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THEORETICAL AND EXPERIMENTAL INVESTIGATIONS ON THE DIFFUSIVITIES OF COMPOSITES[1]

Hannu KOIVUROVA* and András SZEKERES**

*Department of Mech. Engn.
Univ. of Oulu, PO Box Q44, FIN 90571 Oulu, Finland
e-mail: hannu.koivurova@oulu.fi
**Department Applied Mechanics
Research Group of Continuum Mechanics, HAS
Technical University of Budapest
H–1521 Budapest, Hungary
e-mail: szekeres@mm.bme.hu

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Abstract

Dealing with the tailoring of thermo-hygro-elastic (THE) composites the powerful tool of anisotropy is multiplied by taking into account the crosscoupling effects (Soret and Dufour) of heat conduction (Fourier) and moisture diffusion (Fick). In this case four diffusivities arise ($k/\rho c$, d_m, d_m^T, d_T^m), which are somehow related depending all of them on the microstructure of material, but the two latter ones are equal according to the Onsager's relations.

Based on our previous investigations [1, 2] we introduced the governing equations of the problem and solved them numerically for $1D$ with given initial and boundary conditions. With a special definition of the coupled coefficient of thermal/moisture expansion (CT/ME), we got the displacement ($L(t)$) and temperature ($T_L(t)$) versus time at the end of the rod.

The experiment was performed with a rod type specimen made of heat conducting and moisture absorbing composite matrix material. The conditions modelled thermal shock, perfect insulation and different moisture content.

The comparison of the numerical and experimental values of $L(t)$ and $T_L(t)$ enables us to conclude to the diffusivities. Comparing with values of literature [3, 4] we have certain hope to separate the different diffusivities.

Keywords: moisture, diffusivity, coupling, composites, heat.

Contents

[1]Based on the presentation at 31st SOLID MECHANICS CONFERENCE SolMec' 96, Mierki, Poland. September 10–14, 1996.

1. Introduction

In most cases the hygroscopic property of composites is disadvantage. The moisture content or the change in moisture concentration cause degradation which may lead, similarly to temperature effect, finally to failure.

But there are exceptions, e.g. if we want to tailor the fiber reinforced composites (FRCs) to increase the number of parameters involved into process of engineering, the moisture absorbing property is very advantageous. Taking into account the hygroscopic feature of FRCs gives a broader opportunity. If we do not neglect the coupled thermo-hygro-mechanical process, the possibility of tailoring is wider than expected. But in this case several material properties are needed, e.g.

CTE, CME, CT/ME	coefficients of thermal, moisture and coupled thermal/moisture expansion,
k	conductivity,
d_m, d_m^T, d_T^m	moisture and coupled thermal/moisture and moisture/thermal diffusivity.

Most of these parameters are available in the handbooks, but not all of them. Our purpose was to define the coupled coefficients CT/ME, d_m^T and d_T^m and to get some numerical results on them.

The method was a combined one. Based on our theoretical works we performed numerical calculations and experiments. We get the final numerical values of parameters by comparison of experimental and numerical results.

2. Theoretical Background

Dealing with the coupled heat conduction and moisture diffusion problem the governing equations are based on the Fourier's law of heat conduction, on the Fick's law of moisture diffusion and on the Dufour and Soret cross-coupling effects (see *Fig. 1*).

Concerning the general form of the governing equation we refer to [1] and to the strain relations when neglecting the mechanical effects.

For our purpose let us make the following assumptions.

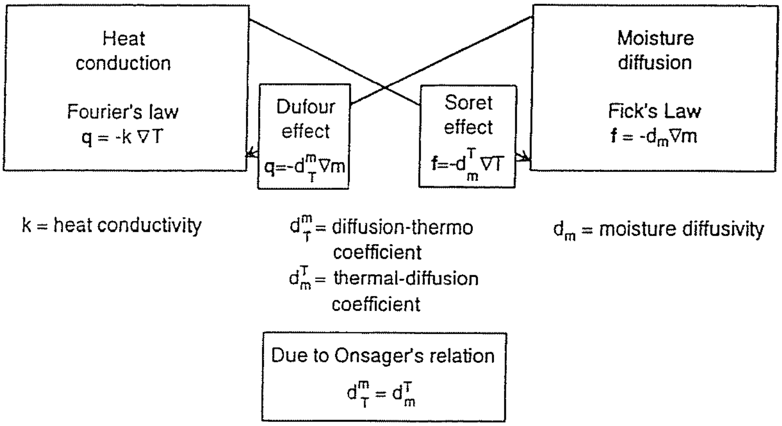

Fig. 1. Coupling of heat conduction and moisture diffusion

- 1-Dimensional conduction and diffusion $\Leftarrow$ Perfect insulation
- Non-stationary process $\Leftarrow$ Step loading
- Linear model $\Leftarrow$ Material parameters do not depend on temperature or moisture

According to this conditions the governing equations are:

$$\rho c \frac{\partial T(x,t)}{\partial t} = -k \frac{\partial^2 T(x,t)}{\partial x^2} - d_T^m \frac{\partial^2 m(x,t)}{\partial x^2}\,, \tag{2.1}$$

$$\frac{\partial m(x,t)}{\partial t} = -d_m^T \frac{\partial^2 T(x,t)}{\partial x^2} - d_m \frac{\partial^2 m(x,t)}{\partial x^2}\,, \tag{2.2}$$

$$\frac{\partial U}{\partial x}(x,t) = \alpha \Delta T(x,t) + \beta \Delta m(x,t)\,. \tag{2.3}$$

The initial and boundary conditions are as follows.

$$T(x,0) = T_\infty\,, \tag{2.4}$$

$$m(x,0) = m_0\,, \tag{2.5}$$

$$U(x,0) = 0\,, \tag{2.6}$$

$$T(0,t) = T_0 \quad \& \quad -k \frac{\mathrm{d}T}{\mathrm{d}x}\Big|_{x=L} = h\left(T(L,t) - T_\infty\right)\,, \tag{2.7}$$

$$m(0,t) = 0 \quad \& \quad \frac{\mathrm{d}m}{\mathrm{d}x}\Big|_{x=L} = 0\,, \tag{2.8}$$

$$U(0,t) = 0 \quad \& \quad \left.\frac{\mathrm{d}U}{\mathrm{d}x}\right|_{x=L} = 0\,. \tag{2.9}$$

By solving the system (2.1 ... 9) we obtain the desired definitions of coupled coefficients of thermal/moisture expansion. In a stationary case

$$\chi_m^T = \alpha + \beta\frac{\Delta m}{\Delta T}\,,$$

$$\chi_T^m = \alpha\frac{\Delta T}{\Delta m} + \beta\,, \tag{2.10}$$

and in dynamical case

$$\chi_m^T = \alpha + \beta\frac{\int_0^1 \Delta m(\xi,t)\mathrm{d}\xi}{\int_0^1 \Delta T(\xi,t)\mathrm{d}\xi}\,,$$

$$\chi_T^m = \beta + \alpha\frac{\int_0^1 \Delta T(\xi,t)\mathrm{d}\xi}{\int_0^1 \Delta m(\xi,t)\mathrm{d}\xi}\,, \tag{2.11}$$

Here $\xi = x/L$ and x is the spatial co-ordinate and L is the length of the rod. We have to emphasize that in stationary case the coefficients depend on x and in dynamical case they depend on t and are the average value along the rod. It means that, strictly speaking, due to our definition they are not only material properties but also process features.

3. Experimental Setup and Experiments

There were two major questions concerning the experiments. First, how we are able to fulfil the ideal assumptions of the theoretical model; step loading and perfect insulation of test pieces. By doing some trials we found out a quite simple setup to do the tests within reasonable accuracy. Another question was to find a suitable material for experiments, a plastic which is at the same time an appropriate heat conductor and a good water absorber. Polyamide seems to fulfil the requirements best. It can absorb water up to 10% and it conducts heat in adequate manner.

The experiments were made with Polyamide PA 6. Specimens were machined to slender rods with 6 mm and 10 mm diameter and 70 mm in length. The experimental setup is shown in *Fig. 2.* The step loading was produced by a heat source in which the samples are attached at beginning of the test. The temperature was adjusted by controlling the operating

Table 1
Material parameters of test material polyamide PA 6 [6, 7]

Heat conductivity	$k = 0.28$	W/(m°C)
Heat capacity	$c = 1570$	J/(kg°C)
CTE	$\alpha = 95 \cdot 10^{-6}$	1/°C
Moisture diffusivity	$d_m = 55 \cdot 10^{-12}$	m^2/s
CME	$\beta = 0.002$	

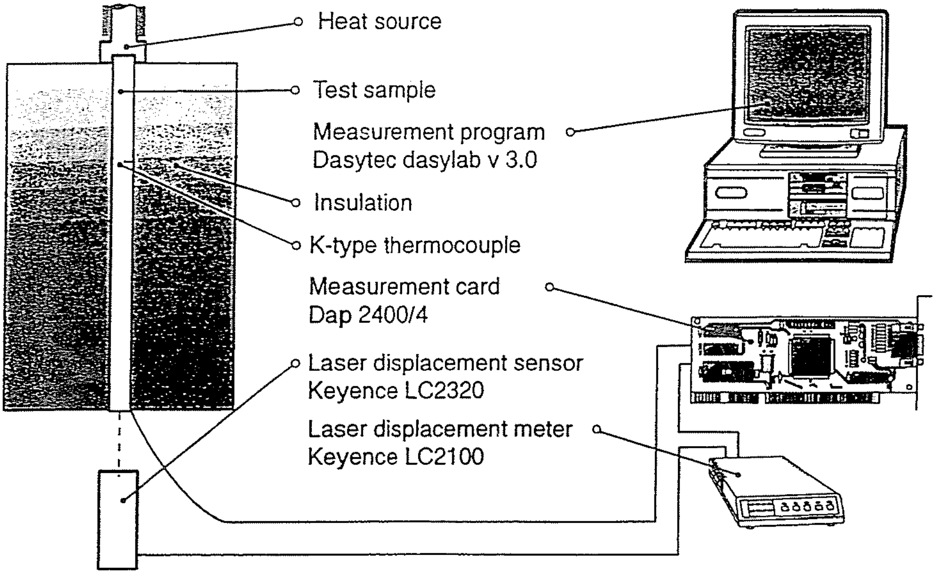

Fig. 2. Experimental setup

voltage of heat source. The insulation was achieved by mineral wool and polyurethane insulation materials.

Temperature measurements were performed with K type thermocouples which were immersed into the test samples at the positions 20 mm and 30 mm from loading end. Length change of sample was measured with laser displacement sensor, Keyence LC2320, by measuring the displacement of the free end of the test specimen. Measurements were carried out by measurement card of Microstar laboratories, Dap 2400/4 and measurement program of Dasytec, Dasylab V.3.0.

The samples were measured in three different moisture contents, 0%, 5% and 8%, which were achieved by sinking them to warm water for different time periods. The moisture content was measured by weighing.

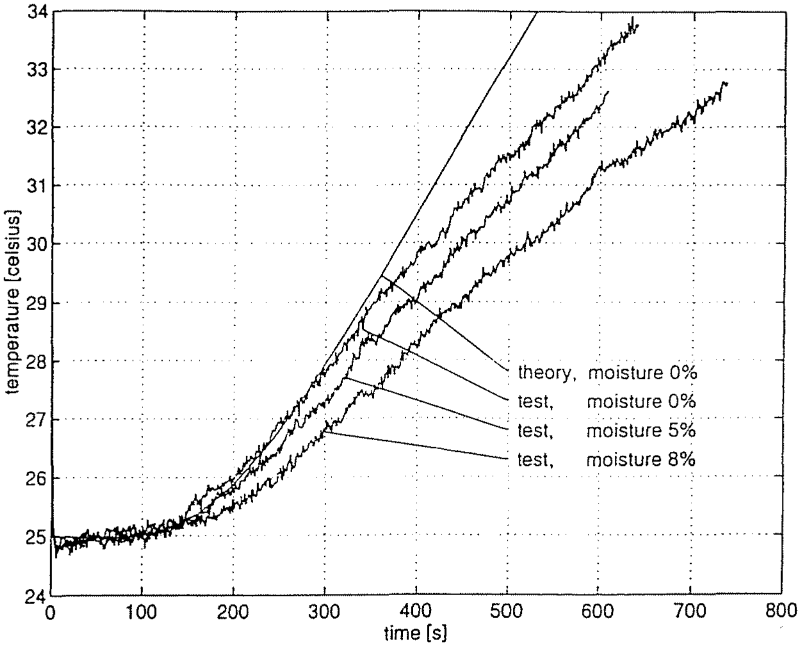

Fig. 3. Temperature vs. time at position 20 mm from the heated end

4. Numerical Method

The governing equations of the problem (2.1 ... 3) were solved numerically with given initial (2.4 ... 6) and boundary conditions (2.7 ... 9). The FEM program PDE2D was used. The program is developed for solving elliptic, parabolic and hyperbolic partial differential equations and eigenvalue problems by the finite element method [5]. The test specimen was modelled with an axisymmetric model and by triangular elements. Temperature and moisture response was integrated with Crank–Nicolson method.

5. Evaluation and Comparison

The next figures illustrate the calculated and measured results. *Figs. 3* and *4* show temperature response curves at position 20 mm and 30 mm from the heated end of the test specimen. Tests were made with the moisture contents of 0%, 5% and 8%. Results show quite good agreement with the dry samples. The difference seems to grow in process of time and the cause of error is most probably the leak of insulation. There are no calculated results for moist test pieces, because they did not differ from the

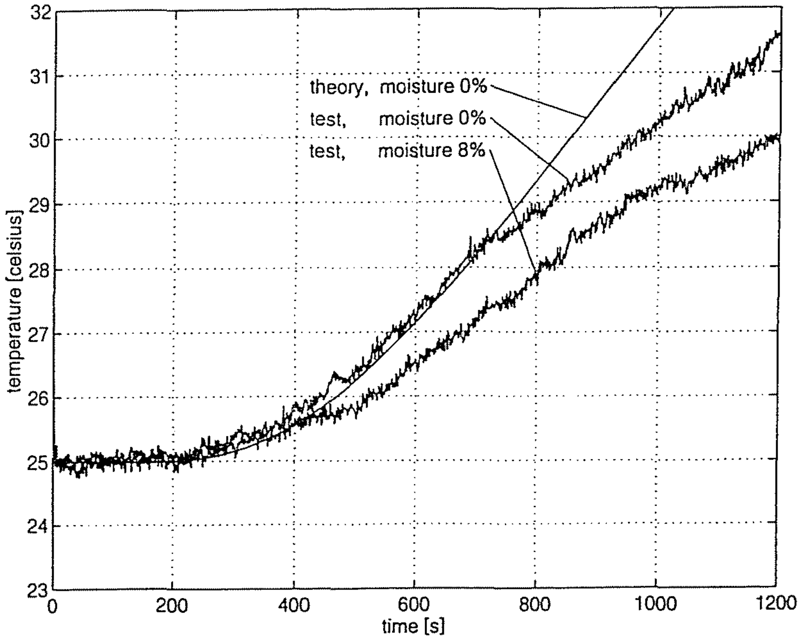

Fig. 4. Temperature vs. time at position 30 mm from the heated end

result of dry sample with reasonable values of coupled thermal/moisture and moisture/thermal diffusivities.

Fig. 5 shows the displacement vs. time of free end of test specimen. In this case the calculated results for moist test pieces differ from the dry sample. The coupled diffusivities were selected to be equal to moisture diffusivity.

6. Conclusion and Summary

According to our aim, the temperature and displacement responses were measured and calculated for different places and moisture contents. To estimate the coupled diffusivities and expansion coefficients, the values were compared. Inspite of characteristic qualitative results, the numerical separation of diffusivities and of expansion coefficients was impossible. To obtain quantitative results for valuable conclusions further experiments and numerical calculations are needed.

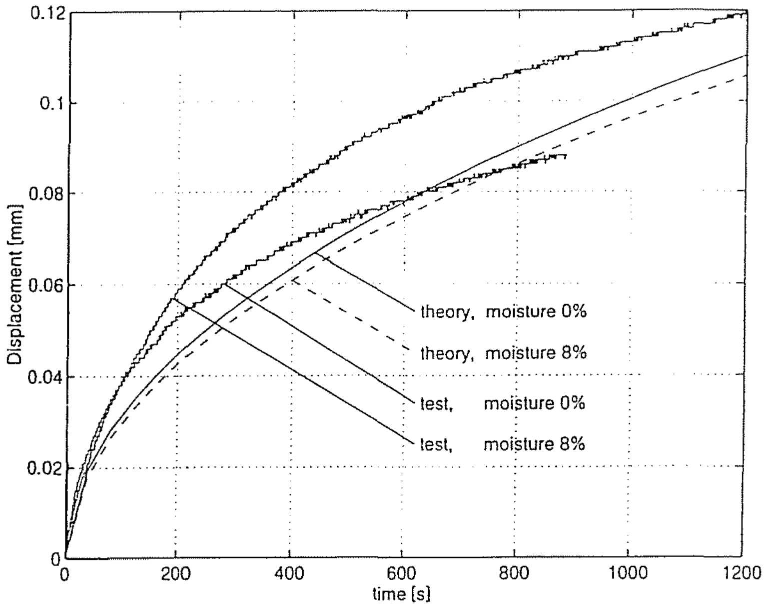

Fig. 5. Displacement vs. time at the free end of test specimen

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