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TWO-PHASE FLOW IN THE VERTICAL PIPELINE OF AIR LIFT

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Abstract

In this paper physical parameters necessary to dimensioning equipment of pneumatic conveying of greater mass flow conveying materials to high vertical distances have been investigated.

Keywords: two-phase flow, pneumatic conveying, pressure drop, particle velocity, concentration.

Introduction

Within the framework of a Research and Development co-operation established between the Erőterv – Waagner Bíró Environment Protection Co., Ltd. (EWB) and the Department of Fluid Machinery of the Technical University of Budapest theoretical and experimental research and development works have been done to study physical relationships of solid material conveying in pipes and channels. The program of this Research and Development is matched to the main profile and professional activity of the EWB Company that are planning of equipment for conveying fly ash, slag, bed ash and limestone. This Research and Development includes experimental investigation of conveying parameters depending on the type of materials, computer aided elaboration of basic data obtained from measurements necessary to the planning of equipment as well as development of methods serving for calculating longitudinal pressure and velocity distributions of conveying pipelines of optional diameters and tracks. The basis of the research work is a pneumatic transport equipment with top/bottom discharge blow tank and air lift installed at the Department of Fluid Machinery but being property of the EWB as well as softwares developed for evaluating measuring results as well as for determining main dimensions and working parameters.

The authors take the pipe-line cross section reducing effect of solid materials into consideration and their calculation method published earlier [1] is supplemented with this consideration. This effect cannot be neglected for low density materials being present in high concentrations. The authors demonstrate the modified values of parameters calculated with the two methods through an actual example.

1. Determining Characteristic Parameters of the Two-Phase Flow

1.1. Restrictions in Formulating the Equations

a) Flow velocity distribution within the pipe cross-section is considered to be uniform.
b) Force originating from impacts of particles.
c) Change of state of the gas is supposed to be isotherm.
d) Effects originating from rotation of particles are not taken into consideration.
e) Particles are considered to be spherical.

1.2. Equations Formulated for the Solid Material Particles

Fig. 1 shows the scheme of pipeline section of cross section 'A', surrounded by the control surface, as cut out from the vertical conveying pipeline.

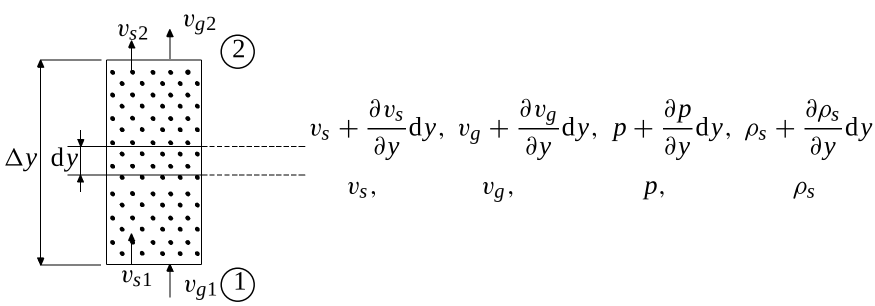

Fig. 1. Elementary pipe section cut from a vertical pipeline

Material concentration in the 'Ady' volume element, and the change of the concentration:

$$\rho_s = \frac{q_s\,\mathrm{d}y}{A\,\mathrm{d}y} = \frac{\dot{m}_s}{Av_s}; \qquad \mathrm{d}\rho_s = -\frac{\dot{m}_s}{A}\frac{\mathrm{d}v_s}{v_s^2}. \tag{1}$$

Continuity equation formulated for the moving particle in this control surface:

$$\rho_s v_s A = (\rho_s - \mathrm{d}\rho_s)(v_s + \mathrm{d}v_s)A. \tag{2}$$

Momentum equation written for particles being in the control volume cut out from a vertical air lift conveying pipe shown in *Fig. 1* is as follows:

$$-\rho_s v_s^2 A + (\rho_s - \mathrm{d}\rho_s)(v_s + \mathrm{d}v_s)^2 A = \mathrm{d}F - \mathrm{d}F_f - \mathrm{d}G_s. \tag{3}$$

In *Eq.* (3) '$\mathrm{d}F$' is the drag force, '$\mathrm{d}F_f$' is the breaking force originating from collision of particles with the pipe wall, '$\mathrm{d}G_s$' is the weight of particles being in the elementary control volume.

After reduction and taking *Eq.* (2) into consideration the left side of *Eq.* (3) can be written as follows:

$$\rho_s v_s A \, \mathrm{d}v_s = \mathrm{d}F - \mathrm{d}F_f - \mathrm{d}G_s. \tag{4}$$

Drag force acting on solid particles is:

$$\mathrm{d}F = \mathrm{d}N \frac{\rho_g}{2} A_0 C_D (v_g - v_s)^2. \tag{5}$$

'$\mathrm{d}N$' in this formula is the number of particles being in the control volume which can be expressed with 'm_1' particle mass in the following way:

$$\mathrm{d}N = \frac{\mathrm{d}m_s}{m_1} = \rho_s \frac{A}{m_1} \, \mathrm{d}y. \tag{6}$$

'C_D' drag coefficient after KASKAS [3] is as follows:

$$C_D = \frac{24}{\mathrm{Re}} + \frac{4}{\sqrt{\mathrm{Re}}} + 0.4; \qquad \mathrm{Re} = \frac{(v_g - v_s)\mathrm{d}_p}{\eta_g} \rho_g.$$

It results from conservation of the gas mass flow that:

$$\dot{m}_g = A_g \rho_g v_g = (A - A_s) \rho_g v_g = (A - A_s) \frac{\rho_{g0}}{p_0} p v_g. \tag{7}$$

By denoting density of the solid material with 'ρ_s^0' the $A_s = \frac{\dot{m}_s}{v_s \rho_s^0}$ cross-section indicates the cross-section reducing effect caused by the material. The value of this can be determined so that particles conveyed with 'v_s' velocity are considered as material flow with reduced cross-section enclosed into a compact flow of 'A_s' cross-section which flows parallel with the gas flow of 'v_g' velocity and of $A_g = A - A_s$ cross-section.

Solving formula (7) for 'v_s' gas velocity, and using the symbol $\beta = \frac{A_s}{A} = \frac{\dot{m}_s}{A v_s \rho_s^0}$ it is obtained that

$$v_g = \frac{\dot{m}_g p_0}{(A - A_s)\rho_{g0} p} = \frac{\dot{m}_g p_0}{A(1-\beta)\rho_{g0} p}. \tag{8}$$

By taking the above formulas into consideration, the drag force is as follows:

$$\mathrm{d}F = \frac{\pi_0}{2} \frac{\dot{m}_s}{A v_s} \rho_{g0} \frac{p}{p_0} \left(v_{g0} \frac{p_0}{p} - v_s \right)^2 \mathrm{d}y. \tag{9}$$

Force 'F_f' – breaking one particle – results from collisions with pipe wall and is supposed acting continuously. Following the thoughts of PÁPAI [2] this force loses a fraction 'ξ' of their kinetic energy, that is:

$$F_f = \frac{\xi}{D} \frac{m_1 v_s^2}{2} = k_i \frac{m_1 v_s^2}{D}. \tag{10}$$

In this formula 'k_i' is a factor characteristic of the conveyed material and is to be determined experimentally, and 'D' is the pipe diameter.

After these considerations the elementary breaking force affecting material particles found in a control volume is:

$$\mathrm{d}F_f = \frac{k_i}{D}\dot{m}_s v_s\,\mathrm{d}y. \tag{11}$$

The weight of material particles found in the control volume is:

$$\mathrm{d}G_s = \frac{\dot{m}_s}{v_s} g\,\mathrm{d}y. \tag{12}$$

Taking the above equations into consideration the following differential equation is obtained:

$$\frac{\mathrm{d}v_s}{\mathrm{d}y} = \pi_2 \frac{p}{v_s}\left(\frac{\dot{m}_g p_0 v_s \rho_s^0}{(A v_s \rho_s^0 - \dot{m}_s)\rho_{g0} p} - v_s\right)^2 - \pi_3 v_s - \frac{g}{v_s}. \tag{13}$$

1.3. *Equations Formulated for the Conveying Gas*

Momentum equation written for the gas found in the volume enclosed into the control surface is:

$$-\rho_g v_g^2 A + (\rho_g - \mathrm{d}\rho_g)(v_g + \mathrm{d}v_g)^2 A = -pA + (p - \mathrm{d}p)A - \mathrm{d}F - \mathrm{d}F_{fD} - \mathrm{d}G_s. \tag{14}$$

In *Eq.* (14) '$\mathrm{d}F_{fD}$' is friction of the conveying gas on the pipe wall. The equation established for the elementary pipe section takes the following form:

$$\mathrm{d}F_{fD} = \pi_1 \rho_g v_g^2\,\mathrm{d}y. \tag{15}$$

The change of the gas velocity can be obtained by means of *Eq.* (8). It can be written that:

$$\mathrm{d}v_g = \frac{\partial v_g}{\partial v_s}\mathrm{d}v_s + \frac{\partial v_g}{\partial p}\mathrm{d}p = \alpha_1\,\mathrm{d}v_s + \alpha_2\,\mathrm{d}p. \tag{16}$$

After reduction *Eq.* (14) established for gas will take the following form:

$$\frac{\mathrm{d}p}{\mathrm{d}y} = \frac{\dfrac{\rho_s^0}{A v_s \rho_s^0 - \dot{m}_s}\left[\pi_5 p \left(\dfrac{\dot{m}_g p_0 v_s \rho_s^0}{(A v_s \rho_s^0 - \dot{m}_s)\rho_{g0} p} - v_s\right)^2 - \pi_1 \dfrac{\dot{m}_g^2 p_0 v_s^3 \rho_s^{02}}{(A v_s \rho_s^0 - \dot{m}_s)^2 \rho_{g0} p} - \pi_7\right]}{1 - 3\dfrac{\dot{m}_g^2 p_0 v_s^2 \rho_s^{02}}{(A v_s \rho_s^0 - \dot{m}_s)^2 \rho_{g0} p^2}} +$$

$$+\frac{\dfrac{\dot m_g^2 p_0 \dot m_s \rho_s^{02}}{(Av_s\rho_s^0-\dot m_s)^3\rho_{g0}p}\left[\pi_2 p\left(\dfrac{\dot m_g p_0 v_s \rho_s^0}{(Av_s\rho_s^0-\dot m_s)\rho_{g0}p}-v_s\right)^2-\pi_3 v_s^2-g\right]}{1-3\dfrac{\dot m_g^2 p_0 v_s^2\rho_s^{02}}{(Av_s\rho_s^0-\dot m_s)^2\rho_{g0}p^2}}. \tag{17}$$

The flow parameters of the two phase mixture flowing in the vertical pipe are described by differential equations (13) and (17).

The differential equation system

$$\begin{aligned}\frac{\mathrm{d}p}{\mathrm{d}y} &= f_1(y, p, v_s),\\ \frac{\mathrm{d}v_s}{\mathrm{d}y} &= f_2(y, p, v_s),\end{aligned}$$

with boundary conditions $v_s = v_{s1}$ and $p = p_1$ at $y = 0$ were solved using the Runge-Kutta method embedded into an iteration cycle. p_1 at $y = 0$ is not known, $p_L = p_0$, $y = L$. In the iteration cycle p_1 is altered until $p_L = p_0$ is satisfied within a given error range.

2. Results Obtained as Solution of the System

2.1. DE. System Has Been Solved for the Demonstration Example by Using the Following Data:

Conveyed material;	PVC powder
Length of the vertical conveying pipe:	$L = 50$ m
Pipe diameter:	$D = 150$ mm
Mass flow of the solid material:	$\dot m_s = 25$ t/h
Mass flow of the conveying gas:	$\dot m_g = 0.27$ kg/s
Average mass of one particle:	$m_1 = 2.21 \cdot 10^{-9}$ kg
Average particle size:	$d_p = 150 \cdot 10^{-6}$ m
Absolute viscosity of the air ($t = 20\,°$C)	$\eta_g = 1.85 \cdot 10^{-5}$ kg/m·s
Pressure at the pipe end:	$p_2 = p_0 = 10^5$ Pa
Air density at the pipe end:	$\rho_{g0} = 1.2$ kg/m^3
Impact factor:	$k_1 = 0.01$
Pipe friction factor:	$f = 0.02$

2.2. On Diagrams Demonstrating Results of Solution of the DE System the Following Physical Quantities Have Been Used:

a. 'P_{pol}' polytrope power introduced at $y = 0$ (identical with the useful compressor power):

$$P_{\text{pol}} = \frac{p_0}{\rho_{g0}} \dot{m}_g \frac{n}{n-1} \left[\left(\frac{p_1}{p_0} \right)^{(n-1)/n} - 1 \right]. \tag{18}$$

b. 'e' specific energy consumption of conveying is given by the following equation:

$$e = \frac{P_{\text{pol}}}{L \dot{m}_s}. \tag{19}$$

c. 'μ' mixing ration is interpreted in the following way:

$$\mu = \frac{\dot{m}_s}{\dot{m}_g}. \tag{20}$$

d. 's' slip originating from velocity difference between solid material and gas is defined as:

$$s = \frac{v_g - v_s}{v_g}. \tag{21}$$

2.3. Diagrams Obtained as Result of the Solution

Comparison of results obtained with and without taking cross-section reducing effect into consideration.

In *Fig.* 2 change of 'p' pressure can be seen along the conveying pipe-line. *Fig. 3* shows the same parameters relating to the initial section of the pipe. From the figures it can be concluded that pressure in the initial part of the pipe for model taking cross-section reducing effect into consideration is:

$$p_1 = 160.756\,\text{kPa} \qquad \beta > 0$$

and for model without considering cross-section reducing effect it is:

$$p_1 = 159.336\,\text{kPa} \qquad \beta = 0.$$

Change of pressure along the pipeline is the highest value in the initial part of the pipeline for both models, because acceleration of the material is the highest here.

Fig. 4 shows change of 'v_s' material velocity along the pipe while in *Fig. 5* velocity can be seen in the initial section of the pipe. Increase in material velocity in the initial section of the pipe is significant due to the acceleration of material while in the further section of the conveying pipe material velocity increases gradually, nearly linearly due to the expansion of gas. On the basis of comparing the two

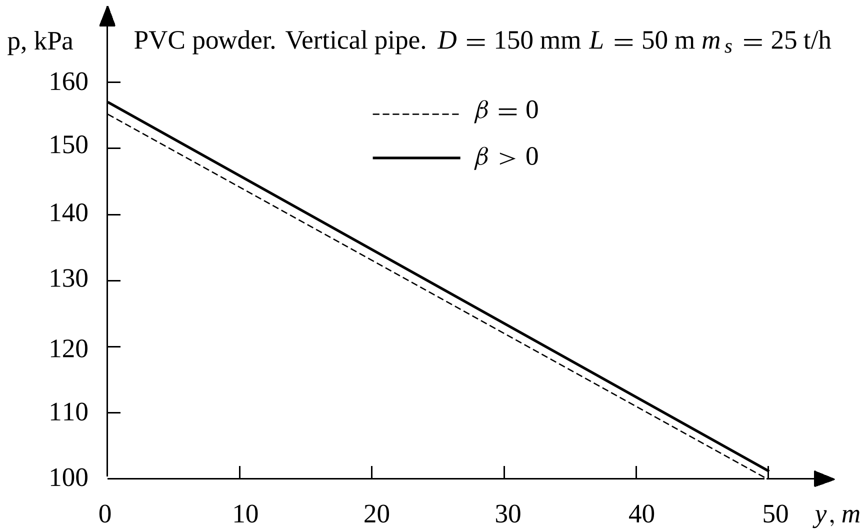

Fig. 2. Calculated pressure as function of the pipe length

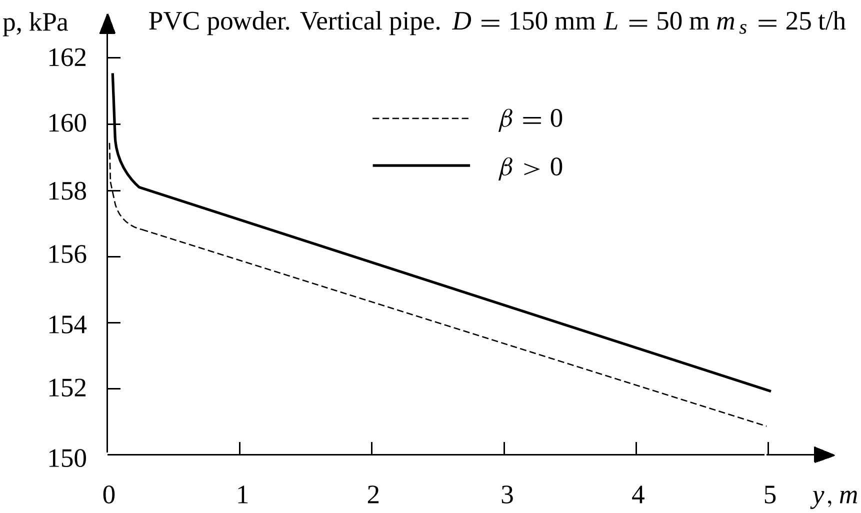

Fig. 3. Calculated pressure as function of the pipe length in the initial section of the pipe

models it can be stated that material velocity increases to a higher value when taking presence of material into consideration than in the case if model is calculated without cross-section reduction.

In *Fig. 6* 'v_s' gas velocity can be seen along the pipe length while this velocity is shown for the initial section of the pipe in *Fig. 7*. From *Fig. 7* it can be concluded

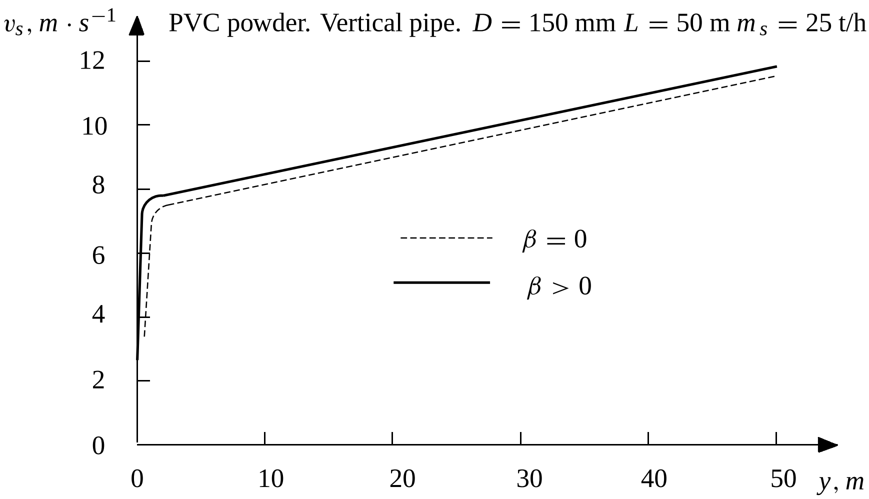

Fig. 4. Material velocity as function of the pipe length

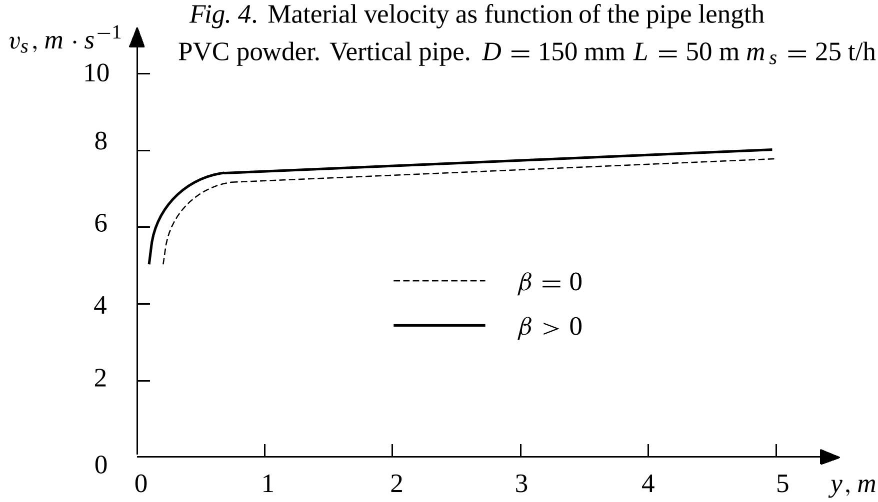

Fig. 5. Material velocity as function of the pipe length in the initial section of the pipe

that gas velocity has a minimum at $y = 0.35$ m in the demonstration example for model taking diameter reduction into consideration with a value of $v_{s\min} = 8.45$ m/s. Gas velocity decreases rapidly in the initial part of the pipe due to the considerable reduction in concentration. This means that the free cross-section will be higher and higher for the gas in this pipe section and as a consequence of this the gas velocity will decrease similarly as if the gas would flow in a diffusor. After the extreme value effect of expansion is dominating that is the gas velocity increases nearly linearly along the pipe length up to the pipe end.

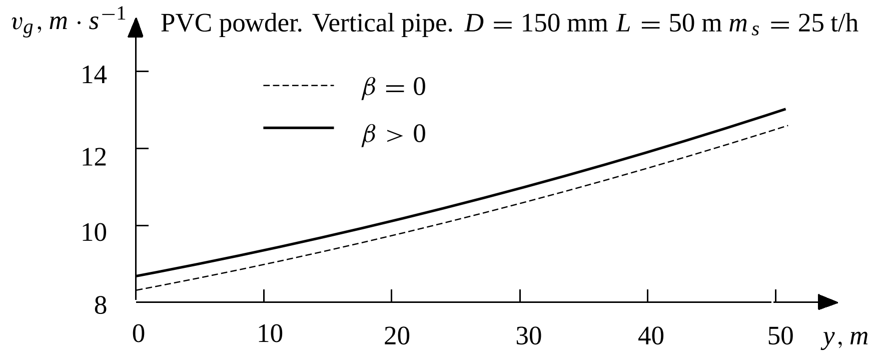

Fig. 6. Gas velocity along the pipe length

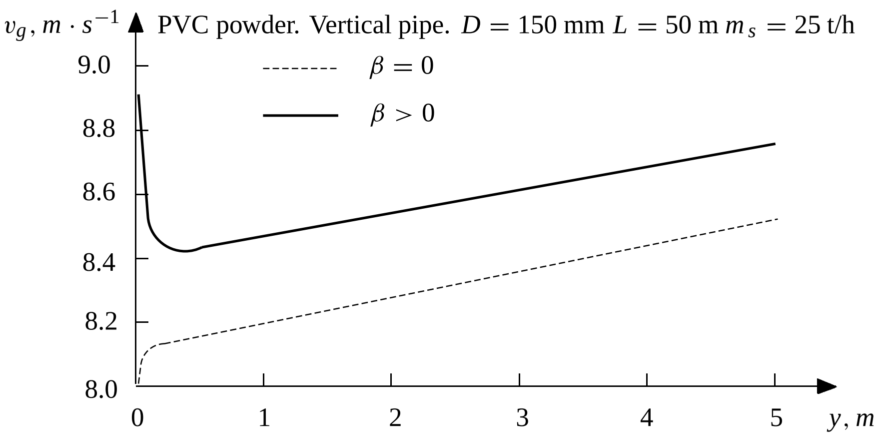

Fig. 7. Gas velocity along the pipe length in the initial section of the pipe

In *Fig. 8* change of the $\beta = A_s/A$ cross-section ratio can be seen along the length of the pipe line. In *Fig. 9* the same connection can be seen for the initial pipe section. Value of 'β' decreases from a value of $\beta = \beta_1 = 0.31$ belonging to the initial pipe section to a value of $\beta = 0.04$ within the first one metre while the change is nearly linear for the subsequent pipe section and this value at the pipe end is equal to $\beta = \beta_2 = 0.026$.

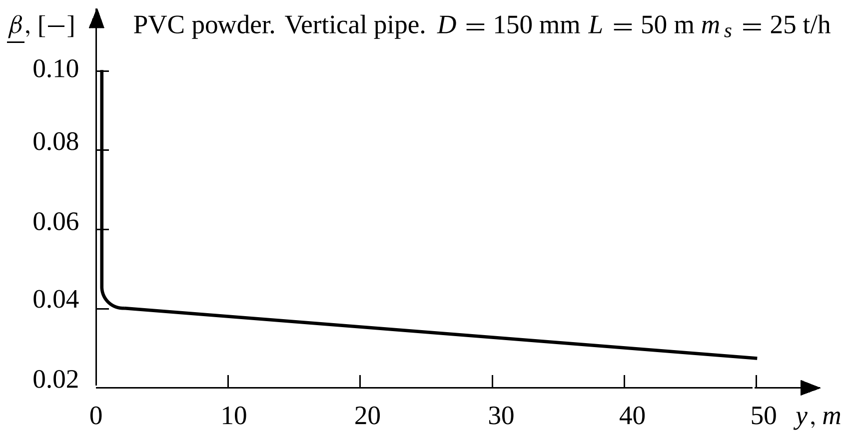

Fig. 8. Change of 'β' as function of the pipe length

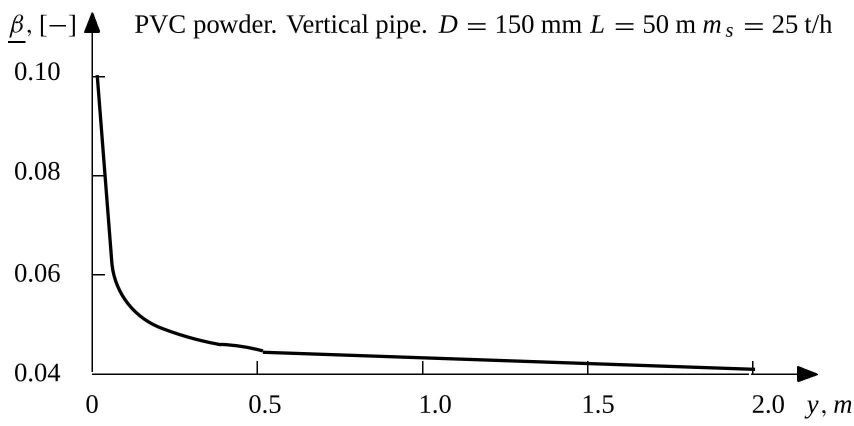

Fig. 9. Change of 'β' as function of the pipe length in the initial section of the pipe

In *Fig. 10* change of slip can be seen along the pipe length. According to data of the demonstration example the value of the slip decreases rapidly in the first one metre of pipe section from a value of $s = s_1 = 0.65$ to a value of $s = 0.09$ while subsequent reduction up to the end of the pipe is negligibly low ($s = s_2 = 0.079$).

If the conveying task demonstrated in the above example is calculated for various gas quantities the characteristic curve of the conveying pipe that is the

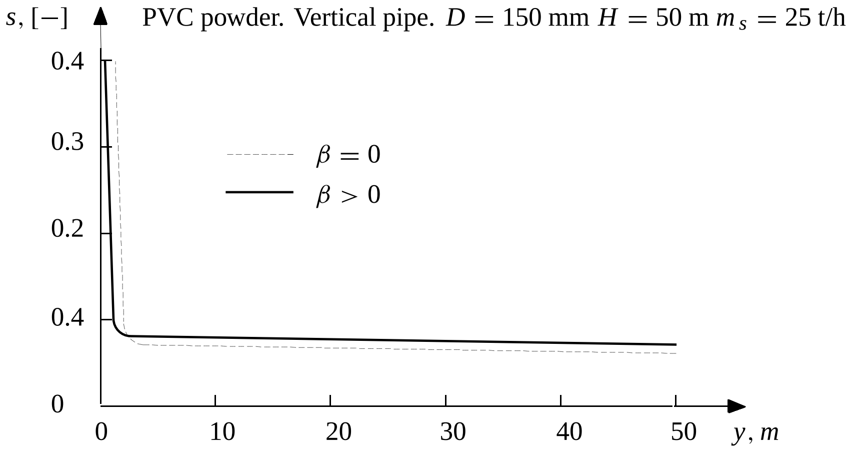

Fig. 10. Change of slip as function of the pipe length

change of the pressure drop $\Delta p = p_1 - p_2 = p_1 - p_0$ in the conveying pipe as function of the '$\dot{m}_g$' gas mass flow can be plotted for two models as seen in *Fig. 11*. As it can be seen from the figure the conveying task has an optimal solution where pressure drop in the pipeline is the lowest value. Lower border of the characteristic curve is close to the possible lowest conveying velocity that is the gas mass flow value is close to the choking limit velocity which can be determined experimentally.

In *Fig. 12* 's_0' slip values relating to the pipe end obtained for various gas

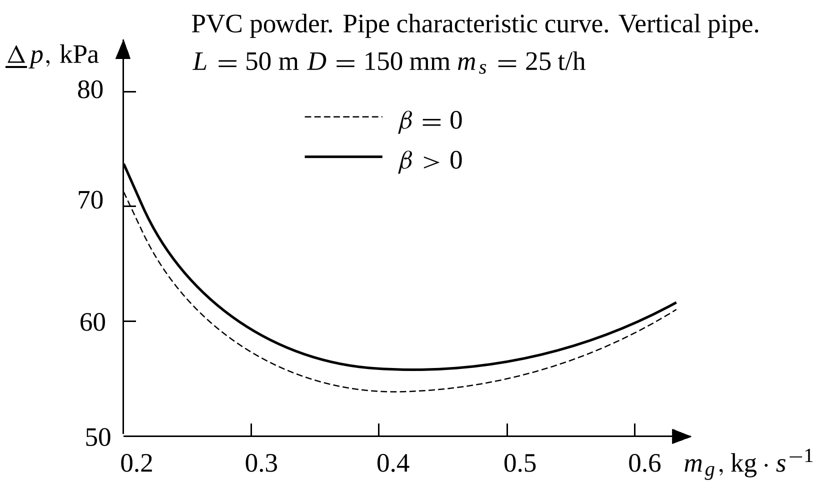

Fig. 11. Pressure drop in the conveying pipe as function of the gas mass flow

mass flow values are shown. In the given conveying task this characteristic curve shows also extreme value as seen in the above figure.

In *Fig. 13* change of 'e' specific energy consumption and 'μ' mixing ratio are shown based on calculations performed for various air mass flow values.

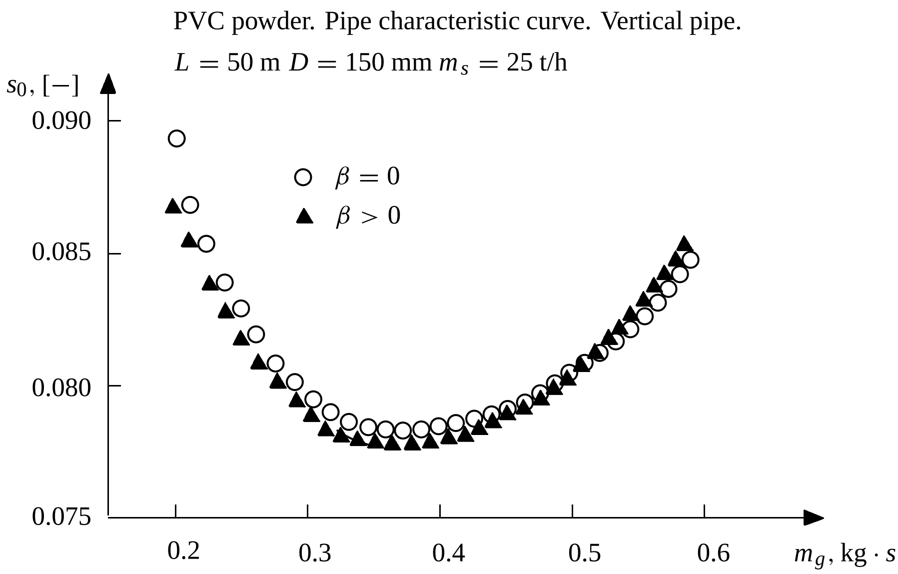

Fig. 12. Slip at the pipe end as function of the gas mass flow

Finally in *Fig. 14* 'β_0' values relating to the pipe end obtained for various gas mass flow values are shown.

By summing up it can be stated that the model developed for two-phase flow describes the change of the characteristic flow parametersz more accurately with the experimental results than the calculation procedure published in out last paper [4] relating to constant conveying conditions and taking constant slip value into consideration.

PVC powder. Pipe characteristic curve. Vertical pipe.

$L = 50$ m, $D = 150$ mm, $m_s = 25$ t/h

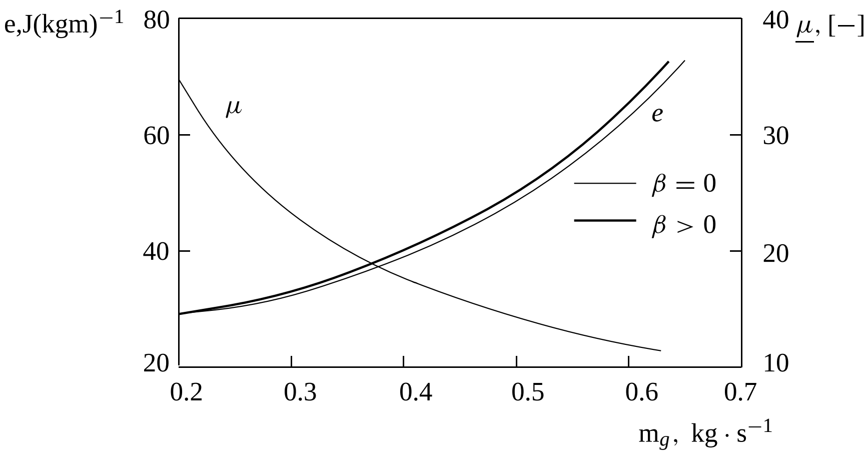

Fig. 13. Specific energy consumption and mixing ratio as function of the gas mass flow

PVC powder. Pipe characteristic curve. Vertical pipe.

$L = 50$ m, $D = 150$ mm, $m_s = 25$ t/h

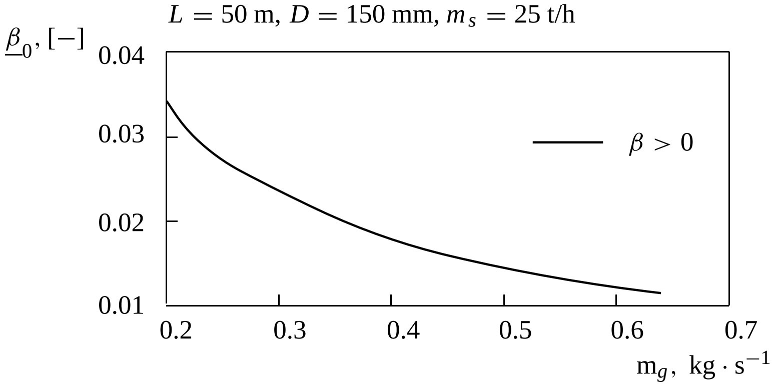

Fig. 14. Change of 'β_0' as function of the gas mass flow

List of Symbols

A	[m^2]	Cross-section of the conveying pipe
A_0	[m^2]	Cross-section at right angles to the flow of one solid particle
C_D	[-]	Drag coefficient
D	[m]	Pipe diameter
$\mathrm{d}N$	[-]	Number of solid particles
d_p	[m]	Average diameter of the solid particle supposed to be spherical
e	[J/kgm]	Specific energy consumption
F	[N]	Drag force on a particle
F_f	[N]	Breaking force
F_{fD}	[N]	Friction force generated on the pipe wall
f	[-]	Friction factor
G_s	[N]	Weight
g	[m/s^2]	Acceleration of gravity
$k_i = \frac{\xi}{2}$	[-]	Impact factor
L	[m]	Length of the conveying pipeline
$\dot{m}_g$	[kg/s]	Mass flow of the conveying gas
$\dot{m}_s$	[kg/s]	Mass flow of the solid material
m_1	[kg]	Mass of one particle
n	[-]	Polytropic exponent
P_{pol}	[W]	Polytropic power
p	[Pa]	Pressure
$p_2 = p_0$	[Pa]	Pressure at $y = L$ place. Atmospheric pressure
p_1	[Pa]	Pressure at $y = 0$ place
$\Delta p = p_2 - p_1$	[Pa]	Pressure drop in the conveying pipeline
Re	[-]	Reynolds number
$s = \frac{v_g - v_s}{v_g}$	[-]	Slip
s_0	[-]	Slip at $y = 0$ place
t	[s]	Time
v_g	[m/s]	Velocity of the gas
vs	[m/s]	Velocity of the particle
$v_{g2} = v_{g0}$	[m/s]	Velocity of the gas at $y = L$ place for $p_2 = p_0$
$w = v_g - v_s$	[m/s]	Relative velocity
$\alpha_1 = -\dfrac{\rho_s^0 \dot{m}_g \dot{m}_s p_0}{(A v_s \rho_s^0 - \dot{m}_s)^2 \rho_{g0} p}$	[-]	Constant
$\alpha_2 = -\dfrac{\rho_s^0 \dot{m}_g p_0 v_s}{(A v_s \rho_s^0 - \dot{m}_s)^2 \rho_{g0} p^2}$	[m^{2}s/kg]	Constant

$\beta = \frac{A_s}{A} = \frac{\dot{m}_s}{A v_s \rho_s^0}$	[-]	Cross-section ratio
β_0	[-]	Cross section ratio at $y = L$ place
η_g	[kg/ms]	Absolute viscosity of gas
μ	[-]	Mixing ratio
ξ	[-]	Factor characteristic of the conveyed material to be determined experimentally
$\pi_0 = \frac{A}{m_1} A_0 C_D$	[m^4/kg]	Constant
$\pi_1 = \frac{Af}{2D}$	[m]	Constant
$\pi_2 = \frac{\pi_0}{2} \frac{\rho_{g0}}{A p_0}$	[s^2/kg]	Constant
$\pi_3 = \frac{\xi}{2D}$	[1/m]	Constant
$\pi_4 = -\pi_1 \rho_{g0} p_0 v_{g0}^2$	[kg^2/ms^4]	Constant
$\pi_5 = -\frac{\pi_0}{2} \frac{\dot{m}_s}{A} \frac{\rho_{g0}}{p_0}$	[s]	Constant
$\pi_6 = v_{g0}^2 \rho_{g0} A p_0$	[kg^2/s^4]	Constant
$\pi_7 = \dot{m}_s g$	[kgm/s^3]	Constant
ρ_g	[kg/m^3]	Gas density
ρ_{g0}	[kg/m^3]	Gas density at $y = L$ place. Density of gas at atmospheric pressure
ρ_s	[kg/m^3]	Concentration
ρ_s^0	[kg/m^3]	Solid material density

Indexes

D	Pipe wall, drag
g	Gas
s	Solid material
1	One particle
i	Impact
f	Friction
0, 1, 2	Characteristic parameters along the pipeline length

References

[1] KOVÁCS, L. – VÁRADI, S.: Pneumatic Conveying in the Vertical Pipeline of Air Lift. *Periodica Polytechnica Ser. Mech. Eng.* Vol. 42, No. 2, 1998.

[2] PÁPAI, L.: Pneumatikus és fluidizációs anyagszállítás (Pneumatic and Fluidized Material Transport). Mérnöki Továbbképző Intézet (Institute of Continuing Engineering Education) 1977. J. 4864.

[3] BRAUER, HEINZ: Grundlagen der Einphasen- und Mehrphasenströmungen. Verlag Sauerländer, AArau und Frakfurt am Main, 1971.

[4] KOVÁCS, L. – PÁPAI, L. – VÁRADI, S.: Determination of Pressure Loss of Dust-Air Flowing in a Pipe with a View on Expansion. *Proceedings of the 7th Conference on Fluid Machinery*. Budapest. 1983. pp. 477–483.