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# CONTRIBUTIONS TO MEASURING OF SCREW SURFACES

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**Abstract**

The derivation of geometrical parameters of the screw surfaces will be illustrated with the exact mathematical forms. We give the method for the calculating of pitch diameter and the lead.

*Keywords:* screw surface, coordinate measuring, Maple V.

## 1. Introduction

The helicoidal surface is one of the most important formS of nature. Typical sampleS of screw surfaces are DNA double helix, marine shells, animal horns et al.

In the ornamental arts the application of helicoidal forms is frequent also.

The first screw applications were in the Neolithic time. The Eskimos made their arrows from seal-tooth by threading the twisted root into the arrow shaft.

The first technical application of screw was in the 24$^{th}$ century BC. On old Egyptian pictograms one can see extraction of oil from seeds by twisting woven fabric bags. The inventor of the first working mechanical application of screw was Archimedes, the famous Greek mathematician and physicist of antiquity. In the Roman Empire the screw was often applied to lift water. The great Greek engineer and inventor Heron from Alexandria made ingenious instruments with screws and worms.

By the fall of the Roman Empire the technical decadence began, but during the migration of nations the fixing of jewelry by screws was used. In the Middle Ages the parts of knights armor were held in place by screws. A particular application of the screw was in torture, used by hangmen from XIV. to XVIII. century. The printing of books from XV-XIX. century used screw presses based on Gutenberg origin design.

Leonardo da Vinci was the most famous engineer of all times, but unfortunately his inventions were not recognized. In his machinery designs, screw frequently played a part.

The general application of the screw begins from the invention of Henry Maudslay in 1797. With the 'English lathe' one could now produce accurate bolts. The first standards of thread and the efficient manufacturing of screws was invented by Joseph Whitworth at middle of the XIX. century.

The important book about screws is ‘Gewinde’ (in German) from Georg Berndt (1880-1972). The first publication of measuring screw threads by 3-wire method can be read in it. This is a generally accepted method, as for example in the American Standards.

The known and applied calculation of the 3-wire method is approximate. The total geometric solution is simplified in order to make calculations possible, so some parameters have to be ignored.

The measurement of geometrical parameters of screws is an important problem, but the calculation of its correct mathematical aspects is very difficult. By the use of a symbolic computation system (e.g. MapleV.) exact solutions can be found for the problem.

## 2. Ball or Wire Measuring of Pitch Diameter

Let’s give the characteristic curves of the screw surface in the form of

$$f_i = \begin{bmatrix} f_{ix}(u_i) \\ f_{iy}(u_i) \\ f_{iz}(u_i) \end{bmatrix} \qquad i = 1, 2. \tag{1}$$

The axis of the screw is in the $z$ axes of the coordinate system. The rotating matrices of the generating surfaces are

$$M_i = \begin{bmatrix} \cos(\varphi_i) & -\sin(\varphi_i) & 0 \\ \sin(\varphi_i) & \cos(\varphi_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad i = 1, 2. \tag{2}$$

The vector of the moving axial direction is proportional by rotating angle of $\varphi$.

$$w_i = \begin{bmatrix} 0 \\ 0 \\ \dfrac{p \cdot \varphi_i}{2 \cdot \pi} \end{bmatrix} \qquad i = 1, 2. \tag{3}$$

The general equation of screw surfaces is the following:

$$r_i = M_i \cdot f_i + w_i \qquad i = 1, 2. \tag{4}$$

The normal vectors of the surfaces are defined by the cross product of the partial derivatives

$$n_i = \frac{\partial r_i}{\partial u_i} \times \frac{\partial r_i}{\partial \varphi_i} \qquad i = 1, 2. \tag{5}$$

The measuring element (cylindrical wire or ball) touched the screw surfaces on the two opposite sides at two points. At the contact points the normal vectors of the screw surfaces and of the measuring element are identical.

The radius of the measuring element is $\rho$. The equation of the surfaces from the screw surface in the direction of the normal vector by $\rho$ distance is

$$R_i = r_i + \rho \cdot \frac{n_i}{|n_i|} \qquad i = 1, 2. \tag{6}$$

The common part of the two surfaces is the intersection line that can be expressed with the vector *Eq.* (6)

$$R_1 - R_2 = 0. \tag{7}$$

Let's given the parameter of profile $u_1$ to the pitch diameter of the screw and the rotation angle $\varphi_1$. By substituting them in the *Eq.* (4) the coordinates of the required point can be calculated. By substituting in the *Eq.* (5) the coordinates of normal vector of the required point in the surface can be calculated. By solving *Eq.* (7) the $u_2$, $\varphi_2$ parameters of the opposite side of the screw, and the radius $\rho$ of the measuring element can be calculated. This measuring element touches the two opposite (left and right) screw profiles in the pitch diameter. Coordinates of touched points in the opposite profiles can be calculated by *Eq.* (6).

If the value of $\rho$ is given, by substituting $\varphi_1$ = optional value in to the *Eq.* (7) the unknown $u_1$, $u_2$ and $\varphi_2$ parameters of the touching points in every two opposite profiles can be calculated. The three-ball or three-wire measurement of the screw is by the way of

$$T = 2 \cdot \left( \sqrt{r_{i,1}^2 + r_{i,2}^2} + \rho_x \right) \qquad i = 1, 2. \tag{8}$$

This calculating method is independent from the form and lead angle of screw profile, illustrated by Maple V program in Appendix I.

## 3. Coordinate Measuring of Lead

Let's find the lead of a given screw by coordinate measurement. Give the values of coordinates of center of the measuring ball by touching the opposite side of the surfaces of the screw

$$v_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \qquad i = 1, 2. \tag{9}$$

We find in the next equation the approximate screw line, because the center of the measuring ball during the measuring process moved in the actual screw line

$$r = \begin{bmatrix} R \cdot \cos(\varphi) \\ R \cdot \sin(\varphi) \\ p \cdot \varphi \end{bmatrix}. \tag{10}$$

It's evident that the original screw surface and the measuring data have derivations. *Eq.* (10) is just an approximation. The smallest vector from the measuring points (9) to the theoretical screw line (10) is the error vector

$$\delta_i = \begin{bmatrix} x_i - \rho \cdot \cos(\varphi_i) \\ y_i - \rho \cdot \sin(\varphi_i) \\ z_i - p \cdot \varphi_i \end{bmatrix} \qquad i = 1, 2, \ldots, n. \tag{11}$$

By the angle parameter $\varphi_i$ the equation of tangent vector of the screw line (10) is

$$t_i = \frac{\mathrm{d}}{\mathrm{d}\varphi} r \bigg|_{\varphi=\varphi_i} \qquad i = 1, 2, \ldots, n. \tag{12}$$

In the required points of screw line (10) the tangent vector (12) and the error vector (11) are perpendicular.

$$t_i \cdot \delta_i = 0 \qquad i = 1, 2, \ldots, n. \tag{13}$$

Expand the expression (13) by (12) and (11)

$$-R \cdot \sin(\varphi_i) \cdot x_i + R \cdot \cos(\varphi_i) \cdot y_i + p \cdot z_i - p^2 \cdot \varphi_i = 0 \qquad i = 1, 2, \ldots, n. \tag{14}$$

Let's make the starting value of $R = R_0$ and $p = p_0$. With this the numerical solution of *Eq.* (14) to $\varphi_i$ can be calculated.
The quadratic sum of error vectors is

$$H = \sum_{i=1}^{n} \delta_i^2 \qquad i = 1, 2, \ldots, n. \tag{15}$$

By expanding the *Eq.* (15) we get

$$H = \sum_{i=1}^{n} (x_i^2 - 2\rho \cos(\varphi_i) x_i + \rho^2 + y_i^2 - 2\rho \sin(\varphi_i) y_i + z_i^2 - 2 z_i p \varphi_i + p^2 \varphi_i^2). \tag{16}$$

The quadratic sum of error vectors is minimal, if

$$\begin{aligned} \frac{\partial H}{\partial p} &= 0, \\ \frac{\partial H}{\partial \rho} &= 0. \end{aligned} \tag{17}$$

By expanding the *Eq.* (17) we get

$$\begin{aligned} \frac{\partial H}{\partial p} &= \sum_{i=1}^{n} (-2 \cdot z_i \cdot \varphi_i + 2 \cdot p \cdot \varphi_i^2), \\ \frac{\partial H}{\partial \rho} &= \sum_{i=1}^{n} \{-2 \cdot x_i \cdot \cos(\varphi_i) + 2 \cdot \rho - 2 \cdot y_i \cdot \sin(\varphi_i)\} \end{aligned} \tag{18}$$

they are solvable to unknown $p$ and $\rho$ in a closed form

$$
\begin{aligned}
p &= \frac{\sum_{i=1}^{n} z_i \cdot \varphi_i}{\sum_{i=1}^{n} \varphi_i^2}, \\
\rho &= \frac{\sum_{i=1}^{n} \{x_i \cdot \sin(\varphi_i) + y_i \cdot \cos(\varphi_i)\}}{n}.
\end{aligned}
\tag{19}
$$

Substitute these values $p = p_1$ $R = R_1$ and repeat the calculation from step (14), while

$$\left|R_{k+1}^2 - R_k^2\right| + \left|p_{k+1}^2 - p_k^2\right| \geq \varepsilon_{\text{error}}. \tag{20}$$

The method is illustrated by Maple V program of the Appendix II.

## References


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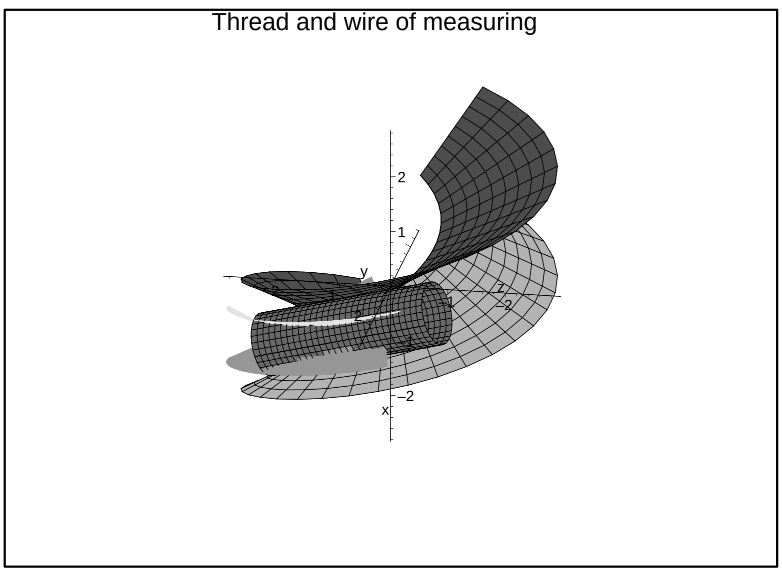
## Appendix I.

$restart$ ; with($linalg$) ;
with($plots$) ; with($plottools$) ;
$p := 4$ ; $a := \frac{3}{10}$ ; $\theta := \arctan(\frac{a}{2})$ ;
$d.tu := 1.1$ ; $\rho := \frac{d.tu}{2}$ ;
$b.1 := -.35$ ; $b.2 := .35$ ;
$c.1 := .5$ ; $c.2 := -.5$ ;
**for** $i$ **to** 2 **do**
$RR.i := \mathrm{evalf}(\frac{p\,\tan(\theta)\,\phi.i\,\cos(\theta)}{2\pi})$ ;
$f.i := \mathrm{vector}(3,\ [0,\ RR.i + u.i,$
$\frac{p\,\phi.i\,\cos(\theta)}{2\pi} + b.i + c.i\,u.i])$ ;
$M.i := \mathrm{matrix}(3,\ 3,$
$[\cos(\phi.i),\ -\sin(\phi.i),\ 0,$
$\sin(\phi.i),\ \cos(\phi.i),\ 0,$
$0,\ 0,\ 1])$ ;
$r.i := \mathrm{multiply}(M.i,\ f.i)$ ;
$e.i := \mathrm{map}(diff,\ r.i,\ u.i)$ ;
$f.i := \mathrm{map}(diff,\ r.i,\ \phi.i)$ ;
$n.i := \mathrm{evalm}(\mathrm{crossprod}(e.i,\ f.i))$ ;
$N.i := \mathrm{norm}(n.i,\ 2)$ ;
$R.i := \mathrm{evalm}(\frac{\rho\,n.i}{N.i})$
**od** ;
$axial := -.04\,p$ ;
$szog := \mathrm{evalf}(\frac{axial\,2\pi}{p\cos(\theta)})$ ;
$qq := \{\phi.1 = szog\}$ ;
$szog.1 := szog - 1$ ;
$szog.2 := szog + 1$ ;
$Q.1 := \mathrm{evalm}(r.1 - R.1)$ ;
$Q.2 := \mathrm{evalm}(r.2 + R.2)$ ;
**for** $s$ **to** 3 **do**
$Bb.s := \mathrm{evalm}(\mathrm{subs}(qq,\ Q.1_s))$ ;
$h.s := \mathrm{evalm}(Bb.s - Q.2_s)$
**od** ;
$mego := \mathrm{fsolve}($
$\{h.1 = 0,\ h.2 = 0,\ h.3 = 0\},$
$\{u.1,\ u.2,\ \phi.2\},$
$\{u.1 = 0..20,\ u.2 = 0..20,$
$\phi.2 = szog.1..szog.2\},\ fulldigits)$ ;
$hely := \{mego_1,\ mego_2,\ mego_3,\ qq_1\}$ ;

**for** $t$ **to** 3 **do**
  $w.1_t := \mathrm{evalf}(\mathrm{subs}(\mathit{hely},\ Q.1_t))$ ;
  $w.2_t := \mathrm{evalf}(\mathrm{subs}(\mathit{hely},\ Q.2_t))$ ;
**od** ;
$T := 2\,(\sqrt{w.1_1{}^2 + w.1_2{}^2} + \rho)$ ;

$$T := 5.039346158$$

$v := \mathrm{vector}(3,\ [])$ ; $w := \mathrm{vector}(3,\ [])$ ;
**for** $i$ **to** 3 **do**
  $v_i := \mathrm{evalf}(\mathrm{subs}(\mathit{hely},\ n.1_i))$ ;
  $w_i := \mathrm{evalf}(\mathrm{subs}(\mathit{hely},\ n.2_i))$
**od** ;
$t := \mathrm{crossprod}(v,\ w)$
$\phi 1.\mathit{min} := -2.75$ ;
$\phi 1.\mathit{max} := 2.75$ ;
$u.\mathit{min} := .7$ ;
$u.\mathit{max} := 2.7$ ;
$\phi 2.\mathit{min} := -1.5$ ;
$\phi 2.\mathit{max} := .2$ ;
$q.1 := \mathrm{plot3d}(r.1,$
  $u.1 = u.\mathit{min}..u.\mathit{max},$
  $\phi.1 = \phi 1.\mathit{min}..\phi 1.\mathit{max},$
  $\mathit{color} = \mathit{red},\ \mathit{grid} = [10,\ 30])$ ;
$q.2 := \mathrm{plot3d}(r.2,$
  $u.2 = u.\mathit{min}..u.\mathit{max},$
  $\phi.2 = \phi 1.\mathit{min}..\phi 1.\mathit{max},$
  $\mathit{color} = \mathit{cyan},\ \mathit{grid} = [10,\ 30])$ ;
$q.3 := \mathrm{plot3d}(Q.1,$
  $u.1 = u.\mathit{min}..u.\mathit{max},$
  $\phi.1 = \phi 2.\mathit{min}..\phi 2.\mathit{max},$
  $\mathit{color} = \mathit{yellow},\ \mathit{grid} = [10,\ 30],$
  $\mathit{style} = \mathit{patchnogrid})$ ;
$q.4 := \mathrm{plot3d}(Q.2,$
  $u.2 = u.\mathit{min}..u.\mathit{max},$
  $\phi.2 = \phi 2.\mathit{min}..\phi 2.\mathit{max},$
  $\mathit{color} = \mathit{green},\ \mathit{grid} = [10,\ 30],$
  $\mathit{style} = \mathit{patchnogrid})$ ;
$\mathit{wire} := \mathrm{tubeplot}([$
  $w.1_1 + \lambda t_1,\ w.1_2 + \lambda t_2,\ w.1_3 + \lambda t_3,$
  $\lambda = -.5...5,\ \mathit{radius} = \rho,$
  $\mathit{color} = \mathit{magenta},\ \mathit{grid} = [30,\ 30],$
  $\mathit{style} = \mathit{patchnogrid}])$ ;
$\mathrm{display}([q.1,\ q.2,\ q.3,\ q.4,\ \mathit{wire}],$
  $\mathit{axes} = \mathit{normal},$
  $\mathit{scaling} = \mathit{constrained},$
  $\mathit{labels} = [x,\ y,\ z],$
  $\mathit{orientation} = [100,\ 70],$
  $\mathit{style} = \mathit{patch},\ \mathit{title} =$
  *Thread and wire of measuring*) ;



## Appendix II

Symbolical extraction of approximate screw line

$restart$ ; with($linalg$) ;
with($stats$) ; with($plots$) ;
$x$ := vector(3, [$x.0$, $y.0$, $z.0$]) ;
r($p$, $\phi$) := vector(3,
 [$\rho$ cos($\phi$), $\rho$ sin($\phi$), $p\,\phi$]) ;
d($p$, $\phi$) := map($diff$ , r($p$, $\phi$), $\phi$) ;

$$\mathrm{d}(p,\ \phi) := [-\rho\ \sin(\phi),\ \rho\ \cos(\phi),\ p]$$

$\delta(p, \phi)$ := evalm($x - \mathrm{r}(p, \phi)$) ;

$$\delta(p,\ \phi) := [x0 - \rho\ \cos(\phi),\ y0 - \rho\ \sin(\phi),\ z0 - p\,\phi]$$

$c$ := expand(
 dotprod($\delta(p, \phi)$, d($p$, $\phi$))) ;

$$c := -\,\overline{\rho}\ \sin(\overline{\phi})\,x0 + \overline{\rho}\ \sin(\overline{\phi})\ \rho\ \cos(\phi) + \overline{\rho}\ \cos(\overline{\phi})\,y0 - \overline{\rho}\ \cos(\overline{\phi})\ \rho\ \sin(\phi) + \overline{p}\,z0 - \overline{p}\ p\,\phi$$

$u.1(p, \phi)$ := expand($\delta(p, \phi)_1{}^2 + \delta(p, \phi)_2{}^2 + \delta(p, \phi)_3{}^2$) ;

$$\mathrm{u1}(p,\ \phi) := x0^2 - 2\,\rho\ \cos(\phi)\,x0 + \rho^2\ \cos(\phi)^2 + y0^2 - 2\,\rho\ \sin(\phi)\,y0 + \rho^2\ \sin(\phi)^2 + z0^2 - 2\,z0\ p\,\phi + p^2\,\phi^2$$

u($p$, $\phi$) := combine($u.1(p, \phi)$, $trig$) ;

$$\mathrm{u}(p,\ \phi) := x0^2 - 2\,\rho\ \cos(\phi)\,x0 + \rho^2 + y0^2 - 2\,\rho\ \sin(\phi)\,y0 + z0^2 - 2\,z0\ p\,\phi + p^2\,\phi^2$$

$p.1(p, \phi)$ := expand($\frac{\partial}{\partial p}$ u($p$, $\phi$)) ;

$$\mathrm{p1}(p,\ \phi) := -2\,z0\,\phi + 2\,p\,\phi^2$$

$p.2(p, \phi)$ := expand($\frac{\partial}{\partial \rho}$ u($p$, $\phi$)) ;

$$\mathrm{p2}(p,\ \phi) := -2\ \cos(\phi)\,x0 + 2\,\rho - 2\ \sin(\phi)\,y0$$

$\rho.1 := 10$ ; $p.x := 5$ ;
$\phi.1 := .314$ ; $N := 20$ ;
$a.1 := -1$ ; $a.2 := 1$ ;
$\varepsilon := 10^{(-6)}$ ; $S := 0$ ;
$sys$ := [$\rho.1$ cos($k\,\phi.1$),
 $\rho.1$ sin($k\,\phi.1$), $p.x\ k\,\phi.1$] ;
$points$ := [seq([$\rho.1$ cos($k\phi.1$),
 $\rho.1$ sin($k\phi.1$) +
 $random_{uniform_{a.1,a.2}}(1)$,
 $p.x\ k\,\phi.1$ +
 $random_{uniform_{a.1,\,a.2}}(1)$],
 $k = 1..N$)] ;
$u$ := pointplot3d(
 $points$, $axes = normal$,
 $color = red$, $connect = false$,
 $symbol = DIAMOND$) ;
$v$ := tubeplot($sys$, $k = 1..N$,
 $numpoints = 100$, $radius = .1$,
 $color = blue$, $style = patchnogrid$,
 $scaling = constrained$,
 $orientation = [30, 30]$) ;
display({$u$, $v$}) ;
THE PICTURE COMES HERE
$\rho := 9.9$ ; $p := 4.98$ ;
$ppp := 5.1$ ; $rrr := 10.2$ ;
**while**
 $\varepsilon < (p - ppp)^2 + (\rho - rrr)^2$
**do**
 $S := S + 1$ ;
 $p := ppp$ ;
 $\rho := rrr$ ;
 **for** $i$ **to** $N$ **do**
 $\theta_i$ := fsolve(

$$- \rho \ \sin(\phi)\, points_{i,\,1}$$
$$+ \rho \ \cos(\phi)\, points_{i,\,2}$$
$$+ p\, points_{i,\,3}$$
$$- p^2\, \phi = 0,$$
$$\phi,\ \phi = 0..6.3,\ fulldigits)$$

**od** ;
**for** *aa* **to** *N* **do**

$aaa_{aa} := points_{aa,\,1}$ ;
$bbb_{aa} := points_{aa,\,2}$ ;
$ccc_{aa} := points_{aa,\,3}$

**od** ;

$$ppp := \frac{\sum_{o=1}^{N} ccc_o\, \theta_o}{\sum_{oo=1}^{N} {\theta_{oo}}^2}\ ;$$

$$rrr := \frac{\sum_{bb=1}^{N} (\sin(\theta_{bb})\, bbb_{bb} + \cos(\theta_{bb})\, aaa_{bb})}{N}$$

**od** ;
print(*Number of iteration steps* =, $S$) ;
print(*Radius* =, *rrr*) ;
print(*Lead* =, *ppp*) ;

$$Number\ of\ iteration\ steps = 4$$
$$Radius = 9.974139820$$
$$Lead = 5.038533456$$