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Isolation Valve Placement Optimization of Water Distribution Networks to Reduce Vulnerability

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Abstract
Due to the growing pace of urbanization and the rapid expansion of the population, the utilities nowadays are facing a more challenging task than ever before, to maintain the functionality of their massive Water Distribution Networks. In this paper, three different techniques are presented for the maximization of network robustness against random pipe-bursts. This paper analyses the vulnerability of segments of WDNs from the viewpoint of the consumers that is the product of the failure rate and the relative demand loss. Topological parameters could approximate the vulnerability. During the investigation, 27 real-life WDNs were analyzed from the region of Western Hungary. In two networks, a complete isolation valve replacement optimization is presented using these low-cost topological parameters as fitness functions. It is proved that by rearranging the isolation valves of the WDN, using a standard genetic algorithm and a fast-evaluated fitness function, the nature of the vulnerability distribution can be changed to a more favorable one. This means that it is possible to increase the robustness of a WDN even without using a calibrated hydraulic model; since these topological parameters can be extracted from simple databases.

Keywords
isolation valve placement, segment graph, topological approximation, vulnerability, water distribution networks

1 Introduction

Nowadays, the population of humankind is continuously increasing; thus, the need for essential resources is also increased to a new level. Besides this process, the pace of urbanization is also growing. On the one hand, it means that sometimes densely populated suburban areas are becoming abandoned. On the other hand, the city centers and the agglomeration areas' population density is increasing on a larger scale. In either case, due to these rapid population changes, the utilities are facing with new challenges. Since the water distribution networks (WDNs) serve the essential needs of human settlements, these complex systems are one of the most exposed services to these phenomena. The complicated and heterogeneous structure of the WDNs makes evaluating the effect of a pipe-burst a challenging task. In contrast, the previously developed central sections of the networks became older and more exposed to the randomly appearing pipe bursts, according to the work of [1].

When a pipe burst is detected, the reconstruction personnel must segregate the network section that beholds the damaged pipe. First, the nearest isolation valves have to be localized and closed. As a result, that the utility companies are constantly fighting with the shortage of the budget and with the aging of the system; every pipeline does not have its isolation valves at both ends. Even if the isolation valves are present, they might malfunction, according to the work of [2] and [3]. These properties lead to the phenomenon that not just the damaged pipe section but whole network parts must be segregated from the system's main body, causing service outages for residential and industrial consumers.

The standard recommendation is to place n (or $n-1$) number of isolation valves in an intersection if n represents the connecting pipelines see [4]. Practically, in real-life WDNs, an approximating $n-1$ rule is followed. The focus of the current study is to identify the optimal way to distribute the isolation valves throughout the network. In other words, if one applies an approximately $n-1$ rule, which pipeline should not have an isolation valve? Since an optimization technique typically requires

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numerous hydraulic simulations, we also present three topology-based fitness functions, as avoiding the simulation might save significant computational time.

2 Hydraulic simulation

For hydraulic simulation, an in-house solver, called STACI, is utilized that was already used to present several results to the literature [5–7]. The hydraulic results were validated, ensuring that the differences in the outputs (e.g., pressure, volume flow rate) are below 1% compared to the well-known EPANET solver. The reason behind the application of an in-house made solver is twofold; on the one hand, performing hydraulic calculations while some parts of the network are isolated, and on the other hand, the control of the source code. Although the standard EPANET solver is not capable of the former one, commercially available programs are, see [8].

Altogether 27 different real-life water distribution networks were analyzed from the region of Western Hungary. The ranges of the topological parameters of these networks can be seen in Table 1.

To present the topological differences of the analyzed networks, we have selected networks 10 and 11 for a detailed presentation. As Fig. 1 and Table 2 depict, these two networks are exactly different from the viewpoint of topological properties. While Network 10 is an urban until, Network 11 serves a rural area, as Fig. 2. indicates.

Table 1 General information about the examined real-life WDNs, representing the comprehensiveness of the networks

Property	Min	Max
Number of nodes	301	7188
Number of segments	12	717
Overall pipe length [km]	4.59	112
Total nominal consumption [m³/h]	0.4	540

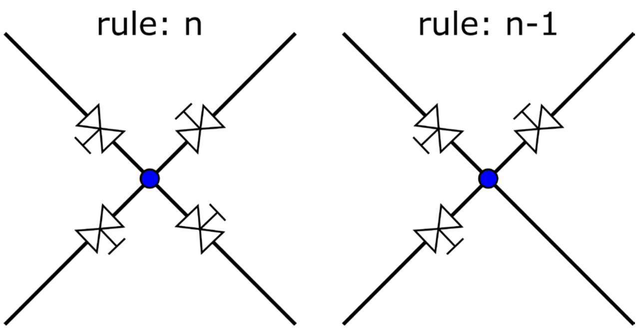

Fig. 1 Representing the n and $n-1$ rules for isolation valve placement

Table 2 The parameters of the networks

ID	Sum. pipe length [km]	Sum. consumption [m³/h]	Number of nodes	Number of isolation valves
10	17.62	6.152	1294	99
11	19.49	8.658	1081	92

3 Segment graph

The results of [4] revealed that the "classic" link-node representation of WDNs could not describe the behavior of the WDNs adequately in the case when the focus of the analysis is the network's reliability against random pipe bursts. However, there are different representations of WDNs capable of overcoming such issues, like the technique proposed by [4]. During this analysis, the same approach was used.

The process of the segment graph building of a small synthetic WDN can be seen in Fig. 3, where the steps of the procedure are the following:

- First, the isolation valves are identified (upper left panel) in the network; after that, they are removed from the system; thus, the whole WDN falls apart into a smaller portion of edges (upper right panel).
- These small "islands" of edges and nodes are identified with a graph mapping algorithm named Breadth-First Search (BFS) technique [9]; see lower left panel.
- Finally, these identified network portions became the nodes of the new graph. They are called "segments", while the edges of the new graph represent the previously removed isolation valves which can be seen on the bottom right panel of Fig. 1.

This new graph is called a segment graph. One of the main advantages of this representation is that when a pipe burst is identified in the system, a node of the segment graph is lost. Thus, all the edges where service outages occur are already identified. It means that by a simple visual analysis of the graph, one can determine which segments will be lost their connection with the system's main body. Thus, which segments will become segregated and be out of service. However, as proved by [10] and [11], not just the directly but the partially segregated segments can also lose the ability to serve the demands of the area. Based on these previous studies [12], applied a pressure-dependent solver [13] and introduced a new metric for the network's vulnerability against randomly appearing pipe bursts. This extension makes it possible to identify the segments where a direct service outage will happen and where the decreased system pressure will cause indirect service outage or comfort loss.

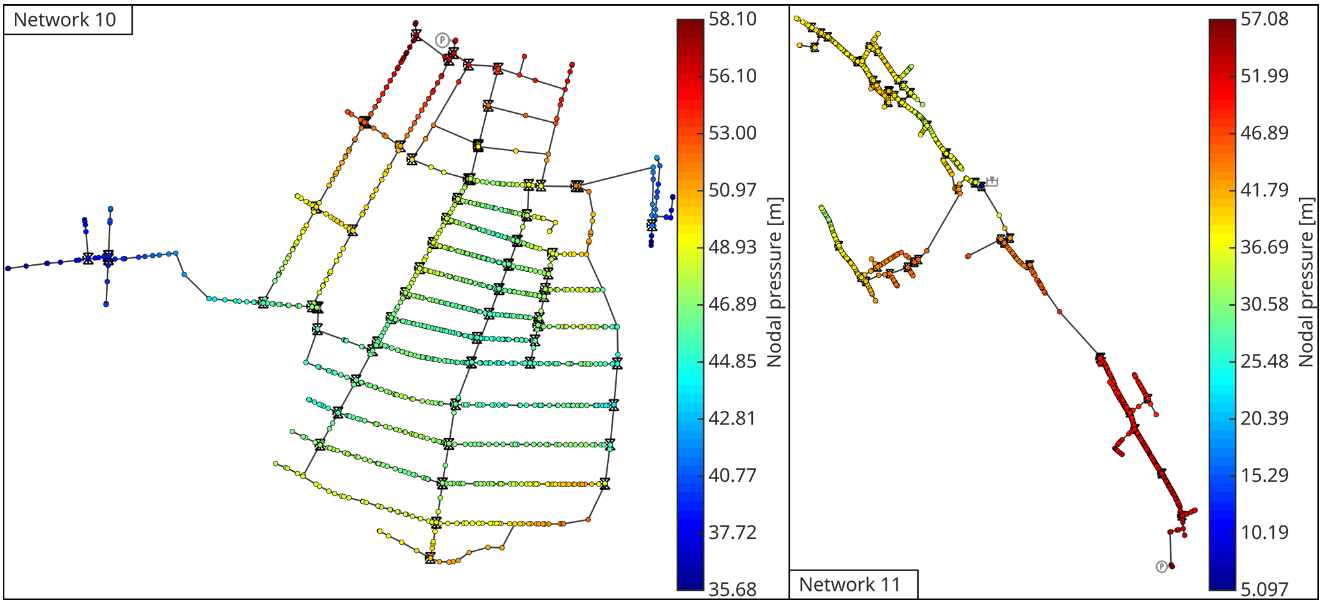

Fig. 2 The nodal pressure distribution of Network 10 and 11

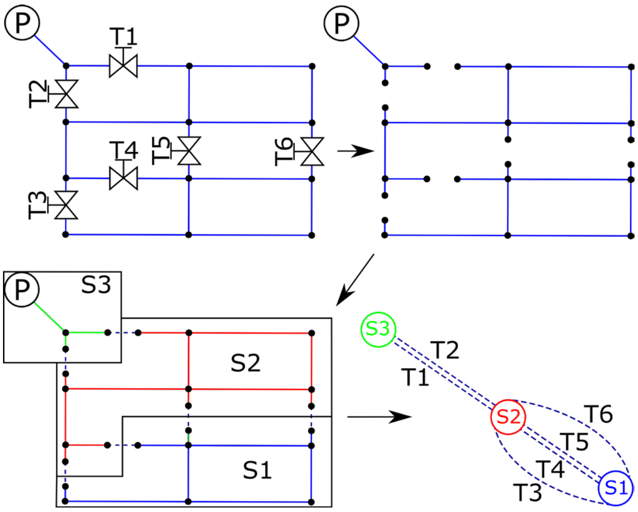

Fig. 3 The process of network segmentation

4 Vulnerability analysis

In a previous study [12], the authors published the vulnerability analysis of the segment graphs of real-life WDNs. It revealed that segment graphs show the nature of random, connected planar graphs, i.e., strictly from a topological viewpoint, they can be considered robust, as the removal of several nodes (isolation of segments) cannot significantly impact the overall integrity of the graph. However, the vulnerability analysis from the demand perspective, where the hydraulics is also considered, highlighted the exposed nature of real-life WDNs, i.e., some segments have a superior role in terms of the reliable functionality of the system. This paper follows the idea of vulnerability, and it analyses its connection with the network topology parameters.

In order to calculate the preliminary introduced vulnerability parameter, first, the failure rate (α_i) has to be considered, which is based on real-life pipe failure statistics, the pipeline length, material, and age. Several other factors can be considered as well, e.g., the pH value of the surrounding soil or the density of traffic at the surface. This quantity represents the possibility of having a random pipe burst in the i^{th} segment, i.e., α_i is between 0 and 1. Besides the failure rate, the amount of drinking water that cannot be provided in the case of the segregation of the i^{th} segment is calculated using the 1D hydraulic solver, STACI, i.e.

$$b_i = \sum d_i - \sum c_i ,$$

where d_i indicates the nominal demand and c_i represents the actual amount of served water according to the model. The variable b_i is also normalized using the overall nominal demand, i.e.

$$\beta_i = \frac{b_i}{\sum d_i} ,$$

which shows the relative demand loss caused by the isolation of the i^{th} segment. Using the preliminarily defined metrics, the failure rate (α_i) and the relative consumption loss (β_i), the local vulnerability of a WDN with respect to the i^{th} segment is the product, that is

$$\gamma_i = \alpha_i \beta_i .$$

According to this dimensionless metric, the local vulnerability of a segment is high if it contains a significant amount of pipelines (presumably with poor quality material), and the relative loss in the drinking water service is also high in the case of its isolation. Let us consider two extreme situations: first, a segment closes to a major source where most of the water inflows, but there are only a few meters of pipelines, the local vulnerability is low, as the probability of having a random pipe burst is small. Second, there is a segment at the edges of a large WDN with hundreds of meters of pipes; although its isolation affects only a few consumers, thus the vulnerability is still negligible. For the complete vulnerability analysis, a hydraulic simulation is required with every segment closed individually to determine the relative demand losses, i.e., β_i values. It means that, for a large WDN, thousands of calculations are necessary to perform, making it computationally inefficient.

5 Approximation of the vulnerability

5.1 Local vulnerability

To perform a complete vulnerability analysis, N_{seg} independent hydraulic simulation needs to be carried out with different topological layouts as different segments are closed in each calculation. For the local vulnerability, two quantities are required: the failure rate and the relative demand loss for each segment. While the first metric is a computational free sorting of a database (if available), but determining the consumption losses with numerous hydraulic simulations could require significant CPU time. Furthermore, to calculate the hydraulics, a well-built model is necessary that might not be available due to the WDN being in the design phase or, even in the case of operating networks, building a well-functioning accurate model requires a significant amount of engineering work. For these two important reasons (computational time and model being unavailable), an approximation of the relative consumption loss is relevant.

As more computational efficiency is needed, and relevant data might be not available, applying a topological approximation could be suitable. Fortunately, in terms of graph theory, numerous topological indicators appeared for describing real-life networks [9]. During recent years, engineers and researchers have been utilizing these tools for finding connections or correlations between topological and hydraulic aspects [14–16], e.g., algebraic connectivity and resilience [17] or betweenness centrality and the location of pipe bursts [18]. First, the idea was to find similar relationships between the vulnerability and a topological property, but besides using the "traditional" link-node representation of the network, also using the segment graph, where the nodes are segments and the isolation valves are the edges, see [4, 7, 12]. However, a significant correlation was not achieved; thus, a different approach was applied.

The exact topology of the segments is indispensable, i.e., the layout of the segments and the isolation valves between them must be known. The relative consumption loss of a segment comes from three components:

- the segment itself,
- the unintentionally closed segments,
- and the topologically connected areas where the demands cannot be fulfilled due to insufficient pressure.

Independently from the exact demand distribution, the first part can be easily calculated. At the same time, the second can also be determined without the need for a detailed hydraulic model since only the unintentionally closed segments must be determined by basic graph tools. Overall, the approximated relative demand loss equals the sum of the nominal demands from all closed segments [19]. The question is what information is available about the distribution of the demands between the segments. This paper considers three scenarios possible depending on what data is available:

1. only topological information (i.e., the segment graph),
2. topology and pipeline lengths,
3. topology and nominal demand distribution (e.g., from billing system in the case of operating networks).

The first scenario means a uniform demand distribution between the segments, i.e., each segment contains the same amount of demand. If pipeline length data is available, it can be used as a weight factor to distribute the overall demands, meaning segments with more meters of pipelines consist of more demand proportionally. The third scenario is equivalent to a standard pressure-driven simulation, as it is also not capable of handling pressure-dependent demands. These approaches are summarized in Table 3.

The approximated relative demand losses were determined for every 27 real-life WDN with each approach: segment ($\beta_{seg,i}$), length ($\beta_{len,i}$), and demand ($\beta_{dem,i}$) based, then the Pearson correlation coefficients [20] were determined between the local vulnerability (calculated by hydraulic simulation) and the approximations. The results can be

Table 3 Scenarios for approximating the relative demand loss without a hydraulic simulation using topological data

Name	Notation	Additional data	Assumption for demands
Segment	$\beta_{seg,i}$	None	Uniform distribution
Length	$\beta_{len,i}$	Pipeline lengths	Weighted distribution
Demand	$\beta_{dem,i}$	Demand distribution	None

seen in Fig. 4 using a correlogram [21]. The three columns represent the three approximations, while the rows belong to different WDNs. As it depicts, in most of the WDNs, the correlation coefficients of demand-based approaches equal one, i.e., the approximation works perfectly. It means that in these networks, every connected segment in each isolation scenario is fully served, i.e., modeling pressure-dependent demands are not required. A similar analysis was performed in [19] for two smaller networks with the same conclusions. Moreover, the segment- and length-based approaches also give strong correlation coefficients, except Network 12, 20, 21, and 24. The reason behind the differences is that these WDNs contain several large demands, e.g., a plant producing mineral water, and the assumption of a uniform demand distribution is severely inaccurate.

5.2 Network vulnerability

Besides the importance of the distribution of local vulnerabilities inside a network, it is also useful to evaluate the overall quality of a network compared to other WDNs. The utility company's decision-making can be supported in the optimal allocation of maintenance and development resources. Therefore, the network vulnerability Γ is introduced as the weighted average of the relative demand losses (β_i), similarly to [11, 19], where the weights are equal to the failure rate (α). Since the failure rate is a normalized value ($\sum \alpha_i = 1$), it can also be rearranged:

$$\Gamma = \frac{\sum_i \beta_i \alpha_i}{\sum_i \alpha_i} = \sum_i \beta_i \alpha_i = \sum_i \gamma_i \ .$$

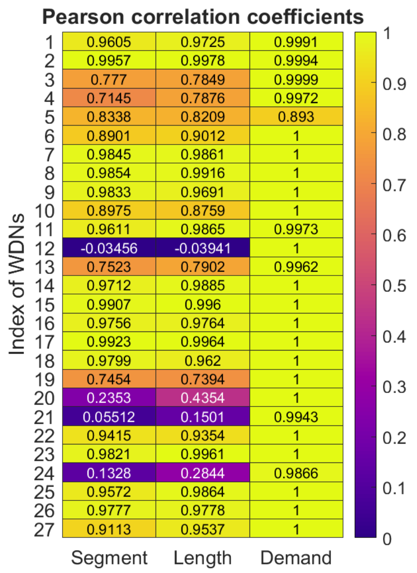

Fig. 4 Pearson correlation coefficients between the relative consumption loss (β_i) and the topological approximations ($\beta_{seg,i}$, $\beta_{len,i}$, $\beta_{dem,i}$)

It means that the sum of the local vulnerabilities is equal to the network vulnerability. Γ is the expected value of the amount of water loss in the case of a single, accidental pipe break according to the hydraulic model.

The network vulnerability can be estimated using the approximated relative demand losses in all three scenarios, mathematically

$$\Gamma_{seg} = \sum_i \beta_{seg,i} \alpha_i$$

$$\Gamma_{len} = \sum_i \beta_{len,i} \alpha_i$$

$$\Gamma_{dem} = \sum_i \beta_{dem,i} \alpha_i \ .$$

The $\beta_{seg,i}$, $\beta_{len,i}$, and $\beta_{dem,i}$ are the approximated outage values in segment-, length- and demand-based scenarios. Fig. 5 represents the results for the approximation of the network vulnerability, where the original hydraulic-based is on the horizontal axis, and the segment, length, and demand-based ones are on the vertical axis. The legend at the upper-left corner contains the Pearson correlation coefficients. As it shows, even the segment-based approximation shows a strong correlation, while the demand-based is close to a perfect match. It suggests that the vulnerability can be approximated without a detailed hydraulic model or numerous hydraulic calculations. It can be a powerful tool during the design or re-design phase of real-life WDNs.

5.3 Isolation valve layout

The distribution of the vulnerability revealed a highly unfavorable phenomenon; that is, the local vulnerability of some segments from every real-life WDN (also the artificial ky networks) is higher with several orders of

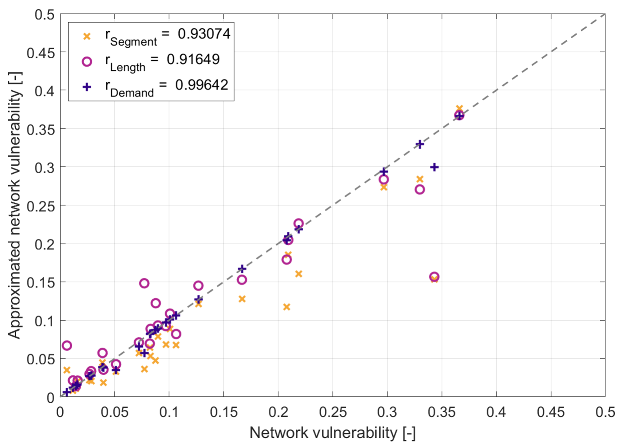

Fig. 5 Approximated (Γ_{seg}, Γ_{len}, and Γ_{dem}) and original (Γ) values of the network vulnerability. The legend at the upper-left corner contains the Pearson correlation coefficients.

magnitudes than the average. The question is: it is possible to achieve a more beneficial, e.g., uniform or normal, distribution of the local vulnerability? The isolation valve placement decisively influences the major properties of WDNs, e.g., the resilience [22]. Therefore, the idea in this section is to replace every isolation valve of the WDN in order to minimize the deviation of the vulnerability values. It means that every isolation valve is deleted from the model virtually; then, with the help of an optimization technique, a new, more favorable layout is determined.

The purpose is to decide whether this is possible and not suggest a perfect method for finding isolation valve layout in general. Since optimization techniques typically require countless function evaluations, during the fitness function calculation, that is, the standard deviation of the local vulnerabilities, the demand-based approximation is applied. The objective function (OF) is chosen to be the standard deviation of the local vulnerabilities

$$\mathrm{OF} = \sqrt{\frac{\sum_{i=1}^{N_{seg}} \left(\gamma_{dem,i} - \gamma_{dem,m}\right)^2}{N_{seg} - 1}} ,$$

where $\gamma_{dem,i} = \alpha_i \, \beta_{dem,i}$, i.e., local vulnerability using the demand-based approximation, and $\gamma_{dem,m}$ is the average.

Network 10 and 11 are used for optimization because these networks are large enough to represent a detailed distribution of local vulnerabilities but small enough to keep the run time within days on an average PC. Network 10 is a grid-like network from a topological point, i.e., it is highly looped, while Network 11 is close to a tree graph with negligible redundancy. As the correlation coefficient of Network 10 equals 1, while in the case of Network 11 equals 0.9973 (see Fig. 5), the negligence of using the approximated vulnerability instead of the original hydraulics-based is acceptable. Also, similar results were concluded in [19] during optimizing the isolation valve layout. According to [23], the topology optimization of WDNs has been a well-established research field since the works of [24]. Nowadays, since the work of [25], genetic algorithms have been among the most widely used techniques of WDN topology optimization. According to [26–28], the actual implementation and parameter settings of efficient genetic algorithms (GA) are always strongly connected to the features of the search space. The presented results were reached using a python package, namely *geneticalgorithm 1.0.2*, where the results of the parameter optimization for the different cases can be seen in Table 4.

Table 4 Scenarios for approximating the relative demand loss without a hydraulic simulation using topological data

Property	Value
Population size	50
Mutation probability	0.01
Elit ratio	0.05
Crossover probability	0.8
Parents portion	0.1
Crossover type	uniform
Max iteration without improvement	500

The parameter which was modifiable by the optimization was the placement of the isolation valves. There was one input for every location: the pipeline number, which will have this new valve. The parameter was an integer value selected between 0 and the number of edges in the network.

The results for the networks with the optimized isolation valve layout can be seen in Fig. 6. The left-hand side depicts the results for Network 11, while the right-hand

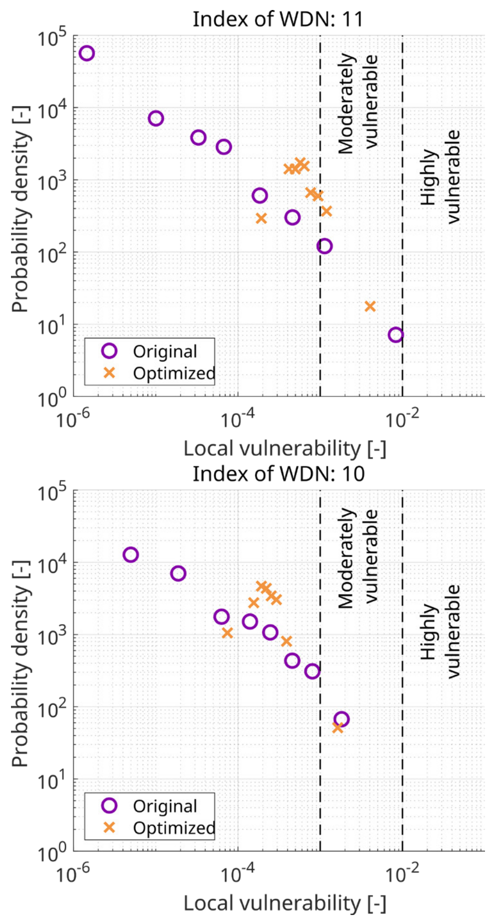

Fig. 6 Local vulnerability distribution for Network 11 (a) and Network 10 (b) with the original ("O" markers) and the optimized ("X" markers) isolation valve layout

side for Network 10. As it shows, the power-law distribution transformed into a significantly favorable one similar to a normal distribution. In the case of the original layout, the amplitude differences between the least and most vulnerable segments are in four orders of magnitudes. In comparison, the optimization could decrease it to two orders. It means the spatial distribution is also more homogeneous. The numerical values of the objective function can be seen in Table 5, and as it indicates, the standard deviation values could be significantly reduced. Moreover, the maximum values of the local vulnerabilities also decreased, e.g., the most exposed part of Network 11 moved from the highly vulnerable region to the moderately vulnerable one.

6 Conclusions

This paper presented an isolation valve placement technique to reduce the vulnerability of highly-exposed real-life WDNs. To determine the hydraulics of the networks, an in-house software called STACI, was utilized to handle pressure-dependent demands and isolated segments. Moreover, three different methods were presented to approximate the relative consumption losses (segment-, length- and demand-based); thus, the vulnerability can be modeled without a detailed hydraulic model. Finally, using a standard genetic algorithm (GA), an optimal isolation valve layout was calculated using two real-life WDNs to reduce the highly exposed areas of networks. Based on the results, the following conclusions can be drawn:

- The approximation of the relative demand losses unveiled that the demand-driven hydraulic simulation is only necessary in the case of only several segment losses, and most cases can be modeled without it. Moreover, if the network does not contain any significant outlier in terms of the demands, the consumption losses can be approximated using a uniform distribution of the demands. This gives an effective tool for the designers, and it can be used during an optimization technique for an objective function as it is computationally more efficient to evaluate.

Table 5 Objective function (OF) values are the standard deviation of the demand-based approximated local vulnerabilities for Network 10 and 11

WDN index	Original OF	Optimized OF	Relatively [%]
10	6.763e-4	2.615e-4	61.33
11	3.075e-3	8.615e-4	71.98

- Finding the isolation valve layout with a standard GA could mitigate the effect of the originally unfavorable power-law distribution. The standard deviation was decreased significantly; moreover, the range between the maximum was reduced with two orders of magnitudes.

Besides the conclusions, there are some open questions and directions for further research, especially regarding the isolation valve placement. Although the GA could prove that replacing the isolation valves can improve the quality of the vulnerability distribution, it is not convenient from a practical perspective. Even with the computationally favorable objective function, the calculation time for a middle-sized WDN (several thousands of pipelines) is in the order of days using a PC with an average performance. It would be helpful for the designers or the utility company to have a direct method with negligible computing time while ensuring favorable vulnerability distribution. Finally, it is a question, how the number of isolation valves affects the deviation of the vulnerability or the network vulnerability, especially if the cost is also considered.

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