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CALCULATION OF SWITCHING SPEED OF A CAPACITIVE MICRO-SWITCH BY ANALYTICAL AND FINITE ELEMENT METHODS

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Abstract

The switching speed of a micromachined switch is calculated by using elastic beam as well as thin plate models. The effect of pre-tightening to achieve higher eigenfrequency of the deflected member is also investigated. A performance parameter is introduced to describe the functioning of the switch in order to take into account not only fast switching, but also the necessary pull-in voltage for the device. Three switch-configurations are analyzed by finite elements and optimum geometrical parameters were searched.

Keywords: MEMS, capacitive micro-switch, design optimization, finite elements.

1. Introduction

Sensors, actuators and other micro-electromechanical systems (MEMS) can be found in almost all modern engineering machines like cars, multimedia-devices, measuring instruments, etc. Micromachined switches are mostly used in radio frequency systems, e.g. mobile phones, phase shifters, smart antennas, etc. Low power consumption and ability to be integrated by other electronic elements make them an attractive alternative to conventional solid-state switches.

In many cases [3, 8, 11, 12, 15] the switch contains an electrostatically actuated membrane, called bridge, deposited on a fixed substrate and a centre conductor facing it. The membrane bridge and the center conductor form a capacitor. During the functioning of the switch this membrane deflects due to electrostatic forces. When the membrane bridge is released, it starts to vibrate. The switching speed (time) is estimated to be proportional to the reciprocal of one-quarter of the lowest eigenfrequency of the membrane. The other important characteristics of a capacitive micro-switch is the critical (maximum) pull-in voltage. A too large voltage across the open switch can fail the switch by either deflecting the membrane and closing the switch, or by causing an electrical breakdown of the gap between the capacitors.

Much efforts are being made to improve the functioning of the switches by increasing their switching speed. Low-cost semi-conductor materials, the productive bulk micromachining technology, design of sophisticated electrical circuits promote the development of micro-switches. Since the device consists of not only electrical, but also mechanical parts, a proper mechanical model is inevitable to design the switch correctly.

The mechanical design of the switch is often carried out by a lumped mass-spring model [2, 3, 10] or by using a simplified one-dimensional beam model [6, 7, 9]. The argument of these models is that the membrane is rectangular and the electrical field between the capacitors is homogeneous, and consequently, the thin elastic membrane deflects rigidly or at least like an elastic beam. Such simple models can be accepted with some criticism. Even for the simplest, rectangular shape of the bridge the beam model is adequate for narrow plates, only. In the case of larger width the cross-sections do not remain unchanged under bending, the boundaries deflect differently, as the middle of the plate (fringing) [14]. Consequently, if one tries to consider shapes other than a rectangle, a two-dimensional plate model is required. In such cases reliable mechanical response can be obtained by only numerical simulation [12, 13].

This paper presents one- and two dimensional models. In the latter case pre-tightening, i.e. in-plane tension of the bridge was also taken into account. In order to get fast switching, the lowest eigenfrequency must be increased. Keeping constant geometrical parameters, it can be achieved by appropriate in-plane forces. The bridge cannot, however, be unbounded stiff. In the case of a stiff membrane large electrostatic force is needed for switching-off, and therefore, the necessary pull-in voltage must also be increased. This leads to more electric consumption, which worsens the performance of the device.

A new performance factor is also introduced taking into account fast switching, as well as acceptable critical pull-in voltage. Using the finite element method, optimal geometrical parameters have been searched and found in the case of two, not rectangular, membrane-shapes.

2. One-Dimensional Model

The schematic view of the micro-switch is shown in *Fig. 1*. The membrane bridge is bonded at the ends on a substrate. It is suspended over a dielectric film deposited on the center conductor. Applying an electrostatic potential between the bridge and the center conductor, the electrostatic force pulls the bridge downwards (pull-in). Because of the elastic behaviour, the bridge starts to vibrate after it has been released (switch-on).

The substrate and rigidly attached center conductor and the ground plates are fixed. The only moving part is the membrane bridge. The schematic cross-sectional view of the structure is shown in *Fig. 2*.

For an accurate evaluation of the switching speed a nonlinear dynamic model capturing electrostatic, residual stress, inertia, air-damping, Van der Waals force, contact and impact would be essential. As a simplified analytical approximation,

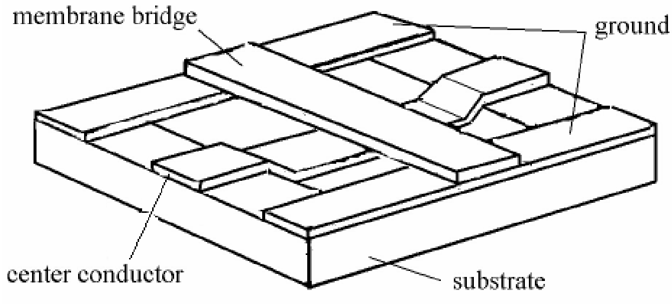

Fig. 1. Capacitive micromachined switch

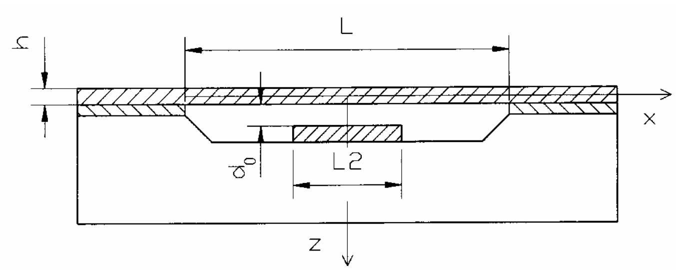

Fig. 2. Cross-sectional view of the micro-switch

the switching speed is estimated by using the first natural frequency of the free, undamped one-dimensional model. The equation of motion is [7]

$$\rho A \frac{\partial^2 w}{\partial t^2} + \tilde{E} I \frac{\partial^4 w}{\partial x^4} - F \frac{\partial^2 w}{\partial x^2} = 0, \tag{1}$$

where $w = w(t, x)$ is the deflection function, ρ is the density, A is the area of cross-section, $\tilde{E} = E/(1 - \nu^2)$ is the equivalent elastic modulus due to plate bending [12] and F is the applied in-plane force. The response function is searched in the form of

$$w(t, x) = \chi(t) X(x), \tag{2}$$

where $X(x)$ is the first mode shape and $\chi(t)$ is obtained by solving the following equation:

$$M\ddot{\chi} + K\chi = 0. \tag{3}$$

The coefficients in *Eq.* (3) are defined by

$$M = \int_0^L \rho b h X^2(x) dx, \text{ and } K = \int_0^L I \tilde{E} X^{(iv)}(x) X(x) dx - \int_0^L F X''(x) X(x) dx. \tag{4}$$

The bridge membrane is considered as a beam fixed at both ends, therefore the boundary conditions are as follows:

$$X(\pm\frac{L}{2}) = 0, \qquad X'(\pm\frac{L}{2}) = 0. \tag{5}$$

The first natural frequency ν_1 is determined by solving the transcendent characteristic equation

$$2(1 - \cosh\lambda_1 L\cos\lambda_2 L) + (\frac{\lambda_1}{\lambda_2} - \frac{\lambda_2}{\lambda_1})\sinh\lambda_1 L\sin\lambda_2 L = 0, \tag{6}$$

where

$$\lambda_1 = \sqrt{\frac{a + \sqrt{a^2 + 4e}}{2}}, \quad \lambda_2 = \sqrt{\frac{-a + \sqrt{a^2 + 4e}}{2}}, \quad a = \frac{F}{\tilde{E}I}, \quad e = \alpha_1^2\frac{\rho bh}{\tilde{E}I} \tag{7}$$

and

$$\nu_1 = \frac{\alpha_1}{2\pi}. \tag{8}$$

Eq. (6) can be solved by using a symbolic algebra program, e.g. Mathematica or Maple. The switch-on time is then estimated simply by $1/(4\nu_1)$.

3. Two-Dimensional Model

Besides the simplest rectangular shape of the membrane bridge, slightly modified geometries have also been investigated. Because of the complicated boundary conditions, no attempt has been made to find an analytical solution. The commercial finite element package COSMOS/M v2.0 [5] is an appropriate tool for the calculation of the eigenfrequencies and the displacement field of the system. It contains also a structural optimization module, by which optimum geometrical parameters could be searched for.

Fig. 3 shows the three types of membrane-shapes. Hashed regions refer to the area, where electrostatic forces are risen (width L_2 of the center conductor, see *Fig. 2*).

The aim is to find best geometrical parameters to achieve large eigenfrequency but also to lower the critical collapse voltage given by [7]

$$V_c = \sqrt{\frac{8K_{\text{eff}}d_0^3}{27\varepsilon_0}}, \tag{9}$$

where d_0 is the initial distance between bridge and centre conductor, ε_0 is the permittivity of air and K_{eff} is the effective stiffness of the beam defined as

$$K_{\text{eff}} = \frac{P}{w_{\max}}. \tag{10}$$

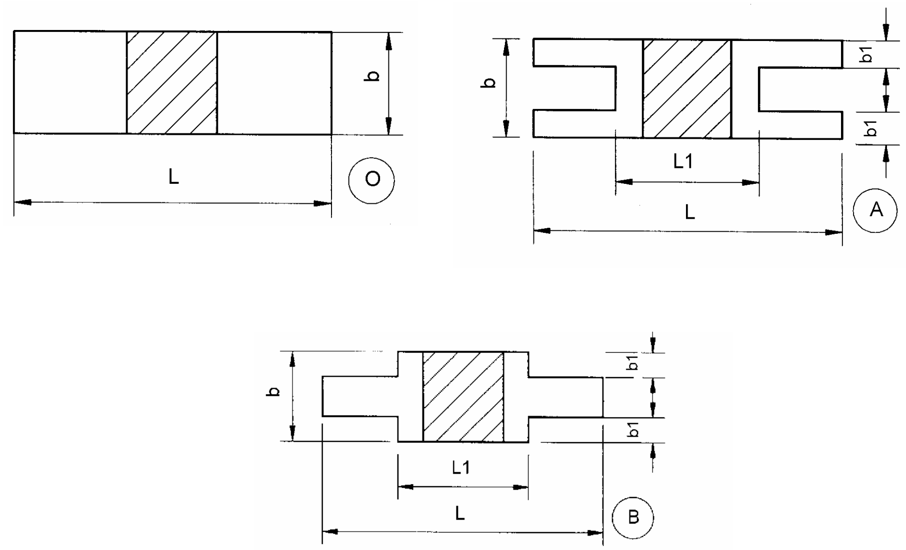

Fig. 3. Different configurations for the bridge membrane

P is the normal pressure load, $w_{\max}$ is the midpoint deflection of the beam. Good performance of the device can be achieved if it has a small critical voltage and a large switching speed. It is seen from *Eqs.* (9) and (10) that

$$V_c = \frac{\text{const.}}{\sqrt{w_{\max}}}. \tag{11}$$

Obviously, large switching speed (small switching time) requires large natural frequency. Consequently, the maximum of the objective function

$$g = \nu_1 \sqrt{w_{\max}} \tag{12}$$

would lead to an optimal structure.

4. Numerical Example

As a numerical example, a micro-switch with the geometrical and material parameters listed in *Table 1* has been analyzed.
In cases A and B $b_1 = b/4$ has been chosen. A homogeneous in-plane tension defined by $p_x = F/bA$ has been applied. *Figs. 4* and *5* show the effect of the tension on the first eigenfrequency and the mid-point displacement, respectively. Dashed line refers to the one-dimensional model in *Fig. 4*. The one-dimensional model gives a quite good estimate for the rectangular model (case O). The finite

Table 1.

L [mm]	0.28
b [mm]	0.09
h [mm]	0.0015
L_2 [mm]	0.12
ρ [kg/m^3]	19300
E [MPa]	76520
ν	0.41

element models consist of 320 four-noded shell element in all the three cases (O, A and B). It is seen that in-plane forces increase the eigenfrequency, however, it decreases the maximum displacement.

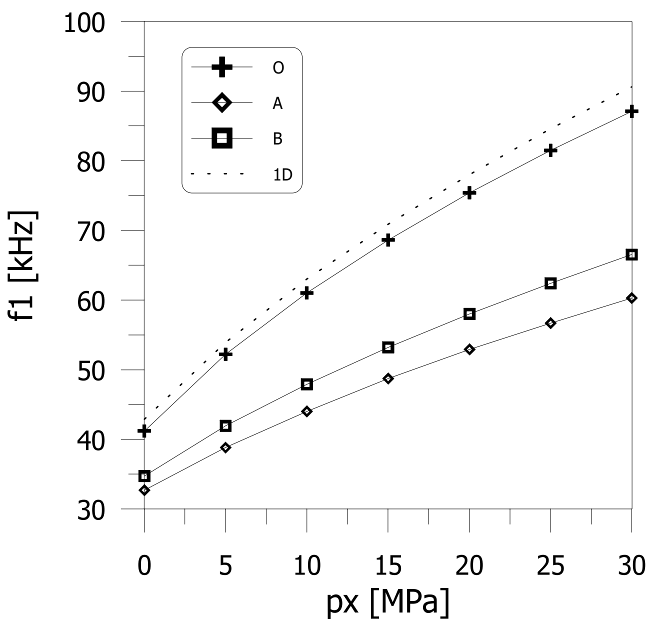

Fig. 4. Effect of in-plane tension on the lowest eigenfrequency

Fig. 6 shows the variation of the dimensionless performance parameter (the value of the objective function divided by 1 kHz·μm) as a function of in-plane tension. It is seen that the curves are about constant and minor differences that can be detected under the different structures.

Maximum occurs at $p_x = 0$, where $g = 128.3$ for structure O, $g = 126.2$ for structure A and $g = 126.5$ for structure B. Keeping L, b, L_2 and all material

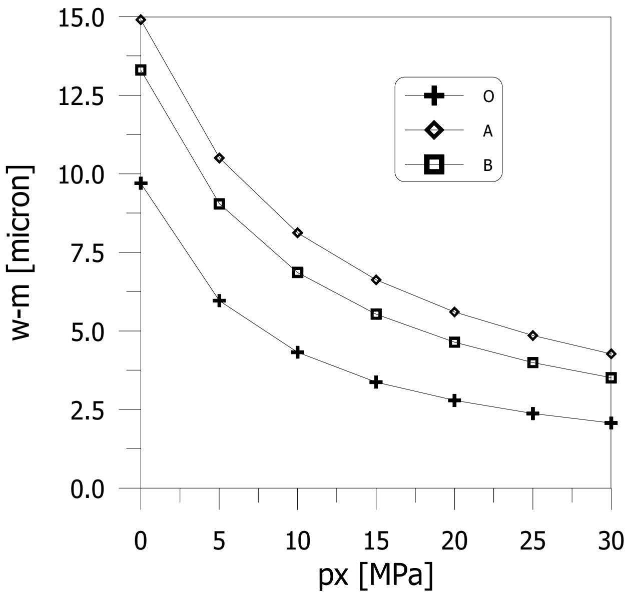

Fig. 5. Effect of in-plane tension on the maximum displacement

parameters constant, three geometrical parameters have been varied as $0.0001 \leq h \leq 0.003$ mm, $0.0001 \leq b_1 \leq b/2$, and $L_2 \leq h \leq L$. *Table 2* shows optimum parameters found by using the OPTSTAR module of the program COSMOS/M.

Table 2.

	A	B
h [mm]	0.00115	0.0027
b_1 [mm]	0.0171	0.0304
L_1 [mm]	0.165	0.160
f_1 [kHz]	53.1	69.6
$w_{\max}$ [μm]	7.15	2.95
g [kHz$\cdot\mu$m]	142.0	180.8

In both cases $g_{\max}$ is much greater than the initial value. It is obvious from *Table* 2 that structure B has better performance.

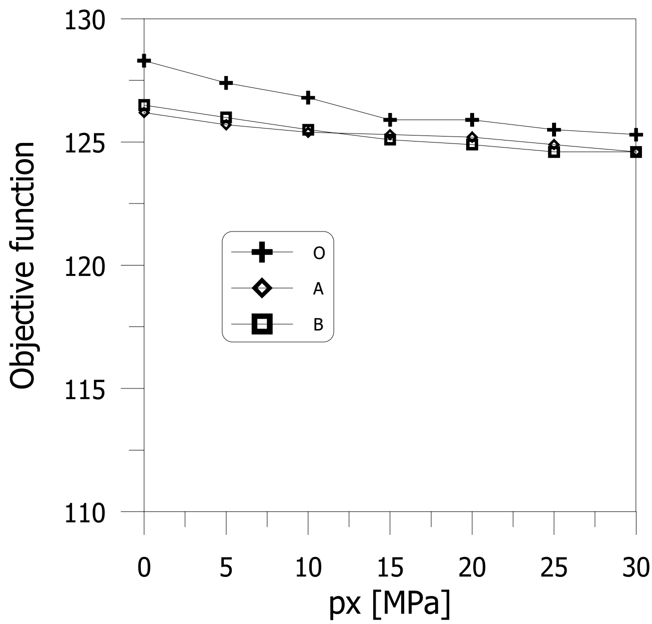

Fig. 6. Effect of in-plane tension on the objective function

5. Conclusions

One- and two-dimensional models can both estimate the eigenfrequency of the membrane bridge in a capacitive micro-switch. Since the switching time is proportional to the lowest eigenfrequency, fast switching can be achieved by having as stiff a membrane as possible. Applying in-plane tension the lowest eigenfrequency can drastically be increased, however, it leads to the increase of the critical pull-in voltage as well. Introducing a parameter equal to the product of the lowest eigenfrequency and the square-root of the maximum deflection of the membrane, the overall performance can be quantified. Using the finite element program package COSMOS/M, optimum geometrical parameters have been found for two structural models of the membrane bridge.

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