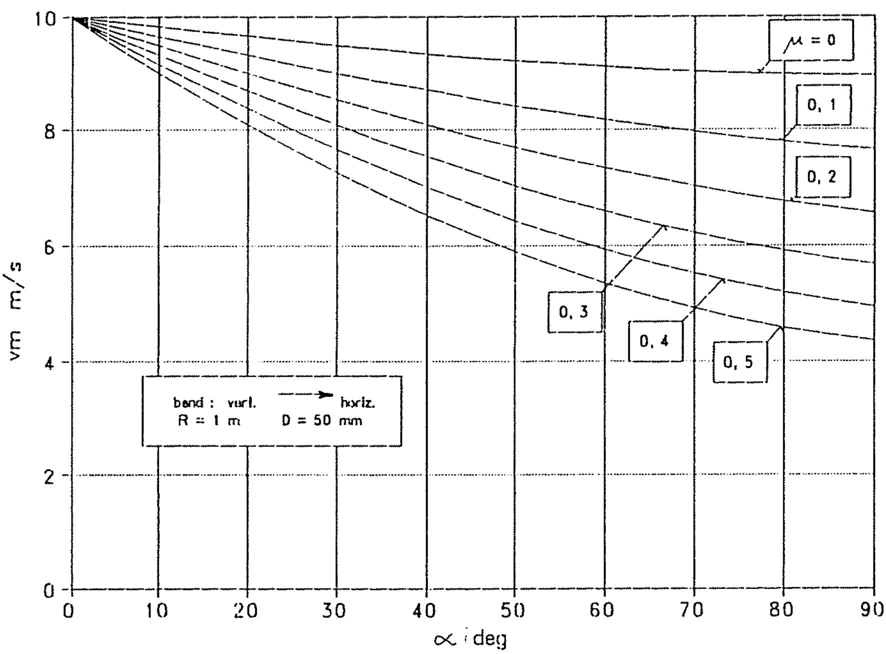

Fig. 6. Material flow velocity change as a function of angle in the case of bend turning from vertical to horizontal.
Parameter: friction coefficient

The influence of the same parameters can be seen in *Figs. 6* and *7*, but at bends of vertical plane vertical start or of horizontal plane, respectively.

In *Fig. 8* 'v_{m2}' velocity values emerging from the horizontal plane bend are plotted in the case of bends with different radii.

In *Fig. 9* '$v_{m1} - v_{m2}$' velocity reduction is plotted at horizontal plane bend. This value is proportional to the pressure drop in the bend.

Explanation for *Figs. 8* and *9* comes directly from the energetical interpretation of the model, surely, when increasing radius, work of the friction force changes because of the increasing of path length.

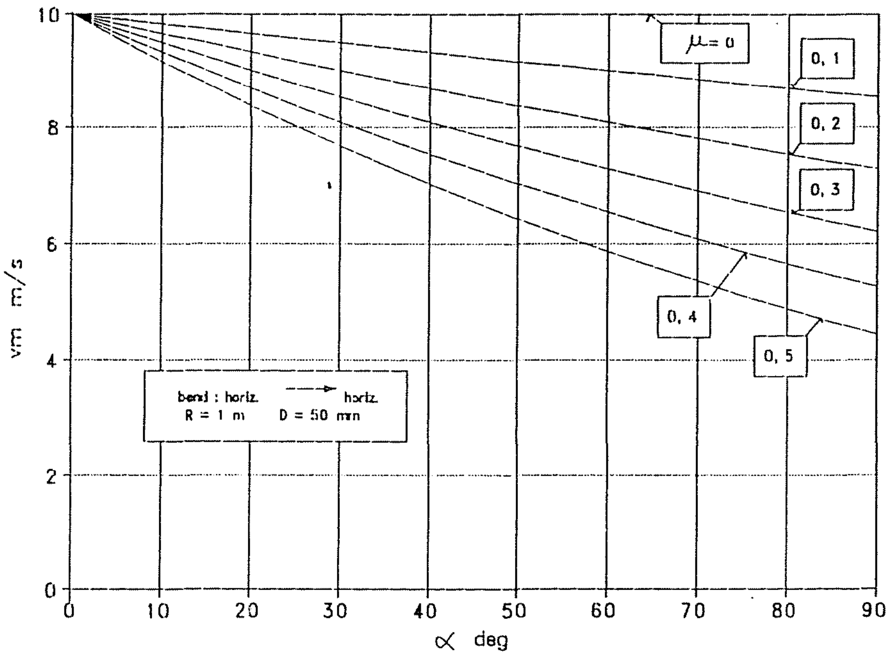

Fig. 7. Material flow velocity change as a function of angle in the case of horizontal plane bend
Parameter: friction coefficient

Numerical Example for the Determination of Pressure Drop in a Vertical Plane Horizontally Starting Bend

Data for the example:

Radius of the bend	$R = 1$ m
Pipe diameter	$d = 38$ mm
Velocity of material entering the bend	$v_{m1} = 10$ m/s
Mass flow of transported material	$\dot{m}_m = 10$ t/h
Central angle of the bend	$\alpha = 90°$
Material velocity in the connecting vertical pipe section (which is long enough)	$v_{mv} = 6$ m/s
Friction coefficient	$\mu = 0.3$

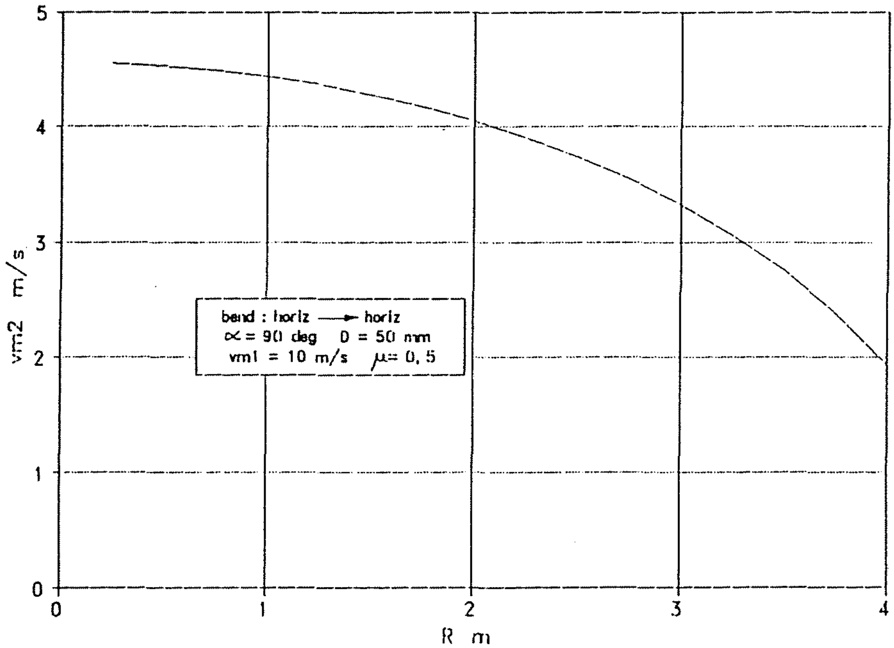

Fig. 8. Velocity of material leaving the bend as a function of radius of the bend

Constant values for *Eq.* (9):

$$\pi_1 = \frac{v_{m1}^2}{gR} = \frac{10^2}{9.81 * 1} = 10.194 \quad [-]$$

$$\pi_2 = \frac{2(1-2\mu^2)}{\pi_1(1+4\mu^2)} = \frac{2*(1-2*0.3^2)}{10.194*(1+4*0.3^2)} = 0.118 \quad [-]$$

$$\pi_3 = \frac{6\mu}{\pi_1(1+4\mu^2)} = \frac{6*0.3}{10.194*(1+4*0.3^2)} = 0.13 \quad [-]$$

From the *Eq.* (9) considering that $\alpha = \pi/2$, $\sin\alpha = 1$; $\cos\alpha = 0$ we obtain:

$$v_m^* = [(1-\pi_2)e^{-2\mu\alpha} - \pi_3]^{1/2} =$$
$$= [(1-0.118)e^{-2*0.3*\pi/2} - 0.13]^{1/2} = 0.462.$$

From this $v_{m2} = v_{m1} * v_m^* = 10 * 0.462 = 4.62$ [m/s].

So the pressure drop in the bend with:

$$A = \frac{D^2\pi}{P4} = \frac{0.038\pi^2 * \pi}{4} = 0.001134 \text{ m}^2$$

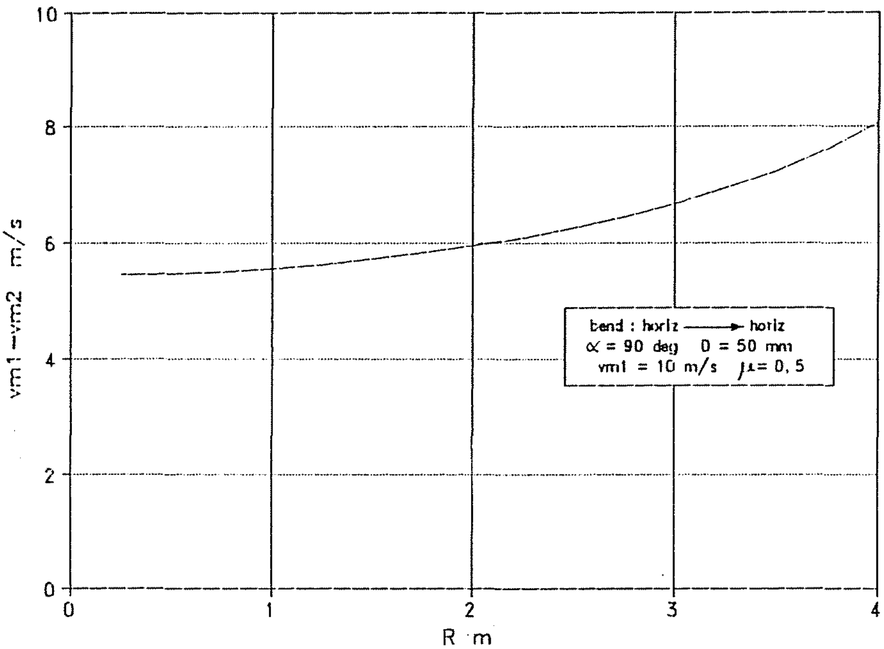

Fig. 9. Change of material velocity as a function of radius of the bend

$$\Delta p_B = \frac{\dot{m}_m}{A}(v_{mv} - v_{m2}) = \frac{10}{0.001134 * 3.6}(6 - 4.62) = 3380 \text{ [Pa]}.$$

Concentration at the beginning of the bend:

$$C_1 = \frac{\dot{m}_m}{v_{m1}A} = \frac{10}{10 * 0.001134 * 3.6} = 245 \text{ [kg/m}^3\text{]}.$$

At the end of the bend:

$$C_2 = C_1\frac{10}{4.62} = 245\frac{10}{4.62} = 530 \text{ [kg/m}^3\text{]}.$$

In *Fig. 10* change of concentration value can be seen as a function of angle 'α'.

In *Fig. 11* the values of function $v_m = f(\alpha)$ in range of $\alpha < \alpha_o$ are calculated from *Eq.* (18). Values in range of $\alpha > \alpha_o$ from *Eq.* (23)

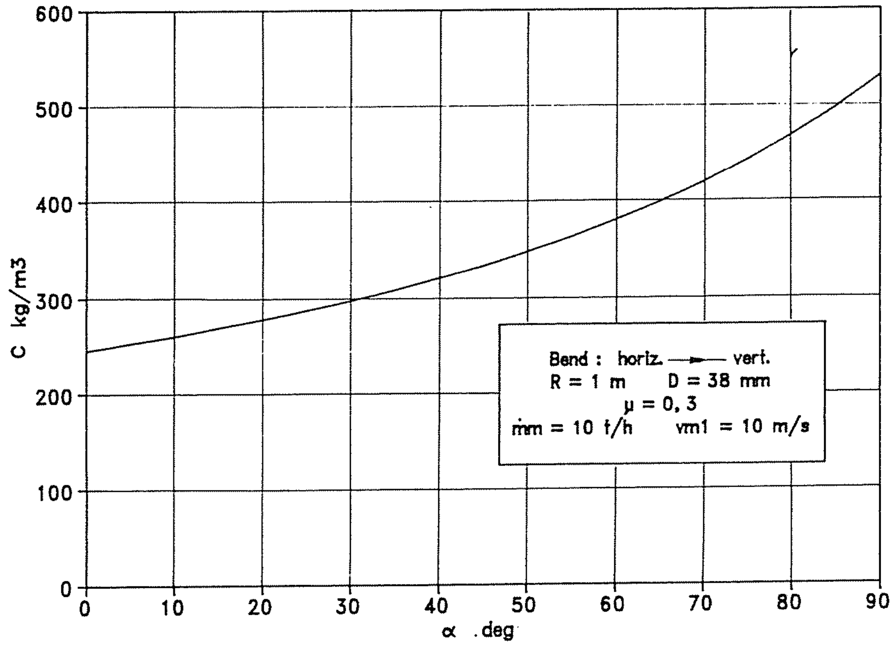

Fig. 10. Distribution of concentration along the bend

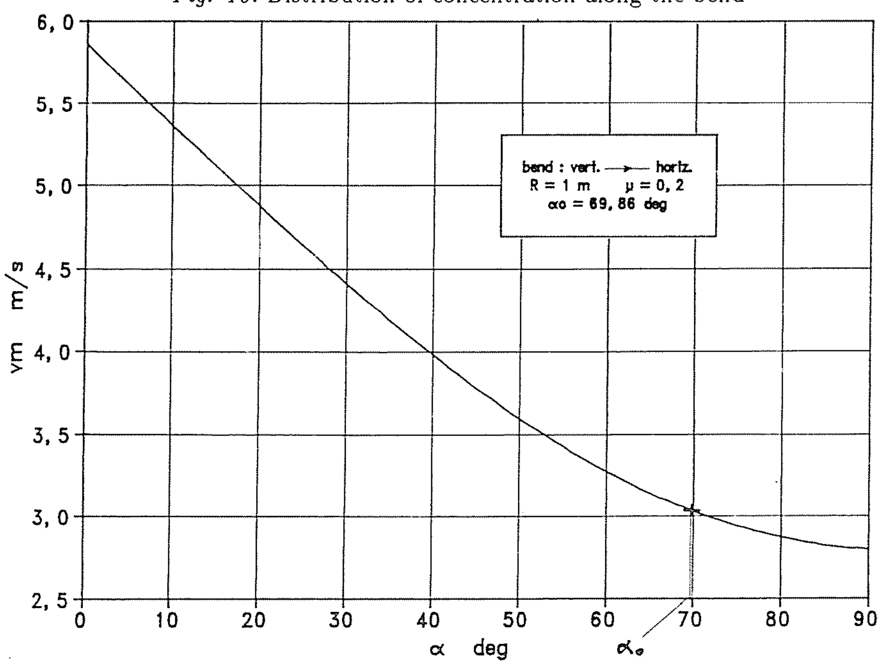

Fig. 11. Change of material velocity as a function of angle of the bend.
In this case, at angle of $\alpha = \alpha_o = 69.86°$ the equality $\frac{v_{mo}^2}{R} = g \sin \alpha_0$ is valid.
The values of function $v_m = f(\alpha)$ in range of $\alpha < \alpha_o$ are calculated from *Eq.* (18),in range of $\alpha > \alpha_o$ are calculated from *Eq.* (23).

Notation

$A = \dfrac{D^2\pi}{4}$	[m^2]	pipe cross-section
C	[kg/m^3]	concentration
D	[m]	pipe diameter
F	[N]	force
g	[m/s^2]	gravitational acceleration
m	[kg]	mass
$\dot{m}$	[kg/s]	mass flow
Δp	[Pa]	pressure drop
$\pi_1 = \dfrac{v_{m1}^2}{gR}$	[–]	constant
$\pi_2 = \dfrac{2(1-2\mu^2)}{\pi_1(1+4\mu^2)}$	[–]	constant
$\pi_3 = \dfrac{6\mu}{\pi_1(1+4\mu^2)}$	[–]	constant
$\pi_4 = \pi_1 + (1+\pi_1^2)^{1/2}$	[–]	constant
t	[s]	time
v_m	[m/s]	material velocity
$v_m^* = \dfrac{v_m}{v_{m1}}$	[–]	dimensionless material velocity
α	[deg;rad]	angle
α_o	[deg;rad]	angle at which $\dfrac{v_{mo}^2}{R} = g \sin \alpha_o$
μ	[–]	friction coefficient

Subscripts

1	beginning of bend
2	end of bend
B	bend
F	friction
h	horizontal pipeline
m	material
N	normal
R	resultant
v	vertical pipeline

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