Periodica Polytechnica
Mechanical Engineering

62(2), pp. 110-117, 2018
https://doi.org/10.3311/PPme.10950

RESEARCH ARTICLE

Experimental and FE Modeling of Mixed-Mode Crack Initiation Angle in High Density Polyethylene

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Received 26 April 2017; accepted after revision 03 January 2018

Abstract

In this paper, an experimental and a numerical analysis were carried out using High density polyethylene (HDPE). Sheets with an initial central crack (CCT specimens) inclined with a given angle are investigated and compared to the loading direction. The kinking angle is experimentally predicted and numerically evaluated under mixed mode (I+II), as a function of the strain energy density (SED) around the crack-tip, using the Ansys Parametric Design Language (APDL).According to the experimental observations and numerical analysis, the plan of crack propagation is perpendicular to the loading direction. Moreover, as suggested by Sih in the framework of linear elastic fracture mechanics (LEFM), the minimum values Sminof the factor S are reached at the points corresponding to the crack propagation direction. These results suggest that the concept of the strain energy- density factor can be used as an indicator of the crack propagation direction.

Keywords

strain energy density, mixed mode, HDPE, crack initiation

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1 Introduction

Thanks to many advantages which the high-density polyethylene (HDPE) has, it is highly used in the networks of drinking water. However, since 2003, several water distributers noted premature failures of connections in HDPE, in particular, when the pipeline is subjected to bending or stretching.

As known, the major source of failure of structural components is crack growth for such reason, defects and pre-cracks may be introduced in HDPE pipelines during installation, repair or by accident. Crack propagation was not considered to predict its trajectory. Nevertheless, at present, studying crack propagations' direction has been made possible by Finite Element Method (FEM), and the prediction of crack initiation angle in components is accessible by the fracture mechanics approaches, according to these approaches, the fracture process is assumed to be initiated from the pre-existing defects cited previously [1]. Therefore, it is necessary to develop a procedure that allows the prediction of crack initiation angle and the propagation direction in HDPE.

In linear elastic fracture mechanics, the various fracture criteria for cracks subjected to mixed mode loading have been introduced for the determination of the propagation direction and the critical stress, such as maximum tangential stress criterion [2, 3-5], maximum principal tangential stress criterion [3], maximum strain criterion [6-7], and strain energy density criterion [3,8]. All these criteria are almost postulated that crack initiation will occur at the crack tip and propagate towards the radial direction.

Sih's [3, 9-11] strain energy density criterion takes the strain energy density to be investigated of an element around the crack tip as the starting point and the strain energy density factor as a basic parameter. It is then postulated that crack initiationwill occur at the crack tip in a radial direction along which the strain energy density factor, S, is minimum, and the crack will begin to propagate when the factor, S, reaches some critical value.

The strain energy density criterion was validated experimentally and numerically on brittle materials [8, 12-20] and ductile materials [21-23]. In the framework of the large deformations, [24, 25] used this criterion on rubber materials to predict the initial crack orientation under mode-I and mixed mode loadings. The SED approach has been successfully used by Torabi et al. [26], Berto et al. [27] and Campagnolo et al. [28] to assess the fracture strength of different materials, characterized by different control volumes and subjected to wide combinations of static loading [29-32]. Moreover, it has been successfully employed for the fatigue strength of welded joints [33-34] and notched components [35, 36]. As shown by Lazzarin et al. [37], the SED can be easily evaluated numerically through finite element analysis by using coarse meshes, and it allows automatically to take into account higher order terms and three-dimensional effects [38, 39]. These considerations are among the main advantages of the SED approach.

The objective of this paper is to present an experimental and numerical modeling of crack initiation angle for ductile fracture of HDPE, under mixed mode (I+II). Using the Ansys Parametric Design Language (APDL), the crack direction is evaluated as a function of the minimum strain energy density $(dW/dV)_{min}$ around the crack tip. The results obtained by numerical approach are compared with those obtained in experiments.

2 Strain energy density theory

The SED fracture criterion locally focuses on the continuum element ahead of the crack,it is based on the notion of weakness or severity experienced by the local material. Failure occurs when a critical amount of strain energy dW is accumulated within the element volume dV and the crack is then advanced incrementally in the corresponding direction [3, 40]. The strain energy density function (dW/dV) is assumed to have the following form

$$\frac{dW}{dV} = \frac{S}{r} \quad (1)$$

Where S is the strain energy density factor and r is the distance from the crack tip. The minimum of the strain energy density factor S_{min} around the crack tip determines the likely direction of crack propagation.

The strain energy density can be determined directly from the relationship.

$$\frac{dW}{dV} = \int_0^{\varepsilon_{ij}} d\sigma_{ij}.d\varepsilon_{ij} \quad (2)$$

Where σ_{ij} and ε_{ij} are the stress and strain components respectively [41].

The strain energy per unit volume dW/dV can be further decomposed into two parts:

$$\frac{dW}{dV} = \left(\frac{dW}{dV}\right)_d + \left(\frac{dW}{dV}\right)_v \quad (3)$$

$(dW/dV)_d$ represents the distortional strain energy per unit volume, corresponding to the deviatoric stress tensor that is associated with distortion (shape change) of an element and is responsible for macroyielding and the creation of slip planes or microcracks. $(dW/dV)_v$ represents the part of the strain energy per unit volume associated with dilatation (volume change) and is responsible for macrofracture (crack growth) and the creation of the cleavage planes which are perpendicular to the direction of tension.

The computed discrete values for S are fitted with approximation function which enables simple determination of the local minimum. The strain energy density function has several local minimums around the crack tip, where the global minimum is not necessarily the true solution, as it can be observed from Fig. 1.

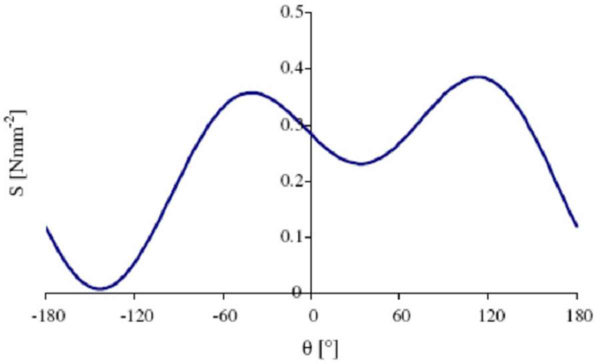

Fig. 1 Strain energy density factor S as a function of the angle θ

The minimum of strain density function S_{min} can be found numerically by incrementalsearch for a local minimum in possible crack extension directions θ_i in the range $\pm\pi$ around the crack tip [42].

Fajdiga et al.[42] used the position of integration points to define the corresponding angle of calculated strain energy density and strain energy density factor S around the crack tip. In this study, the minimum strain energy density $(dW/dV)_{min}$ is computed by introducing a ring of elements around the crack tip. At each crack increment, the crack direction is evaluated as a function of the angle between the centre of the element and the crack axis (Fig. 2).

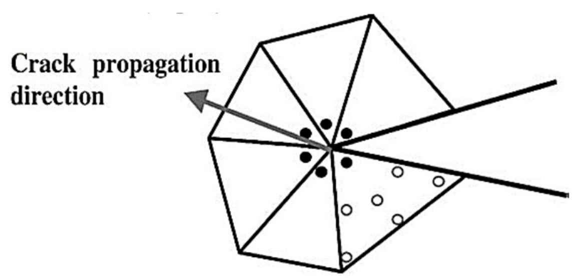

Fig. 2 Direction of propagation using the minimum strain energy density

3 Mixed mode fracture tests

3.1 Materials, specimens, tests

The experiments were carried out in CHIALI Group using HDPE. Fracture tests were performed using specimens containing an inclined central crack introduced by a razor blade.

The dimensions of these specimens are: length h=120mm, width w = 75mm and thickness B = 4mm (Fig. 3). The considered crack length is a = 20mm with four orientations defined by the angle α = {0°, 30°, 45°, 60°}. Specimens were loaded in a uniaxial direction under controlled load F=10KN with a strain rate of v = 50mm/min, but a mixed mode (I+II) could be induced around the crack tip because of the crack inclination. Fig. 3 shows the geometry of the center cracked specimens used in fracture tests under mode-I and mixed mode loadings.

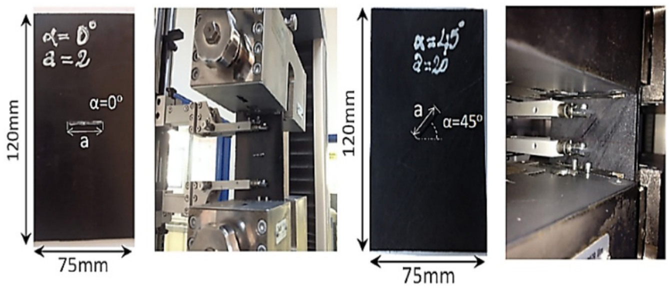

Fig. 3 Geometry of the Centre Cracked Tension specimens (CCT) used in fracture tests

3.2 Crack initiation angle

The crack initiation angle θ_0 is the direction in which the crack propagates from the original crack (Fig. 4(a)). For different α, the value of θ_0 is predicted by each fracture test. Fig. 4(b) shows the crack initiation angle evaluated in this study (Table1).

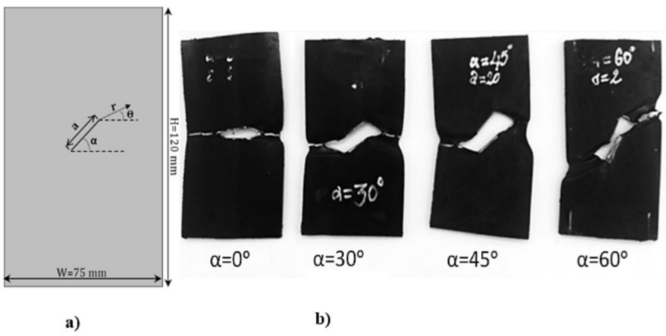

Fig. 4 (a) Description of crack imitation angle θ, (b) Crack initiation obtained from fracture tests

Table 1 Crack initiation angle θ_0 (°) obtained from fracture tests

Specimen code	Crack inclined angle α (°)	Crack initiation angle θ_0 (°)
1	0°	-5°
2	30°	-45°
3	45°	-64°
4	60°	83°

4 Finite elements study
4.1 Model and meshing

The geometry of the centre cracked plate (120 mm x 75mm) with an initial crack (a = 20mm) is considered for 2-Dimensional finite element analysis. The pre-existing central crack is inclined to the horizontal axis with angle α = {0°, 30°, 45°, 60°}.The FE calculation was achieved by gradually increasing of the displacements (d) applied to the nodes located at the top of the plate.

All the above mentioned fracture tests were numerically simulated using the ANSYS® finite element code [43].

For the mesh generation of the cracked plate, the element type 'PLANE183' is used, as shown in Fig. 5(a).

It is a higher order two dimensional, 8-node element having two degrees of freedom at each node (translations in the nodal x and y directions), quadratic displacement behavior and the capability of forming a triangular-shaped element, which is required at the crack tip areas.

Due to the singular nature of the stress field in the vicinity of the crack, the singular elements proposed by Barsoum [44] are considered at each crack tip area. Fig. 5(b) shows the typical finite element mesh used for numerical analysis.

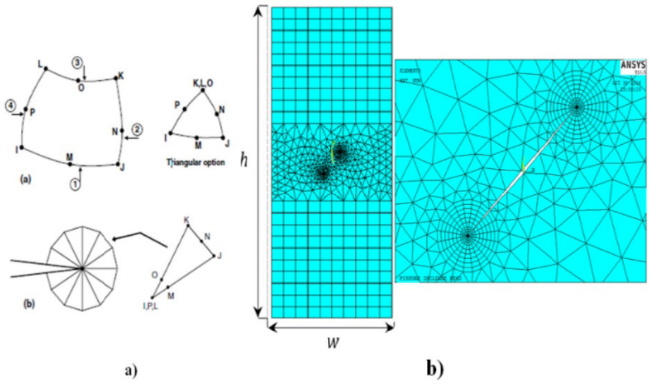

Fig. 5 (a) Element types used in meshing. (b) Example of a selected meshing (α = 45°)

The material properties used in this study were those corresponding to the stress-strain curve obtained from experimental tensile tests obtained by Blaise [45].

The stress-strain curve is modelled by elastic-plastic behavior with multi-linear isotropic hardening according to the experimental plastic curve [46, 47].

The rings of the elements surrounding the tip of the crack were employed. This mesh will be used in order to determine the strain energy density in these elements thus to determine the kinking angle θ in the direction for which this energy density is minimal $(dW/dV)_{min}$.

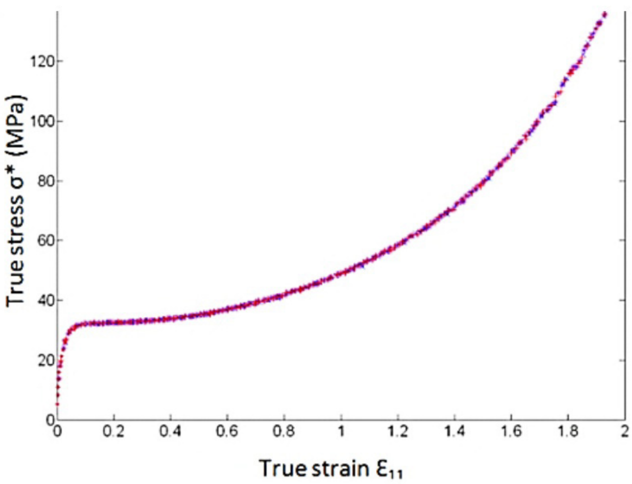

Fig. 6 The stress-strain curve obtained from experimental tensile tests [45]

Fig. 7 illustrates the description of the parameters: r, α and θ. Where: r is the distance from the crack-tip, α is the crack inclination angle and θ is the initial crack propagation.

The specifications of the crack tip mesh and a close up view for crack inclination angle $\alpha = 45°$ are shown in Fig. 8. In this figure, 'r_i' represents the distance between the crack tip and the center of the element 'c'.

5 Results and discussion

5.1 Strain energy density

The obtained results are traced for several values of the radius r_i and for four orientations of initial crack $\alpha = 0°$, 30°, 45° and 60° (Fig. 9).

These figures highlight that, out of the core region surrounding the crack tip (in our case $r \geq r_1$), the minimum of (dW/dV) is reached for a constant value independently of distance r. The angle θ_0 corresponds to the horizontal direction perpendicular to the loading.

In general, a material element is subjected to both distortion and dilatation. Distortion and dilatation vary in proportion, depending on the load history, location, and nonuniformity in stress or energy fields. For the macrocrack under tension, the macrofracture coincides with the direction in which $(dW/dV)_v > (dW/dV)_d$ and macroyielding with the direction in which $(dW/dV)_d > (dW/dV)_v$. In the case of 2-D analysis, the continuum mechanics solution of the stress problem shows that the

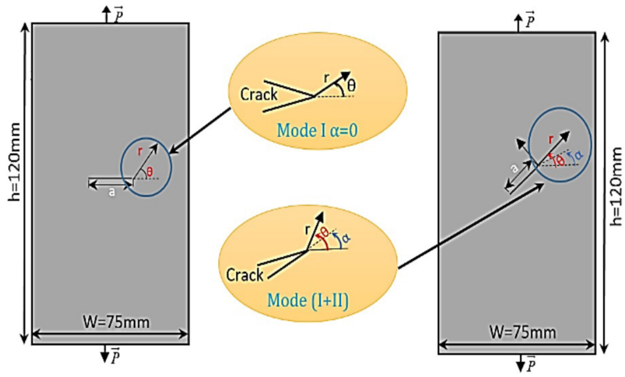

Fig. 7 Description of the parameters r, α and θ

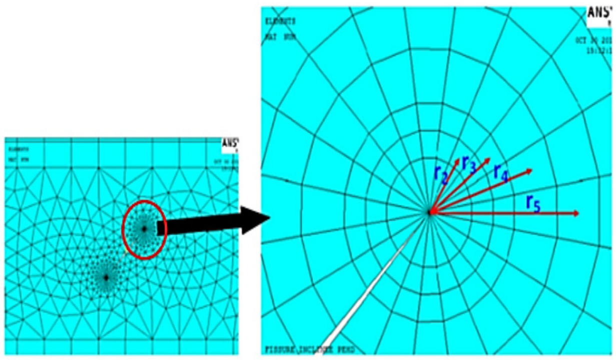

Fig. 8 Ring of elements around the crack tip

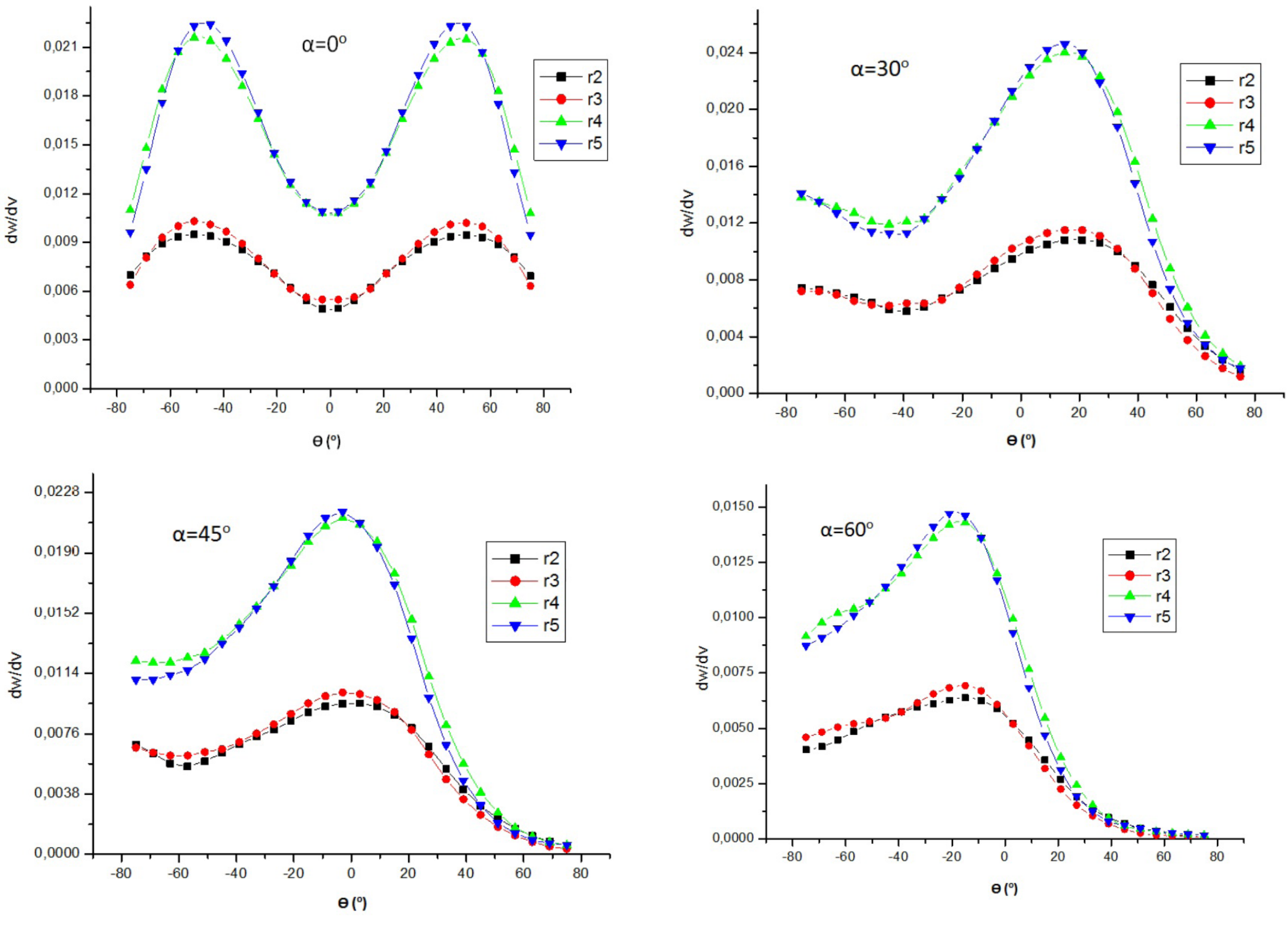

Fig. 9 Evolution of dW/dV as a function of θ for several values of the radius r (with $\alpha = 0°$, 30°, 45° and 60°)

principal stresses σ_1 and σ_2 are equal (σ_1=σ_2) in the element ahead crack, under this condition the greatest change in volume occurs ($(dW/dV)_v > (dW/dV)_d$).

That means the domination of the macrodilatation, which is responsible to the cleavage process in the plane perpendicular to the loading direction [41].

The relative local minimum of dW/dV corresponds to large change in volume and is identified with the region dominated by macrodilatation leading to fracture i.e. the appearance of cleavage planes perpendicular to the direction of tension.

The angle values obtained numerically are compared with those predicted experimentally. The obtained result shows a good correlation between the two approaches. Table 2 summarizes the crack initiation angle θ_0 of fracture tests and FE analysis for different crack inclination angles α.

Table 2 Comparison between crack initiation angle θ_0 (°) obtained from fracture tests and numerical results (for r2)

Crack inclination angle α (°)	Crack initiation angle θ_0 (°)	
	Fracture tests	SED criterion
0	-5	0
30	-45	-40
45	-64	-56
60	-83	-78

5.2 Effect of elements number surrounding the crack tip

For the strain energy density criterion, the precision is strongly related to the number of elements surrounding crack tip zone [35]. For this purpose, we examined the influence of the mesh size (or number of the elements) on the strain energy density variation. For that, the density (dW/dV) is evaluated for different number of elements surrounding the crack tip. Inthis study, the numbers of elements considered are 20, 36, 48 and 60 (Fig. 10).

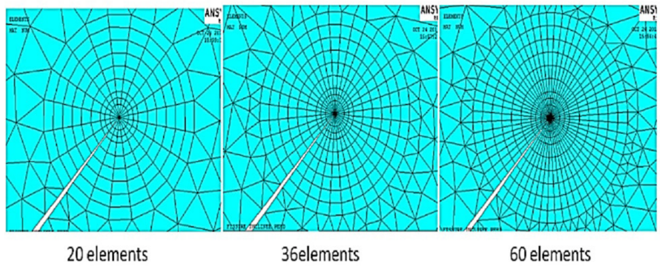

Fig. 10 Elements' sizes around the crack tip

For different number of elements surrounding crack tip zone and for several values of the radius r, Fig. 11 illustrates the evolution of the strain energy density (dW/dV) as function of the initial angle of propagation θ. The results obtained show that the precision is related to the elements sizes around the crack tip.

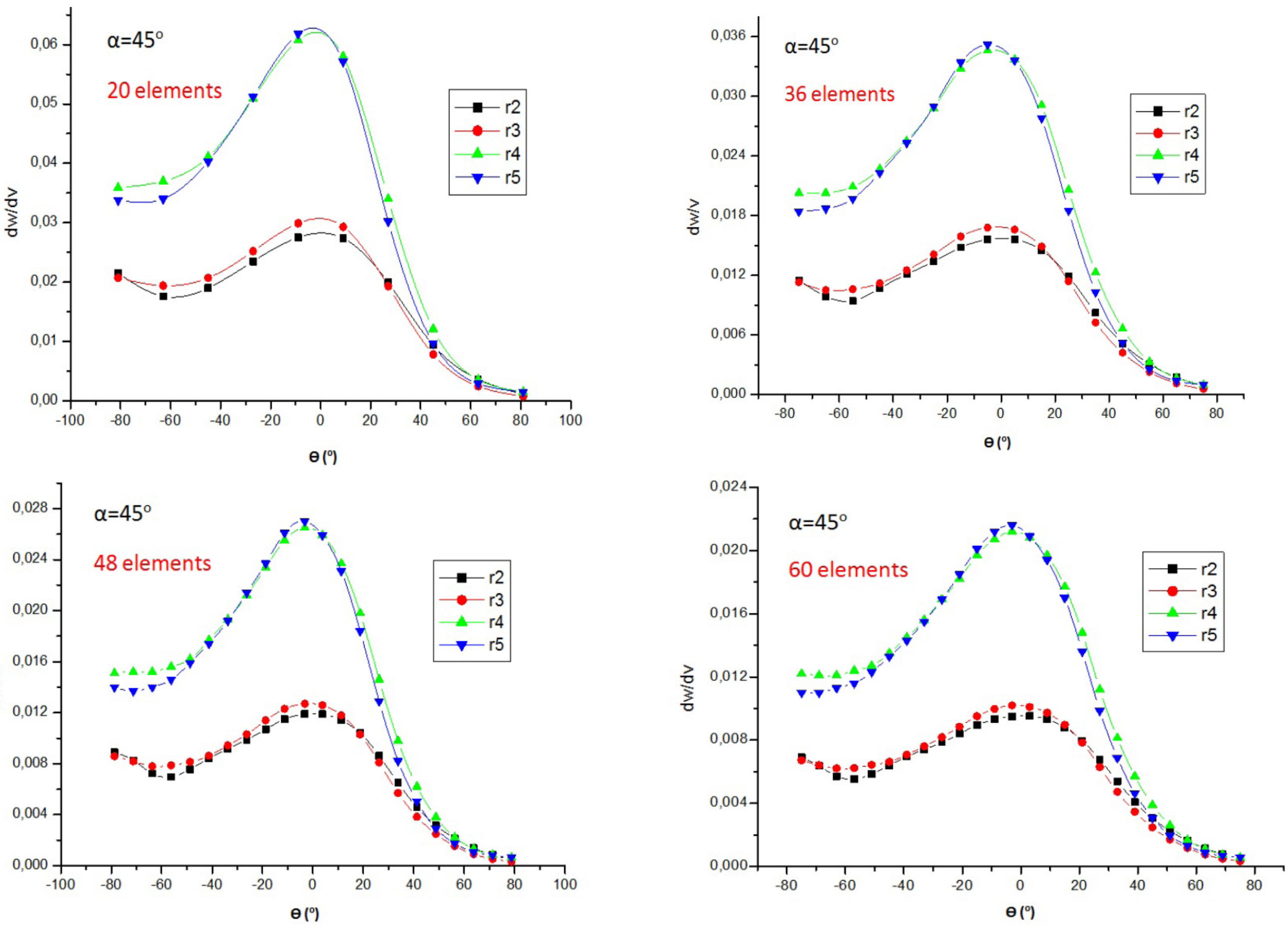

Fig. 11 Influence of meshing on the evolution of dW/dV (for α = 45°)

To better show the influence of the elements sizes around the crack tip on the determination of initial angle of crack propagation θ_0, Fig. 12 illustrates the variation of angle θ_0 as a function of number of elements of elements surrounding the crack tip for various distances.

The curves obtained show that more there will be elements around the crack tip; more the crack direction θ_0 at each crack increment length and the final path of crack propagation will be precise.

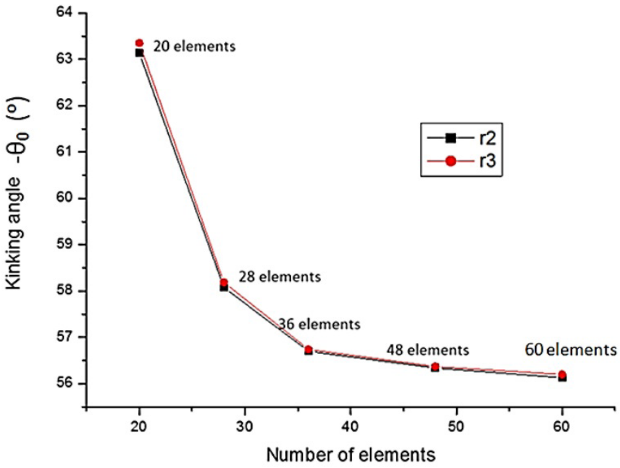

Fig. 12 Influence of meshing on the determination of the angle θ_0 (for $\alpha = 45°$)

6 Conclusion

The study has been conducted to analyze and simulate the initial crack propagation angle in HDPE, under mixed mode (I+II). Using the Ansys Parametric Design Language (APDL), the strain energy density approach is investigated. For various crack inclined angle, the kinking angle is evaluated as a function of the Minimum Strain Energy Density (MSED) around the crack tip. The obtained results allow us to deduce the following conclusions:

- The quarter-point singular elements proposed by Barsoumare used to consider the singularity of stress and deformations fields at crack tip.
- Out of a core region surrounding the crack tip, the minimum of SED is reached for a constant value independently of distance r. The obtained results show a convenient agreement between experimental and numerical approach.
- The minimal value of SED, corresponding to the direction of crack propagation, is always reached in the plan perpendicular to the loading axis, independently of the initial crack orientation.
- The value of the angle of initial crack direction is related to the number of the elements surrounding the crack tip.
- Consequently, the SED approach, developed in the linear elastic fracture problems, could be extended to highly non-linear deformable materials as an indicator of the crack propagation direction.

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