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RESPONSE OF RIGID BLOCK ASSEMBLIES TO EARTHQUAKE EXCITATION

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Abstract

The behaviour of free-standing structures modelled by a mechanical system consisting of two rigid blocks put on each other and subjected to forcing at its supports has been studied. In case of the presence of support motion, the blocks may slide and/or rotate (lift up) on each other and/or on the support. The possible types of initial motion and the contact forces for the structure subjected to inertial force caused by support excitation have been calculated by using the basic law of dynamics. As an application example for calculations and the related computer program the analysis of a standard crane of bridge-like structure under earthquake excitation is presented. The risk of overturning of the upper block on the steady lower one is investigated by applying an analytical approach and a formulation of the upper body as inverted pendulum under horizontal base excitation. Furthermore, the lowest natural frequency related to the vertical vibration is calculated for different configurations.

Keywords: earthquake excitation, stacked rigid blocks, initial rocking and toppling motion.

1. Introduction

For any structure subjected to a certain foundation shaking, an understanding of the response behaviour of these objects is of great importance for safe operations, protection of existing structures, and the design of new equipment. They may be subjected to support excitation due to nearby machine vibrations or earthquake ground motion. In recent years, the seismic safety of nuclear reactors has become a particularly important topic.

The formulation assumes rigid bodies, rigid horizontal foundation, Coulomb friction, planar motion in both vertical planes. The free-standing man-made structures are modelled by rigid non-prismatic blocks. Assemblies consisting of two stacked blocks are considered. The model is applicable to pieces of machinery, land vehicles, water and oil tanks, crane structures, petroleum cracking towers, prefabricated buildings, power transformers, concrete shields around radiation equipment, nuclear heat-exchange boiler, nuclear reactor core made from graphite blocks and other free-standing equipment including even furniture.

The ground excitation is modelled by the acceleration components of the rigid support in both horizontal and the vertical directions. The effect of the support accelerations on the structure is taken into account by the inertial forces reduced in the centres of gravity of the corresponding bodies. These forces are proportional to the accelerations induced by the support motion.

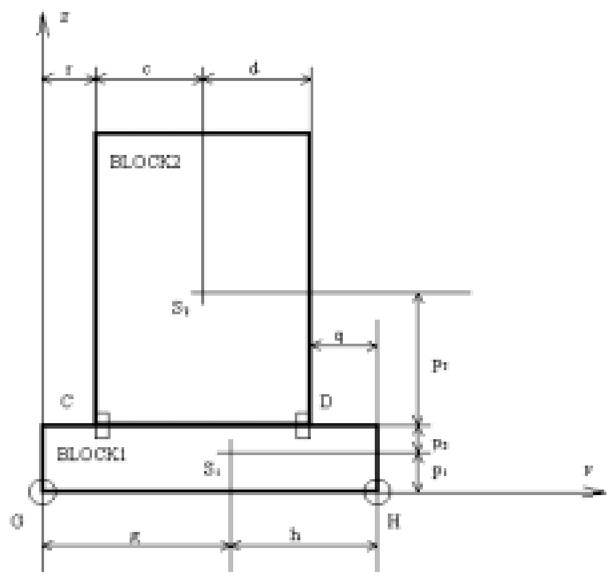

Fig. 1. The two-block model in the $y - z$ plane

The response of the structure to support shaking is given at the initial moment of the motion in terms of the instantaneous state of acceleration of the rigid bodies. The contact forces between the blocks, and between the lower block and the supporting surface are also calculated. The formulation is based on classical dynamics applied to rigid bodies.

An application of the formulation to a crane is presented. The model consists of two rigid blocks stacked on each other. The risk of overturning of the upper block on the steady lower one is also investigated using the classical approach and formulation of HOUSNER [1]. Housner determined the minimum horizontal acceleration required to overturn a block with a single acceleration pulse. This work was reconsidered in the light of modern dynamical systems theory. The rocking block problem turned out to possess extremely complicated nonlinear dynamics (see for example the paper of HOGAN [2]). However, this is not in the scope of this investigation, only the instantaneous state of motion is determined as a response of the structure.

The purpose of this investigation is to give a first assessment of free-standing structures for rocking and toppling behaviour in the simplest possible manner, in order to be able to judge whether further detailed analysis is required.

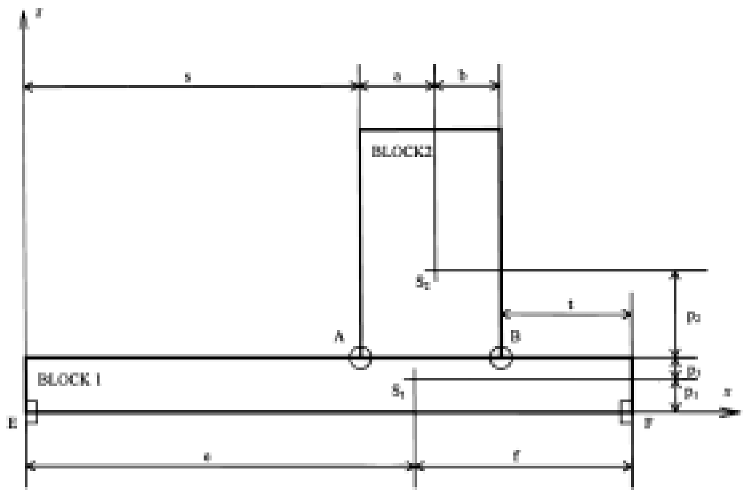

Fig. 2. The two-block model in the $x - z$ plane

2. Description of the Two-Block-System Model

The rigid body model of a composed free standing structure consists of two parts which may move relative to each other and to the supporting surface in the following way: the lower block (BLOCK1) is free to move in the $y - z$ plane (*Fig. 1*) while the upper block (BLOCK2) in the $x - z$ plane (*Fig. 2*). The term 'free to move' includes the five possible types of planar motion: rest, slide, rock, slide-rock and free-flight. The blocks are supported by a horizontal rigid base along their edges. The blocks do not need to be homogeneous so that the centers of gravity S_1 and S_2 do not coincide with the geometric centers. *Figs. 1* and *2* show the sketches of the structure in the $y - z$ and the $x - z$ planes with the geometric dimensions and the notation of the edges. The masses of the bodies are denoted by m_1 and m_2, the mass moments of inertia in the $y - z$ plane are Θ_{S1x} and Θ_{S2x}, the ones in the $x - z$ plane are Θ_{S1y} and Θ_{S2y}, respectively.

The motion of the supporting surface is modelled via the effect of inertial forces acting on the rigid bodies. These forces are proportional to the accelerations induced by the forcing at the supports of the structure. The effects of the acceleration components on the rigid structure are taken into account by the inertial forces reduced to the centres of gravity of the corresponding bodies. These accelerations are considered in both horizontal directions (x, y), a_{hor}, in both senses (left: $-x, -y$, and right: $+x, +y$) together with the vertical acceleration component a_{ver}. In the vertical direction (z), however, the acceleration is considered with negative sense only, when it shows downwards $(-z)$, and the corresponding inertial force (showing upwards) will decrease the weight of the structure. The acceleration upwards increases the stability of the structure and it may not cause sliding or uplifting which would not occur already with the acceleration showing downwards.

The horizontal and vertical components of the acceleration of the supporting surface, a_{hor} and a_{ver}, can be expressed relative to the gravitational acceleration $g = 9.81$ [m/s^2] by the factors $k_x = k_y$ and k_z, $a_{\text{hor}} = k_y \cdot g$, $a_{\text{ver}} = k_z \cdot g$.

3. Analysis of the Initial Motion and the Contact Forces

The basic law of dynamics (or statics) is used to analyse the possible motions (or equilibria) of the structure relative to the supports in all the possible critical cases. By solving the equations of motion (or equilibrium) the acceleration components of the centres of gravity of both blocks, the angular accelerations and the contact forces between BLOCK1 and BLOCK2, and between the support and BLOCK1 are obtained at the very first moment after the supporting surface acceleration has occurred.

3.1. Analysis in the $y - z$ Plane

The two blocks are supported at their corners so that BLOCK1 may execute general plane motion (sliding and/or uplifting about the rear edge), while for BLOCK2 only uplifting may occur.

The free-body-diagram is shown in *Fig. 3*.

The forces acting on BLOCK2 are as follows:

F_2	inertial force, opposite to the acceleration of the supporting surface $F_2 = m_2 \cdot a_{\text{hor}} = m_2 \cdot k_y \cdot g$
G_2	weight reduced by the inertial force in the vertical direction $G_2 = m_2 \cdot a_{\text{ver}} = m_2 \cdot g \cdot (1 - k_z)$
C_y	horizontal component of the contact force between BLOCK1 and BLOCK2 at the front corner and the contact point C
D_y	horizontal component of the contact force between BLOCK1 and BLOCK2 at the rear corner at the contact point D
C_z	vertical component of the contact force between BLOCK1 and BLOCK2 at the front corner at the contact point C
D_z	vertical component of the contact force between BLOCK1 and BLOCK2 at the rear corner at the contact point D

The forces acting on BLOCK1 are as follows:

F_1	inertial force, opposite to the acceleration of the supporting surface $F_1 = m_1 \cdot a_{\text{hor}} = m_1 \cdot k_y \cdot g$
G_1	weight reduced by the inertial force in the vertical direction $G_1 = m_1 \cdot a_{\text{ver}} = m_1 \cdot g \cdot (1 - k_z)$
$CD_y = C_y + D_y$	sum of the horizontal components of the contact forces between BLOCK1 and BLOCK2 at the contact points C and D
C_z, D_z	vertical components of the contact forces between BLOCK1 and BLOCK2 at the contact points C and D
G_y	horizontal component of the contact force between BLOCK1 and the supporting surface at the front corner
H_y	horizontal component of the contact force between BLOCK1 and the supporting surface at the rear corner
G_z	vertical component of the contact force between BLOCK1 and the supporting surface at the front corner
H_z	vertical component of the contact force between BLOCK1 and the supporting surface at the rear corner

3.1.1. Analysis of BLOCK2

First we assume that BLOCK1 is in equilibrium. Then for BLOCK2, there are two possibilities:

3.1.1.1 If $C_z > 0$ then BLOCK2 is in equilibrium C_z can be determined solving the equilibrium equations:

$$m_2 k_y g - (C_y + D_y) = 0, \tag{1}$$

$$D_z + C_z - m_2 g(1 - k_z) = 0, \tag{2}$$

$$C_z c - D_z d + (C_y + D_y) p_2 = 0. \tag{3}$$

3.1.1.2 If $C_z < 0$ then BLOCK2 begins to rotate about the rear edge (about the axis through D) The initial acceleration of the centre of gravity (a_{S2y} horizontal component, a_{S2z} vertical component), the angular acceleration ε, and the contact force components D_y, D_z can be calculated from the two force equations, the moment equation, and the kinematical relation between the acceleration of the centre of gravity and the angular acceleration:

$$m_2 k_y g - D_y = m_2 a_{S2y}, \tag{1}$$

$$-(1 - k_z) m_2 g + D_z = m_2 a_{S2z}, \tag{2}$$

$$-D_z d + D_y p_2 = \theta_{S2} \varepsilon, \tag{3}$$

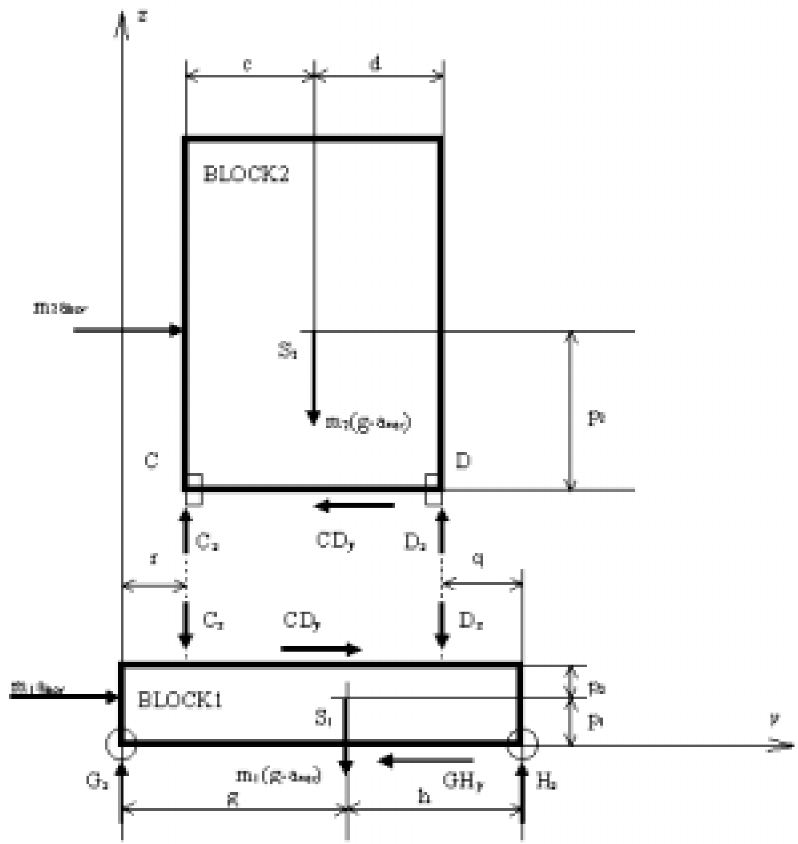

Fig. 3. Free body diagram in the $y - z$ plane

$$\varepsilon p_2 = a_{S2y}, \tag{4}$$
$$\varepsilon d = a_{S2z}. \tag{5}$$

3.1.2. Analysis of BLOCK1

3.1.2.1 If $G_z > 0$ then BLOCK1 may be in equilibrium The equilibrium equations are:

$$C_y + D_y - (G_y + H_y) + m_1 k_y g = 0, \tag{1}$$
$$G_z + H_z - C_z - D_z - m_1 g(1 - k_z) = 0, \tag{2}$$
$$G_z g - H_z h + GH_y p_1 + CD_y p_3 - C_z(g - r) + D_z(h - q) = 0. \tag{3}$$

Solving the above system of equilibrium equations, if $G_z > 0$, the assumption of equilibrium must be verified:

a. *If* $GH_y \leq \mu_0(G_z + H_z)$ *then BLOCK1 is in equilibrium.*
b. *If* $GH_y > \mu_0(G_z + H_z)$ *then BLOCK1 slips.*

For the case of sliding, the initial acceleration a_{S1y} of the centre of gravity, and the contact force components GH_y, G_z, H_z can be calculated from the two force equations, the moment equation about S_1 and the relation between the sliding friction force and the normal force:

$$m_1 k_y g + CD_y - GH_y = m_1 a_{S1y}, \tag{1}$$
$$G_z + H_z - C_z - D_z - m_1 g(1 - k_z) = 0, \tag{2}$$
$$\begin{aligned} G_z g - H_z h + GH_y p_1 + CD_y p_3 + \\ + D_z(h - q) - C_z(g - r) = 0, \end{aligned} \tag{3}$$
$$GH_y = \mu(G_z + H_z). \tag{4}$$

3.1.2.2 If $G_z < 0$ *then general plane motion occurs:*

c. uplifting about H
d. uplifting with sliding

Supposing the uplifting, the initial acceleration of the centre of gravity (a_{S1y} horizontal component, a_{S1z} vertical component), the angular acceleration ε and the contact force components H_y, H_z can be calculated from the two force equations, the moment equation and the kinematical relations between the acceleration of the centre of gravity and the angular acceleration:

$$m_1 k_y g + CD_y - H_y = m_1 a_{S1y}, \tag{1}$$
$$-m_1 g(1 - k_z) + H_z - C_z - D_z = m_1 a_{S1z}, \tag{2}$$
$$\begin{aligned} m_1 k_y g p_1 - m_1 g(1 - k_z)h + \\ + CD_y(p_1 + p_3) - D_z q - C_z(g + h - r) = \theta_H \varepsilon, \end{aligned} \tag{3}$$
$$\varepsilon p_1 = a_{S1y}, \tag{4}$$
$$\varepsilon h = a_{S1z}. \tag{5}$$

Verification of the assumption uplifting:

c. *If* $H_y \leq \mu_0 H_z$ *then the initial motion of BLOCK1 is uplifting.*
d. *If* $H_y > \mu_0 H_z$ *then uplifting with sliding occurs.*

In the latter case, the equations for the general plane motion for the six unknowns are:

$$m_1 k_y g + CD_y - H_y = m_1 a_{S1y}, \tag{1}$$
$$-m_1 g(1 - k_z) + H_z - C_z - D_z = m_1 a_{S1z}, \tag{2}$$
$$\begin{aligned} CD_y p_3 + D_z(h - q) - C_z(g - r) + \\ + H_y p_1 - H_z h = \theta_{S1} \varepsilon, \end{aligned} \tag{3}$$
$$\varepsilon p_1 + a_H = a_{S1y}, \tag{4}$$
$$\varepsilon h = a_{S1z}, \tag{5}$$
$$H_y = \mu H_z. \tag{6}$$

In the case of a negative a_{S1y}, there is no uplifting with the sliding, only sliding occurs.

3.1.2.3 If BLOCK1 is not in equilibrium, then the analysis for BLOCK2 must be modified, because it was performed with the condition that BLOCK1 is in equilibrium. Since the worst case for BLOCK1 is translational sliding, the system of equations to solve will be composed from the three dynamical equations of BLOCK2, from those of BLOCK1, and from the sliding friction condition for BLOCK1:

$$\begin{aligned}
m_2 k_y g - CD_y &= m_2 a_{12}, && (1)\\
-m_2 g(1-k_z) + C_z + D_z &= 0, && (2)\\
C_z c - D_z d + (C_y + D_y) p_2 &= 0, && (3)\\
m_1 k_y g + CD_y - GH_y &= m_1 a_{12}, && (4)\\
G_z + H_z - C_z - D_z - m_1 g(1-k_z) &= 0, && (5)\\
G_z g - H_z h + GH_y p_1 + CD_y p_3 + &\\
+ D_z(h-q) - C_z(g-r) &= 0, && (6)\\
GH_y &= \mu(G_z + H_z). && (7)
\end{aligned}$$

In *Eqs.* (1) and (4) a_{12} is the common translational acceleration of both blocks. The first three equations have been formed with the assumption that BLOCK2 does not move on BLOCK1. It is true if C_z results to be positive. If $C_z < 0$, then BLOCK2 will uplift on BLOCK1. For the latter case, the system of equations can be written as follows:

$$\begin{aligned}
m_2 k_y g - CD_y &= m_2 a_{S2y}, && (1)\\
-m_2 g(1-k_z) + D_z &= m_2 a_{S2z}, && (2)\\
-D_z d + (C_y + D_y) p_2 &= \theta_{S2}\varepsilon, && (3)\\
m_1 k_y g + CD_y - GH_y &= m_1 a_{12}, && (4)\\
G_z + H_z - D_z - m_1 g(1-k_z) &= 0, && (5)\\
G_z g - H_z h + GH_y p_1 + CD_y p_3 + D_z(h-q) &= 0, && (6)\\
GH_y &= \mu(G_z + H_z), && (7)\\
C_z &= 0, && (8)\\
a_{S2y} &= a_{12} + \varepsilon p_2, && (9)\\
a_{S2z} &= \varepsilon d. && (10)
\end{aligned}$$

Because of the non-symmetric nature of the structure, the analysis must be repeated with the opposite sense of the supporting surface acceleration.

3.2. Analysis in the $x - z$ Plane

Now, for BLOCK1 only uplifting is possible, while BLOCK2 may execute a general plane motion. However, the analysis is performed with the assumption that

BLOCK2 on BLOCK1 is in one of its extreme positions, so that there is no sliding outwards for BLOCK2.

The algorithm is analogous with that of the foregoing analysis. Now, the contact points between BLOCK1 and BLOCK2 are denoted by A and B, while E and F denote the contact points between BLOCK1 and the supporting surface. The notation of the forces are to be changed accordingly, as well as the geometric dimensions.

4. Application Example: Seismic Assessment of a Crane Structure

The assembly of a realistic crane structure consists of three main parts which may move relative to each other. These are the bridge, the cart and a vertical actuator bar. The bridge may roll on its wheels in the horizontal direction y on the rails attached to the building structure. The cart may roll on its wheels in the horizontal direction x on the rails attached to the bridge. The actuator bar may move vertically relative to the cart, but this motion is not modelled, and the bar is simply considered in its highest position only, having a rigid connection to the cart.

The crane is modelled as a structure consisting of two rigid blocks shown in *Figs. 1* and *2*. The corresponding data are given in *Table 1*. Under regular working conditions, these blocks either stand steadily when the brakes are applied on the wheels, or they may roll on each other and on the platform. In case of the presence of seismic forces, they may slide and/or rotate (lift up) on each other.

During calculations, we consider that the brakes are applied on the wheels, i.e. the wheels cannot rotate, they can slide only, according to the two-block model supported on the edges. This sliding motion may be obstructed when the cart is in one of the farmost positions on the left or on the right. In these cases the cart may slide inwards only.

The rails supporting the bridge are considered to be fixed rigidly to the building structure.

Figs. 4 and *5* present some typical seismic spectra for the acceleration components in horizontal (a_{hor}) and vertical (a_{ver}) directions. These acceleration values give the acceleration amplitudes of the vibrations of the corresponding one degree-of-freedom systems having the natural frequencies and the damping value presented in the diagrams.

When the crane is modelled as a rigid structure, the corresponding natural frequency tends to infinity, and the corresponding asymptotic values of the spectra are used to identify the maximum acceleration amplitudes. These values are independent, of course, from the damping value (which are not defined for rigid bodies). Thus, the corresponding horizontal acceleration is obtained from *Fig. 4*:

$$a_z = -a_{\mathrm{ver}} = -2.35\ [\mathrm{m/s^2}]$$

and the vertical acceleration is obtained from *Fig. 5*:

$$|a_x| = |a_y| = a_{\mathrm{hor}} = 2.70\ [\mathrm{m/s^2}].$$

Table 1. Data of the two-block-system model of the crane

$Y - Z$ plane	Bridge (BLOCK1)	Cart with actuator bar (BLOCK2)
Mass	$m_1 = 16\,600$ kg	$m_2 = 25\,600$ kg
Mass moment of inertia w.r.t. x axis through centre of gravity	$\Theta_{S1x} = 49\,400$ kgm^2	$\Theta_{S2x} = 151\,600$ kgm^2
Mass moment of inertia w.r.t. x axis through lower left corner	$\Theta_G = 115\,600$ kgm^2	$\Theta_C = 422\,300$ kgm^2
Mass moment of inertia w.r.t. x axis through lower right corner	$\Theta_H = 165\,750$ kgm^2	$\Theta_D = 422\,300$ kgm^2
Geometrical data	$p_1 = 0.98$ m $p_3 = 0.22$ m $g = 1.74$ m $h = 2.46$ m	$p_2 = 2.83$ m $c = 1.6$ m $d = 1.6$ m $r = 0.5$ m $q = 0.5$ m

$X - Z$ plane	Bridge (BLOCK1)	Cart with actuator (BLOCK2)
Mass	$m_1 = 16\,600$ kg	$m_2 = 25\,600$ kg
Mass moment of inertia w.r.t. y axis through centre of gravity	$\Theta_{S1y} = 256\,650$ kgm^2	$\Theta_{S2y} = 124\,100$ kgm^2
Mass moment of inertia w.r.t. y axis through lower left corner	$\Theta_E = 849\,600$ kgm^2	$\Theta_A = 374\,250$ kgm^2
Mass moment of inertia w.r.t. y axis through lower right corner	$\Theta_F = 524\,750$ kgm^2	$\Theta_B = 374\,250$ kgm^2
Geometrical data	$p_1 = 0.98$ m $p_3 = 0.22$ m $e = 5.9$ m $f = 3.9$ m	$p_2 = 2.83$ m $a = 1.33$ m $b = 1.33$ m $s = 5.14$ m (0)* $t = 2$ m (7.15 m)*

* Data in parentheses when the cart is in the leftmost position on the bridge

A conservative estimate is considered for the case when the crane structure is not rigid. It is supposed that the structure has no natural frequency below 20 [Hz] with a vertical vibration mode, but it is supposed to have a vibration mode in

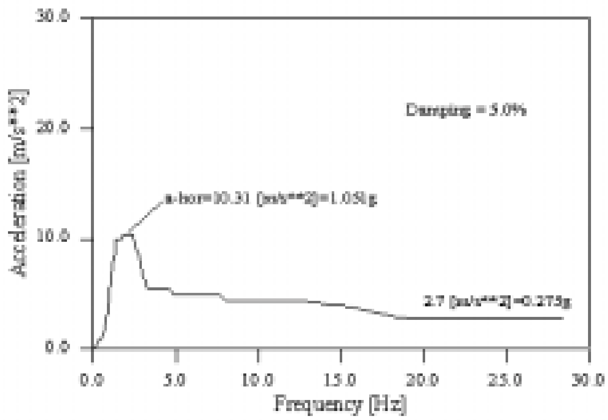

Fig. 4. Typical seismic spectrum for the horizontal acceleration component

horizontal direction with a corresponding natural frequency below 20 [Hz]. If the modal analysis of the crane is not available, the worst case is considered: the whole structure may oscillate horizontally with the most dangerous natural frequency at 2.5 [Hz]. Then assuming a realistic damping value 0.05 for the steel structure, a conservative estimate for the horizontal accelerations come from *Fig. 5*:

$$|a_x| = |a_y| = a_{\text{hor}} = 10.31 \text{ [m/s}^2\text{]},$$

while the vertical acceleration is the same as above.

When the brakes are applied on the wheels of the crane, the dry friction force is modelled between the steel wheels and the corresponding steel rails. Using some standard estimates, the static coefficient of friction between the steady surfaces is considered by $\mu_0 = 0.25$, while the dynamic coefficient of friction between the sliding surfaces is considered by $\mu = 0.20$.

4.1. Results for the Fully Rigid Structure

The results for the accelerations of the centres of gravity and for the angular accelerations are presented in *Table 2* for the seismic accelerations given for the fully rigid structure. The table presents the six extreme geometrical cases which may be one of the most dangerous. The contact forces acting on the two blocks are shown in *Table 3*.

4.2. Results for the Horizontally Elastic Structure

The algorithm of the calculation is the same like that of the fully rigid structure, but the value of the horizontal acceleration component will be different, as described

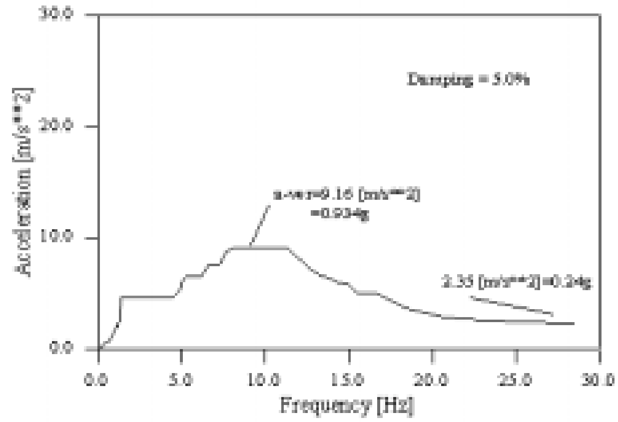

Fig. 5. Typical seismic spectrum for the vertical acceleration component

Table 2. Initial state of acceleration and types of motion of the rigid structure

Plane of motion	Direction of inertial force	Cart position farmost to	Body	Angular acceleration $[1/s^2]$	C of G horizontal acceleration $[m/s^2]$	Type of motion
yz	$+y$	–	cart	0	1.2	bridge slides
			bridge	0	1.2	with cart
yz	$-y$	–	cart	0	-1.2	bridge slides
			bridge	0	-1.2	with cart
xz	$+x$	right	cart	0	0	both in
			bridge	0	0	equilibrium
xz	$+x$	left	cart	0	1.2	cart slides
			bridge	0	0	only
xz	$-x$	right	cart	0	-1.2	cart slides
			bridge	0	0	only
xz	$-x$	left	cart	0	0	both in
			bridge	0	0	equilibrium

in the foregoing section.

The results for the accelerations of the centres of gravity and for the angular accelerations are presented in *Table 4* for the seismic accelerations given for the elastic structure. The constraining forces acting on the two boxes are presented in *Table 5*.

Table 3. Contact forces for the rigid structure

Plane of motion	Direction of inertial force	Cart position farmost to	Body	Vertical, left [N]	Vertical, right [N]	Horizontal (sum) [N]
yz	$+y$	–	cart	$C_z =$ 61661	$D_z =$ 129203	$CD_y =$ 38173
			bridge	$G_z =$ 125342	$H_z =$ 189091	$GH_y =$ 62886
yz	$-y$	–	cart	$D_z =$ 129203	$C_z =$ 61661	$CD_y =$ 38173
			bridge	$H_z =$ 104158	$G_z =$ 210274	$GH_y =$ 62886
xz	$+x$	right	cart	$A_z =$ 21652	$B_z =$ 169211	$AB_x =$ 69062
			bridge	$E_z =$ 81161	$F_z =$ 233271	$EF_x =$ 113775
xz	$+x$	left	cart	$A_z =$ 54652	$B_z =$ 136212	$AB_x =$ 38173
			bridge	$E_z =$ 194027	$F_z =$ 120405	$EF_x =$ 82885
xz	$-x$	right	cart	$B_z =$ 54652	$A_z =$ 136212	$AB_x =$ 38173
			bridge	$F_z =$ 180138	$E_z =$ 134294	$EF_x =$ 82885
xz	$-x$	left	cart	$B_z =$ 21652	$A_z =$ 169211	$AB_x =$ 69062
			bridge	$F_z =$ 67272	$E_z =$ 247161	$EF_x =$ 113775

Table 4. Initial state of acceleration and types of motion for the horizontally elastic structure

Plane of motion	Direction of inertial force	Cart position farmost to	Body	Angular acceleration [1/s^2]	C of G horizontal acceleration [m/s^2]	Type of motion
yz	$+y$	–	cart	0	8.8	bridge slides
			bridge	0	8.8	with cart
yz	$-y$	–	cart	0	-8.8	bridge slides
			bridge	0	-8.8	with cart
xz	$+x$	right	cart	-1.3	3.7	cart lifts
			bridge	0	0	up
xz	$+x$	left	cart	0	8.8	cart slides
			bridge	0	0	only
xz	$-x$	right	cart	0	-8.8	cart slides
			bridge	0	0	only
xz	$-x$	left	cart	1.3	-3.7	cart lifts
			bridge	0	0	up

4.3. Conclusions

In case the crane structure has no natural frequency below 20 [Hz], it may happen that the cart and the bridge slide together with a moderate acceleration of about 1.2 [m/s^2] in the $y - z$ plane, or the cart may slide on the bridge with the same acceleration in the $x - z$ plane.

Table 5. Contact forces for the horizontally elastic structure

Plane of motion	Direction of inertial force	Cart position farmost to	Body	Vertical, left [N]	Vertical, right [N]	Horizontal (sum) [N]
yz	$+y$	–	cart	$C_z =$ 61661	$D_z =$ 129203	$CD_y =$ 38173
			bridge	$G_z =$ 125342	$H_z =$ 189091	$GH_y =$ 62886
yz	$-y$	–	cart	$D_z =$ 129203	$C_z =$ 61661	$CD_y =$ 38173
			bridge	$H_z =$ 104158	$G_z =$ 210274	$GH_y =$ 62886
xz	$+x$	right	cart	$A_z =$ 0	$B_z =$ 235670	$AB_x =$ 168210
			bridge	$E_z =$ 59640	$F_z =$ 299599	$EF_x =$ 339094
xz	$+x$	left	cart	$A_z =$ 54652	$B_z =$ 136212	$AB_x =$ 38173
			bridge	$E_z =$ 181372	$F_z =$ 133061	$EF_x =$ 209056
xz	$-x$	right	cart	$B_z =$ 54652	$A_z =$ 136212	$AB_x =$ 38173
			bridge	$F_z =$ 167482	$E_z =$ 149950	$EF_x =$ 209056
xz	$-x$	left	cart	$B_z =$ 0	$A_z =$ 235668	$AB_x =$ 168210
			bridge	$F_z =$ 36570	$E_z =$ 322669	$EF_x =$ 339094

In case the crane structure has a natural frequency below 20 [Hz] with a horizontal vibration mode, it cannot be excluded in the worst case that the cart lifts up on the steady bridge when it is parked at one of the farmost positions, or the cart slides with a considerable acceleration of about 8.8 [m/s^2] on the bridge, or they slide together with this acceleration in the $y - z$ plane.

5. Checking the Risk of Overturning of the Upper Block

In the preceding analysis, the possible types of initial motion for a crane structure subjected to inertial force caused by earthquake have been investigated. It was shown that the inner (say right) edge of the cart might lift up on the steady bridge during a horizontal outward excitation (directed to the left) when it is parked at one (leftmost) side on the bridge.

The goal of the present study is to find out whether the uplifting cart will turn over or not at this (say leftmost) position on the bridge. The calculations are carried out by applying the classical analytical approach and formulation of HOUSNER [1].

Two different models of earthquake motion are used. The first kind of excitation is a single pulse of constant acceleration lasting for a finite time t_1. Based on the formula of HOUSNER [1], *Fig. 6* gives a relation between the duration (t_1) of the pulse and the magnitude of the constant acceleration pulse (a_1). This relation presents the limit between overturning and tilting only.

For the crane cart model, the value of the constant horizontal acceleration pulse is $a_1 = 10.31$ [m/s^2]. It is seen from the graph of this relation (*Fig. 6*), that the cart will not turn over if the duration of the constant acceleration pulse is less than 0.37 [sec].

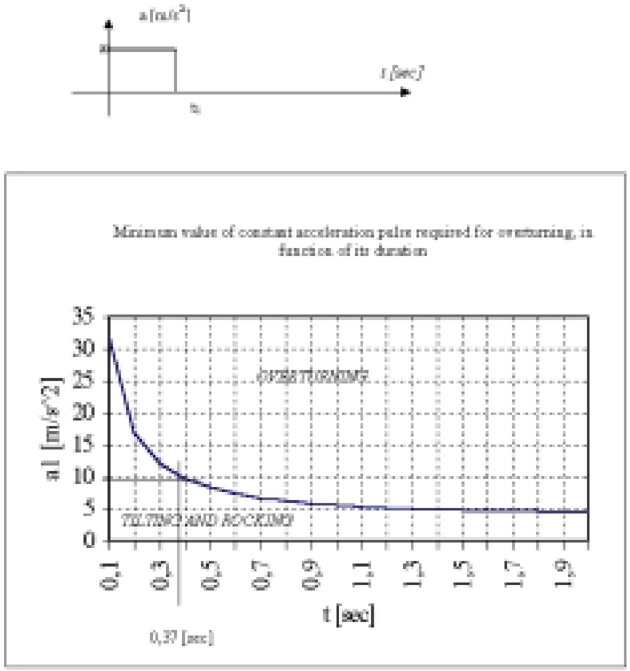

Fig. 6. Ground acceleration model: Single pulse of constant acceleration

Under the same conditions as above, but with the use of a horizontal half sine-wave acceleration pulse with amplitude of 10.31 [m/s^2], the maximum duration required for having no overturning is 0.99 [sec] (see *Fig. 7*) which means that the minimal frequency of the pulse must be 0.5 [Hz].

Fig. 8 shows again the graph of the relation between the peak value and the frequency of the sinusoidal acceleration pulse. Considering the usual relative damping of 5 [%], the seismic spectrum for the horizontal acceleration (*Fig. 4*) has been transformed into the diagram of the limit of overturning in *Fig. 7*. The transformed spectrum falls within this limit (*Fig. 8*). However, the physical interpretation of this result is limited, since it means that there is no overturning if the cart is subjected to a half sine-wave acceleration having the same frequency as the horizontal natural frequency of the cart.

5.1. Discussion of the Results

The single pulse of constant acceleration model is very unrealistic for earthquake motion. With the use of the half sinusoidal pulse acceleration, the calculations carried out for the data for the crane result in the statement that the cart will not overturn on the bridge if the frequency of the pulse is greater than 0.5 [Hz]. As

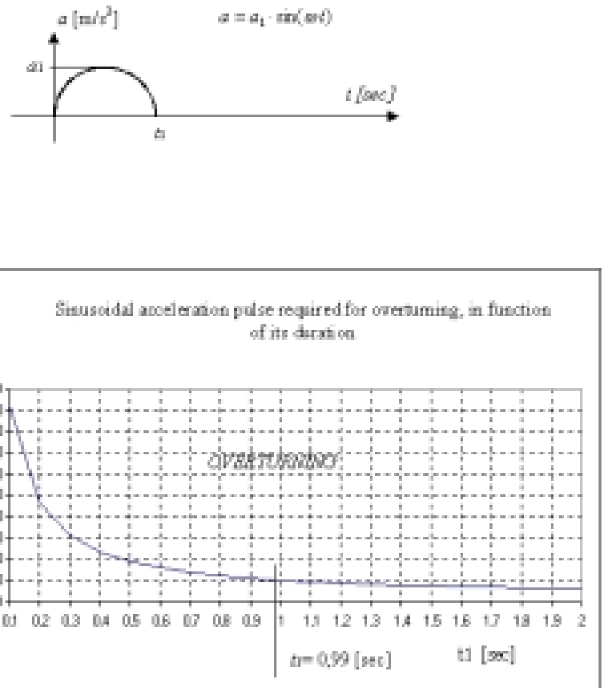

Fig. 7. Ground acceleration model: Single pulse of sinusoidal acceleration

Fig. 8 shows, the corresponding seismic spectrum does not contain small frequency oscillations with accelerations great enough for overturning.

6. Effect of the Cart Position on the Vertical Natural Frequency of the Bridge

The relation between the cart parking position on the bridge and the lowest natural frequency related to the vertical vibration mode in the simplified mechanical model of the crane structure is presented here. The aim of this analysis is to examine if it is possible to prevent the cart from uplifting, or actually, from jumping up, by choosing its position properly.

The crane structure consists of two main parts as described above. The bridge may roll in the horizontal y direction on rails attached to the building structure. The cart with the actuator bar, considered as a rigid body, may roll in the transverse direction x on rails attached to the bridge (*Fig. 2*).

The bridge is modelled as a simply supported pin-pin beam of length $l = 9.8$ [m] with uniform cross section. The area moment of inertia of the cross section with respect to the axis of bending through the centre is $I_y = 0.014$ [m^4]. The modulus of elasticity is $E = 2.1{*}10^{11}$ [N/m^2]. The mass of the bridge is $m_1 =$

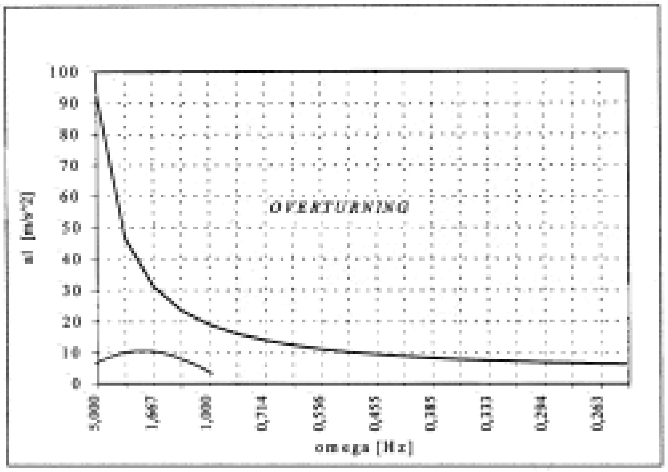

Fig. 8. Single pulse of sinusoidal acceleration – seismic spectrum

16600 [kg], and the cart is modelled by a lumped mass $m_2 = 25600$ [kg]. It may park at any position d in the x direction on the bridge.

The beam may exhibit bending vibration in the vertical $x - z$ plane.

To determine the first natural frequency of the above oscillatory system, Dunkerley's method is used. The system is considered as the superposition of two sub-systems, an infinite degree-of-freedom (DOF) one and a single DOF one. Then the first natural frequency α can be expressed in the following form:

$$\frac{1}{\alpha^2} \cong \frac{1}{\alpha_1^2} + \frac{1}{\alpha_2^2}, \tag{1}$$

where α_1 is the lowest natural frequency of the bending vibrations of the beam as a continuous medium, and α_2 is the natural frequency of the lumped mass m_2 vibrating on an elastic beam of negligible mass.

The solution of the frequency equation is:

$$\alpha_i = i^2 \cdot \pi^2 \cdot \sqrt{\frac{I_y \cdot E}{m_1 \cdot l^3}}. \tag{2}$$

This yields $\alpha_1 = 21.6$ [Hz] for $i = 1$.

The spring constant k of the elastic beam of negligible mass is:

$$k = \frac{d^2 \cdot (l - d)^2}{3 \cdot I_y \cdot E \cdot l}. \tag{3}$$

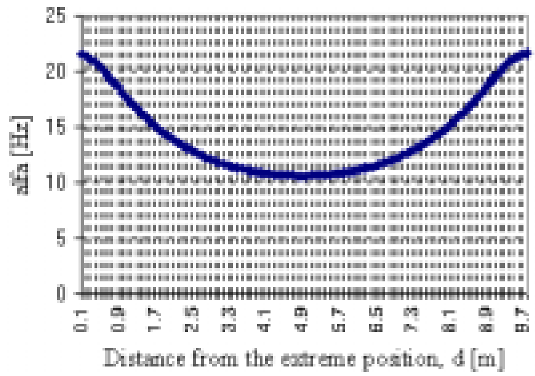

Fig. 9. First natural frecquency in funtion of the cart position on the bridge

The natural frequency α_2 depends on the position of the mass m_2 on the bridge via k:

$$\alpha_2 = \frac{1}{\sqrt{m_2 \cdot k}}. \tag{4}$$

Substituting (2), (3) and (4) into (1), we obtain the first natural frequency of the whole system (α) as function of the position d of the cart on the bridge.

The graph of the function $\alpha - d$ is shown in *Fig. 9*.

6.1. Discussion of the Results

It is seen from the graph in *Fig. 9*, that the lowest natural frequency of the structure vibrating in the $x - z$ plane varies between 10.6 [Hz] and 21.6 [Hz] depending on the cart position. The first value belongs to the cart middle position $d = l/2$, while the second to the extreme positions $d = 0$ and $d = l$. It can be seen that the mass m_2 has no effect on the natural frequency of the structure at the two extreme positions.

Taking into account the dimension of the cart in the x direction, its centre of gravity is placed at 1.33 [m] from the support in the outmost position. Thus, 1.33 [m] $< d < l -$ 1.33 [m], and it is not possible to park the cart in a position in which the lowest natural frequency of the structure would be greater than 18 [Hz], i.e.

$$10.6 \text{ [Hz]} < \alpha < 18 \text{ [Hz]}.$$

As the seismic spectrum of *Fig. 5* shows, the acceleration amplitude in vertical direction may reach the gravitational acceleration g in the range 8–12 [Hz] of the natural frequency of the structure in the presence of 5% relative damping. Since the structure is probably more elastic than it was estimated in the above calculation, and the neglected mass moment of inertia of the cart decreases the calculated natural frequencies even further, it cannot be excluded that the cart will jump up on the bridge and fully lose contact with the bridge.

7. Summary and Conclusions

The analysis described in this paper concerns the response of free-standing man-made rigid structures to support motion. By use of the classical rigid body dynamics formulation, a preliminary assessment can be given for the initial behaviour of structures modelled by a multibody system consisting of two stacked rigid blocks.

The supporting surface motion has been modelled by the acceleration components in both horizontal and the vertical directions. The initial motion of the structure just after the shaking has been formulated in terms of the possible types of two-dimensional motions in both vertical planes. The calculation of instantaneous state of acceleration of the two-block system caused by the inertial forces due to the ground acceleration and the calculation of the contact forces at the same moment has been presented.

The supporting surface shaking may be caused by nearby machine vibrations or earthquake excitation.

To demonstrate the method, the screening analysis of a crane structure has been presented. The assembly has been simplified to a system consisting of two rigid blocks allowed to move relative to each other and to the floor in both horizontal and the vertical directions. By a conservative estimation the horizontally elastic behaviour has been also taken into account.

The result of the analysis has proved that the upper block, the model of the cart, may uplift on the steady bridge. To investigate the rocking or toppling behaviour of the cart, the classical analytical approach and formulation of HOUSNER [1] has been applied. Housner developed a method based on the inverted pendulum model to check the risk of overturning of slender rigid blocks during earthquakes modelled by different types of horizontal ground motion.

According to the calculations based on this formulation, it is more likely that the cart will not overturn after it lifts up at a farmost position on the bridge. The risk of overturning would be minimised if the cart is parked in the mid of the bridge because of possible sliding before uplifting. The estimation of the vertical natural frequency, however, indicates that the cart may lose contact with the rails on the bridge when parking in the mid.

The limits of this model are obvious: earthquake-induced base excitations are much more complex. However, the model presented in this paper may be useful to give a preliminary qualification concerning the possible response to be expected of the structure to the support motion.

References

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[2] HOGAN, S. J.: On the Dynamics of Rigid-Block Motion under Harmonic Forcing, *Proceedings of the Royal Society of London*, Series A, **425**, pp 441–476, 1989.