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Uncertainty's Effects Assessment and the Relevant Parameters on Soil-Pipe Interaction (SPI) System Responses Using Sobol Sensitivity Analysis

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Abstract

Buried pipes play a crucial role in the transportation and distribution of various substances, such as energy, fluids, and waste, influencing daily life and industrial activities. The interaction between soil and pipes significantly affects the distribution of forces and stresses in the soil-structure systems. This study focuses on the soil-pipe interaction (SPI) system, employing the simplified Winkler model to represent SPI. The uncertainty in soil and pipe parameters is a major source of variability in the response model. To address this, five semi-empirical models for computing the soil reaction modulus are analyzed. A global sensitivity analysis, specifically the Sobol method, is employed to quantify the effects of uncertain input parameters on soil reaction modulus and pipe flexural strains. Results indicate that soil parameters, particularly Young modulus, have the most substantial impact on the soil reaction modulus, while pipe parameters exhibit negligible effects. The sensitivity analysis provides insights into the influential factors and their interactions. Moreover, a probabilistic analysis is conducted to assess the variability of soil reaction modulus and pipe displacement, revealing differences among semi-empirical models and soil types. This study contributes to identifying key parameters affecting the SPI system, allowing for better-informed decision-making in pipeline design and management. The findings emphasize the importance of considering uncertainties in soil and pipe properties for accurate predictions of soil-pipe system behavior.

Keywords

probability, epistemic uncertainty, soil reaction modulus, Sobol sensitivity analysis, soil-pipe interaction (SPI)

1 Introduction

In the modern world, buried pipes are an essential part of the lifelines infrastructure elements that transport and distribute energy, fluids, waste water, oil products, and gas. Particularly the pipes enable long-distance transportation of liquid gas, liquid fuel, and water, playing a critical role in daily lives and industrial activities, due to the benefits of simple structure, easy construction, and environmental protection.

The soil which constitutes the environment of these pipes can decisively influence on the distribution of forces, displacements and subsequently stresses and deformations of the pipes. We are then talking about an interaction between the soil and these pipes. Numerous research studies have examined the soil-pipe interaction (SPI) system in various ways, for example [1–5].

The soil-structure interaction is an ever-persistent complex problem, but has not yet been thoroughly studied until now [6]. Depending on the considered viewpoint, the treatment of this phenomenon is perceived differently [7–12].

In our investigation, the SPI was represented by the simplified Winkler model [13]. This model has been widely used to solve numerous soil-structure interaction problems and has produced effective solutions for a variety of

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real-world issues. This is due to its simplicity and the benefit of only requiring one parameter (the soil reaction modulus) to describe the soil elastic response and load-bearing structures. The mechanical parameters of the soil, as well as the mechanical and geometrical characteristics of the structure, determine the soil reaction modulus, which is not an intrinsic property of the soil. All these parameters are uncertain and constitute the main source of uncertainty in the response (output) model.

Natural variations in a soil's physical and mechanical characteristics result from the complexity of natural geological processes (such as erosion, transport, deposition, compaction, and physico-chemical reactions) that gave rise to the soil's formation [14]. The mean, variance, and covariance function in the case of a spatial approach to the natural variability are frequently employed to measure the natural variability and are associated with uncertainty on each parameter [15, 16]. Uncertainties pertaining to a pipe's materials are considered by treating structure parameters as random variables; these elements are represented by probability distributions that can be incorporated into the structure's computation [17].

Analytical models with one or two parameters are used to study the SPI system [18, 19]. The soil reaction modulus k is a typical parameter for these models. The literature contains a variety of semi-empirical formulae for calculating this modulus. Vesić [18] and Biot [19] presented k as a function of the soil elastic properties, the relative rigidity of the soil, and the foundation above it. k depends on the soil's and the structure's stiffness. For a given applied stress, structures of the same size but varied stiffness would produce different values of k. A new model for representing Winkler's k was developed by Basudhar et al. [20], leading to the simultaneous prediction of the maximum values of bending moment and deflection. k is depending on the specifications of the structure and is not constant for a particular soil. According to Farouk and Farouk [21], it was improper to determine k by ignoring the soil-footing system's stiffness.

In this study, the five most popular semi-empirical models for buried pipe designs are selected to calculate the soil reaction modulus k, Section 2.1.1.

The presence of uncertainty in the SPI system leads to the need to characterize it both qualitatively and quantitatively using the concepts of random variables from probability theory [22]. Some of the input variables may be somewhat unknown (having a lot of uncertainties), yet have minimal impact on the model output (low variation). On the other hand, others variables have little uncertainty, but have a significant impact on an output (many variations). Identifying the influential inputs and variables is therefore essential to better understand the model's behavior or detect anomalies: this is the sensitivity analysis.

An important aspect of our study is the global sensitivity analysis. This technique allows studying the effect of uncertain input parameters on their entire range of fluctuation on the model outputs (soil reaction modulus, displacements). It enables simultaneous parameters variation and consideration of the probability distribution for each input. The Sobol approach is the global sensitivity analysis used in this paper. This method is based on the variance analysis of the model outputs to calculate the sensitivity indices. These indices are known to be good indicators of the sensitivity model to its input parameters. The first order index, the total index, and the interaction between the various input parameters can only be calculated using the Sobol method. It enables the model parameters to be categorized according to the degree to which they influence the variability of the outcomes. This ranking determines which elements are irrelevant and which require further research [23].

A variety of procedures to study the uncertainties effects on the SPI response system has been proposed in the literature. Without being exhaustive, one can cite some of the most recent research Imanzadeh et al. [5]. They applied FOSM and SOSM methods to define the coefficient of variation of soil reaction modulus. These methods reveal the effect of soil and pipe parameters uncertainties on the soil reaction modulus. The main effects arise from uncertainties in soil Young modulus, external diameter of buried pipe and Poisson's ratio of soil. Wu et al. [24] determine the sensitivity analysis by ANOVA method (analysis of variance), the results indicate that pipe deformation and soil pressure are highly affected by the elastic modulus and soil Poisson's ratio. Zamanian et al. [25] use Sobol's sensitivity analysis to study the influence of uncertain variables related to material properties and loading to identify the significant variables that affect the leakage and collapse of buried concrete sewer pipes under large truck loads. The results show that concrete and backfill soil properties, along with truckloads variables, significantly impact key pipe responses.

In this similar context, the goal of this present paper is to estimate the variability of soil reaction modulus and pipe settlements (flexure strains) from the soil parameters (Young modulus, soil Poisson's ratio) and pipe parameters uncertainties (Poisson's ratio and shear modulus). To determine the most influential parameters on these two key parameters, Sobol global sensitivity analysis is used for five semi empirical expressions on soil reaction modulus and on pipe displacement applied to SPI system for two distinct soils; Soft Clay soil and Sand and Gravel soil.

Our contribution in this study is not only to identify the most influential parameters but to classify them in order of influence on the model outputs. This step will allow us to detect the uncertain parameters requiring more interest to reduce the uncertainties which are tainted by them. A probability analysis of the soil reaction module and the displacement has also been carried out for the five semi-empirical models and the two cases of soil.

2 Evaluation of uncertainty's effect on SPI system

This research examines the soil reaction modulus and displacement response of a SPI system (SPI). Due to the model's intrinsic uncertainty, it must be statistically and subjectively described as random variables using probability theory.

The processes of an uncertainty analysis are outlined in Fig. 1, which also highlights the approach's iterative character in many applications [26–28]. The main steps are detailed as follows:

- Step A: Specify the physical model and the relevant quantities for the analysis. The soil-pipe system model is described in Section 2.1. The uncertain parameters (Young modulus E_s and Poisson's ratio of soil v_s, shear modulus G_p and Poisson's ratio v_p of the pipe), two soil types Soft Clay, Sand and Gravel are considered.
- Step B: Identify, measure, and depict the system's sources of uncertainty. To account for the entire space of variance, the SPI parameters must be described in a probabilistic context. Uncertainties in both soil and pipe can be assembled into random and epistemic uncertainty. In many cases, it is difficult to clearly distinguish between them.
- Step C: Input uncertainties defined in step B should be propagated through step A's model [29]. Sampling-based methods are widely used for numerical propagation of variability and uncertainty. In this study, the first two statistical moments of the response (mean and standard deviation) are calculated.
- Step D: Uncertainty propagation techniques are usually used to offer information on the precise impacts of the random input parameters on the response unpredictability. In this case, sensitivity analysis is applied, which gives the input parameters prioritization, Section 2.2.

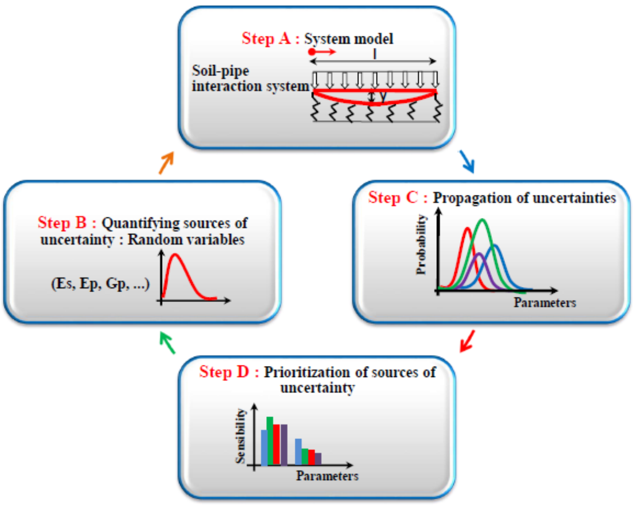


Fig. 1 Detailed steps of the uncertainty quantification

2.1 SPI system modeling

A pipe network is made up of several sections Fig. 2. Each section is made up of a set of nozzles of standardized length, linked together by connecting joints. In most cases, it is buried and rests directly on the soil. Various models have been developed to account for SPI, such as the classic model of Winkler [13] and the modifications made to this model by Filonenko-Borodich [30], Pasternak [17], Hetényi [31], Kerr [32] and Horvath [33]. Despite its limitations, we have chosen to use Winkler's model in this study, because it is easy to use and apply and it is capable of indirectly taking into account soil characteristics variations.

Winkler [13] assumed that the soil reaction under the pipe at each point was proportional to the spread foundation deflections $y(x)$ (displacement), Section 2.1.2, to compute the stresses under the pipe resting on an elastic medium. Consequently, the distinctive vertical deformation of the foundation can be defined using identical, independent, closely spaced, discrete and linear springs. The constant of proportionality for these springs is the soil reaction

Fig. 2 Pipe network

modulus k, Section 2.1.1. Fig. 3 shows a static model of the SPI system. The pipe is considered as uniformly loaded beam $q(x)$ carried elastically along its whole length l.

2.1.1 Semi-empirical estimation of soil reaction modulus

The continuous reaction $R(x)$ between the soil and the pipe components is represented by Eq. (1):

$$R(x) = S_s(x) \times d, \tag{1}$$

where $S_s(x)$ is the stress under the pipe and d is the pipe external diameter. Winkler's model defines the stress as follows:

$$S_s(x) = k \times y(x), \tag{2}$$

where k is the soil reaction modulus and $y(x)$ is the pipe deflection (soil settlement).

According to the theory of elasticity, the response of an elastic element to a loading must be characterized by at least two parameters: the elasticity modulus and Poisson's ratio. As soil is an inelastic and inhomogeneous material, more than two parameters are generally necessary for its mechanical behavior. Thus, using a single parameter, such as soil reaction modulus k, may seem oversimplifying. However, such an approach seems coherent given the variability and uncertainties linked to the characterization of a soil [1].

It is important to note that the pipe stiffness affects the soil reaction modulus k in addition to being a characteristic related to the soil. It depends on a number of variables, including the pipe's width and length, the laying depth, the kind of material utilized, and the bedding type. Only semi-empirical techniques may calculate the k value (Vesić [18]; Biot [19]; Meyerhof and Baike [34], Klöppel and Glock [35], Selvadurai [36]). Since each author came up with a unique expression, the uncertainty around the soil reaction modulus was highlighted. Equations (3) to (7) provide the relevant formulas:

$$\text{Klöppel and Glock: } k = \frac{2E_s}{d(1+\nu_s)}, \tag{3}$$

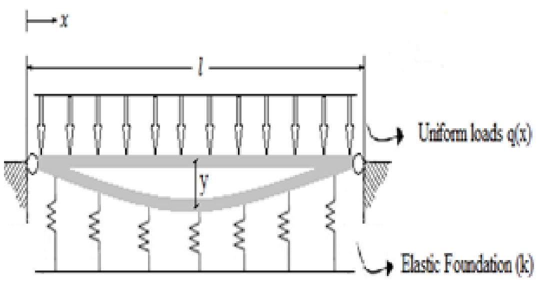


Fig. 3 SPI model

$$\text{Vesić: } k = \frac{0.65}{d} \sqrt[12]{\frac{E_s b^4}{E_p I}} \frac{E_s}{1-\nu_s^2}, \tag{4}$$

$$\text{Meyerhof and Baike: } k = \frac{E_s}{(1-\nu_s^2)d}, \tag{5}$$

$$\text{Biot: } k = \frac{0.95}{d} \left(\frac{E_s b^4}{E_p I} \right)^{0.108} \frac{E_s}{1-\nu_s^2}, \tag{6}$$

$$\text{Selvadurai: } k = \frac{0.65}{d} \frac{E_s}{1-\nu_s^2}, \tag{7}$$

where E_s, ν_s are respectively, Young modulus and Poisson ratio of the soil, d, I, E_p are diameter, inertia moment and Young modulus of the pipe respectively.

The semi-empirical formulas proposed by these authors are not only relatively different, but also result in values that are far apart.

A comparative study was conducted between the five models for the same pipe configuration with external diameter $d = 1.5$ m, thickness $e = 0.15$ m, and Young modulus $E_p = 20$ GPa, soil Poisson's ratio $\nu_s = 0.3$, soil Young's modulus E_s ranges from 0 to 200 MPa. Fig. 4 shows that Vesić's model reflects the lowest values of k, while the Klöppel and Glock's model represents the highest values. When E_s is lower than 40 MN/m^2 Biot and Selvadurai expressions give exactly the same value of k. Vesić and Selvadurai models give almost the same value of k however for the Meyerhof and Baike model, k is nearly twice that of the Vesić model k value for the considered example. The variety of models underlines difficulty to choose a value of soil reaction modulus for a given E_s value.

2.1.2 Determination of displacement using Timoshenko beam theory

Timoshenko method [37] was used in the current study to predict the deflection of uniformly loaded pipe on elastic

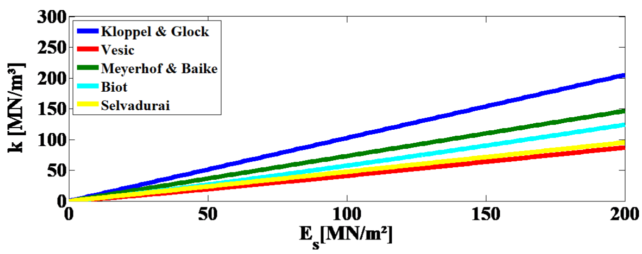


Fig. 4 Evolution of soil reaction modulus k as a function of the soil Young modulus E_s for the five semi-empirical models

foundation. The differential equation of the deflection curve of the pipe is:

$$E_p I \frac{d^4 y}{dx^4} = q - k \times y. \tag{8}$$

Using the notation:

$$\beta = \sqrt[4]{\frac{k}{4E_p I}}. \tag{9}$$

Equation (8) can be directly integrated in the following form to describe the deflection of the pipe:

$$\begin{aligned} y = \frac{q}{k} + A \sin \beta x \sinh \beta x + B \sin \beta x \cosh \beta x \\ + C \cos \beta x \sinh \beta x + D \cos \beta x \cosh \beta x. \end{aligned} \tag{10}$$

Taking the origin of the coordinates at the middle, it is concluded from the condition of symmetry that $B = C = 0$. Substituting this in the solution in Eq. (10) and using the conditions at midpoint of the pipe:

$$(y)_{x=l/2} = 0. \tag{11}$$

It is found that:

$$A = -\frac{q}{k} \frac{2 \sinh \frac{l}{2} \sin \frac{l}{2}}{\cosh l + \cos l}, \tag{12}$$

$$D = -\frac{q}{k} \frac{2 \cosh \frac{l}{2} \cos \frac{l}{2}}{\cosh l + \cos l}. \tag{13}$$

Then the deflection at the middle of the pipe is:

$$y = \frac{q}{k} \left[\begin{aligned} 1 - \frac{2 \sinh \frac{l}{2} \sin \frac{l}{2}}{\cosh l + \cos l} \sinh \beta x \sin \beta x \\ - \frac{2 \cosh \frac{l}{2} \cos \frac{l}{2}}{\cosh l + \cos l} \cosh x + \cos x \end{aligned} \right]. \tag{14}$$

2.2 Global sensitivity analysis using Sobol method

The main goal of the global sensitivity analysis is to provide a comprehensive understanding of which input parameters interacting with each other, Young modulus E_s and Poisson's ratio v_s of soil, and shear modulus G_p and Poisson's ratio v_p of the pipe, have the most significant influence on the soil-pipe system's output: the soil reaction modulus and hence the displacement.

The Sobol method is one of several methods to perform global sensitivity analysis. It has been widely used in the recent research in various fields (in environmental models [38], chemical models [39], biological model [40], ecological models [41]). It explores widely the space of uncertain input variables as it measures not only the individual importance of each parameter but also their interaction effect on the model's output. It provides a clear ranking of parameter importance by order of influence.

The framework in Fig. 5, is based on a probabilistic perspective, considering the following model:

$$\boldsymbol{Y} = f(\boldsymbol{X}), \tag{15}$$

where the output $\boldsymbol{Y}$ is a random vector, $\boldsymbol{X}$ as input described by known probability distributions. These distributions reflect the uncertain knowledge on the system.

In order to quantify the importance of an input factor $\boldsymbol{X_i}$ on the variance of $\boldsymbol{Y}$, the $\boldsymbol{X_i}$ variable is fixed to value x_i^*, so the conditional variance is:

$$V\left(\boldsymbol{Y}\boldsymbol{X}_i = x_i^*\right). \tag{16}$$

By taking all the possible values of $\boldsymbol{X_i}$ with:

$$\boldsymbol{E}\left(V\left(\boldsymbol{Y}\boldsymbol{X}_i\right)\right). \tag{17}$$

We can write the total variance as:

$$V(\boldsymbol{Y}) = V\left(\boldsymbol{E}\left(\boldsymbol{Y}\boldsymbol{X}_i\right) + \boldsymbol{E}\left(V\left(\boldsymbol{Y}\boldsymbol{X}_i\right)\right)\right). \tag{18}$$

The first order sensitivity index for factor $\boldsymbol{X_i}$ from the first term in Eq. (18) is represented by:

$$\boldsymbol{S}_i = \frac{V\left(\boldsymbol{E}\left(\boldsymbol{Y}\boldsymbol{X}_i\right)\right)}{V(\boldsymbol{Y})}. \tag{19}$$

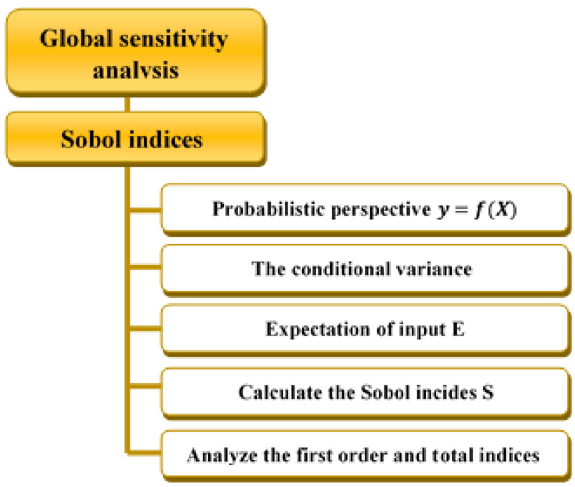


Fig. 5 Global Sobol sensitivity analysis scheme

Thus, the second order sensitivity index which studies the effect of interactions of two parameters on the output is given by:

$$S_{ij} = \frac{V_{ij}}{V(Y)}. \tag{20}$$

Order indices greater than two can also be calculated. Total order sensitivity index studies the effect of single parameter and the effects of its interaction with all other parameters on the variation of the output in Eq. (21):

$$S_{T_i} = \sum_{k \neq i} S_k. \tag{21}$$

3 Case study

3.1 Model data

The example below is based on the SPI system and the pipe is considered as a beam with hinged ends on elastic soil. The load is applied uniformly throughout the pipe's length (q = 40 kN/m for Soft Clay soil and q = 43 kN/m for Sand and Gravel soil), which is caused by the weight of the earth backfill over the pipe and its own weight. The maximum deflection occurs at the middle of the pipe. The uniform law distribution of uncertainty is taken into account in the input parameters (E_s, v_s, G_p, v_p), for numerical propagation in the sensitivity analysis section, it will propagate by normal law distribution with mean M and standard deviation SD.

3.1.1 Soil data

Soil parameters uncertainties and their propagation characteristics are defined in Table 1 [42, 43] for two cases of elastic soil foundation: Soft Clay soil and Sand and Gravel soil with respective densities of 1700 kg/m^3 and 1920 kg/m^3.

3.1.2 Pipe data

Table 2 [44, 45] below gives the parameters uncertainties of a concrete pipe with 1.5 m diameter, 0.15 m thickness and 2400 kg/m^3 density, buried under a soil of 1 m high, (pipe's concrete is isotropic its Young modulus $E_p = 2G_p\ (1 + v_p)$, so G_p and v_p are considered as the pipe's uncertain parameters). For 10,000 simulations, the normal probability law distribution characterizes the parameters uncertainty.

4 Results and discussions

The results developed in Section 4 are based on methodology outlined in Section 2. The first order sensitivity indices estimates the individual influence of the four input parameters (E_s, v_s, G_p, v_p) and the total sensitivity indices take into account interactions with other parameters (the large number of interaction indices prevents them from being presented) on the considered output firstly on soil reaction modulus (obtained with the five semi empirical models), then on the pipe displacement. Sobol indices value is ranged from 0% to 100%, the value of 100% means that only this parameter affects the output variable and the value of 0% means the individual influence is absent.

4.1 Sensitivity analysis of soil reaction modulus of SPI system

The results of k Sobol sensitivity analysis represented in Table 3 and Fig. 6 are given in terms of first and total order indices for the five expressions (Eqs. (1) to (5)): (Klöppel and Glock, Vesić, Meyerhof and Baike, Biot, Selvadurai), for two soil types: Soft Clay soil and Sand and Gravel soil.

We note that in the both soil cases, the soil elastic modulus has the highest total sensitivity indices with the following values (93.1; 92.2; 92.2; 91.6; 91.5)% for Soft

Table 1 Soil input parameters

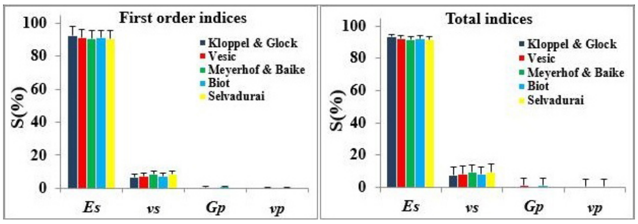
Parameters	Type	Minimum value	Maximum value	Mean	Standard deviation	Reference
E_s (MPa)	Soft Clay	2.07	5.18	3.625	0.897	[42, 43]
v_s		0.20	0.50	0.35	0.087	
E_s (MPa)	Sand and Gravel	69	172.5	120.75	29.87	
v_s		0.15	0.35	0.25	0.058	

Table 2 Pipe input parameters

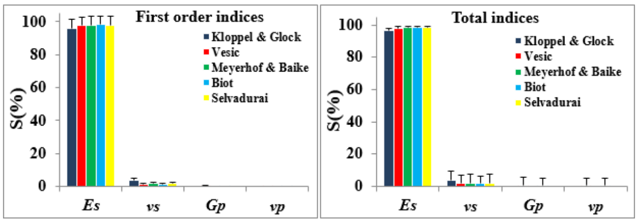
Parameters	Type	Minimum value	Maximum value	Mean	Standard deviation	Reference
E_p (MPa)	Concrete	20000	40000	12794	2654.2	[44, 45]
G_p (MPa)		8196.7	17391.3			
v_p		0.15	0.22	0.185	0.020	

Table 3 Sensitivity indices of the soil reaction modulus k for Soft Clay and Sand and Gravel soils

Soil type	Sensitivity indices	Parameter	Semi-empirical models									
			Klöppel and Glock		Vesić		Meyerhof and Baike		Biot		Selvadurai	
			M (%)	SD (%)	M (%)	SD (%)	M (%)	SD (%)	M (%)	SD (%)	M (%)	SD (%)
Soft Clay	First order	E_s	92.6	5.2	91.3	4.6	90.8	4.9	91.4	4.5	90.7	5.0
		v_s	6.9	1.7	7.4	1.8	8.4	2.1	7.1	1.7	8.5	2.1
		G_p	0.0	0.0	0.6	0.3	0.0	0.0	0.8	0.4	0.0	0.0
		v_p	0.0	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.0	0.0
	Total order	E_s	93.1	1.7	92.2	1.8	91.6	2.1	92.2	1.8	91.5	2.1
		v_s	7.4	5.2	8.2	4.7	9.2	4.9	7.9	4.7	9.3	5.0
		G_p	0.0	0.0	0.9	4.5	0.0	0.0	1.2	4.4	0.0	0.0
		v_p	0.0	0.0	0.5	4.5	0.0	0.0	0.5	4.5	0.0	0.0
Sand and Gravel	First order	E_s	96.0	5.3	97.8	4.9	98.0	5.4	98.4	4.7	98.0	5.4
		v_s	3.7	1.2	1.4	0.8	1.7	0.9	1.4	0.7	1.7	0.9
		G_p	0.0	0.0	0.6	0.4	0.0	0.0	0.1	0.0	0.0	0.0
		v_p	0.0	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.0	0.0
	Total order	E_s	96.3	1.2	98.1	0.9	98.3	0.9	98.6	0.7	98.3	0.9
		v_s	4.0	5.3	1.8	5.0	2.0	5.4	1.6	4.7	2.0	5.4
		G_p	0.0	0.0	0.8	4.8	0.0	0.0	0.2	4.6	0.0	0.0
		v_p	0.0	0.0	0.4	4.9	0.0	0.0	0.2	4.6	0.0	0.0



(a)



(b)

Fig. 6 Sobol indices of soil reaction modulus; (a) Soft Clay soil; (b) Sand and Gravel soil

Clay soil according to expressions of Klöppel and Glock, Vesić, Biot, Meyerhof and Baike, Selvadurai respectively. However for the Sand and Gravel soil (98.6; 98.3; 98.3; 98.1; 96.3)% for Biot, Meyerhof and Baike, Selvadurai, Vesić, Klöppel and Glock respectively.

The soil Poisson's ratio v_s, which has the following total indices values for Soft Clay soil (9.3; 9.2; 8.2; 7.9; 7.4)% and for Sand and Gravel soil (4.0; 2.0; 2.0; 1.8; 1.6)%, is the second most significant parameter. It has a slight influence shows an inverse relationship with increasing soil Young modulus uncertainty. In the same way as analyzed by FOSM method [5], for this current case study, the influence of pipe material characteristics, G_p, v_p is essentially insignificant.

The above results' analysis shows that the soil's Young modulus E_s, with its significant impact, is the most relevant parameter on the soil reaction modulus k for the five models, regardless of whether it contains a large amount of uncertainty (Sand and Gravel soil) or a small amount (Soft Clay soil).

Since the values of the sensitivity indices are close for all semi-empirical expressions, we can conclude that the soil reaction modulus is sensitive to random uncertainties tainted to the soil parameters (E_s, v_s) and not very sensitive to epistemic uncertainties due to the choice of the k computational expression.

Since they are directly related, k grows as E_s grows. The soil stiffness is measured by the soil Young modulus, and the soil stiffness beneath the pipe is characterized by the soil reaction coefficient (higher k values indicate greater stiffness). Thus, the soil Young modulus is the primary component influencing k.

4.2 Sensitivity analysis of SPI system displacement

In Section 4.1, Table 4 followed by the Fig. 7 represent the Sobol sensitivity analysis of the maximum displacement of the pipe as output, as function of the four same parameters (E_s, v_s, G_p, v_p), is shown by first and total order indices in both soil cases.

Table 4 Results of sensitivity analysis of Timoshenko's displacement for Soft Clay soil and for Sand and Gravel soil

Soil type	Sensitivity indices	Parameter	Klöppel and Glock		Vesić		Meyerhof and Baike		Biot		Selvadurai	
			Semi-empirical models									
			M (%)	*SD* (%)	*M* (%)	*SD* (%)	*M* (%)	*SD* (%)	*M* (%)	*SD* (%)	*M* (%)	*SD* (%)
Soft Clay	First order	E_s	96.8	6.6	97.1	13.2	94.0	10.7	96.3	4.8	92.7	5.0
		v_s	3.1	1.4	2.5	1.4	3.6	1.8	2.4	1.3	6.0	1.1
		G_p	0.0	0.0	0.2	0.3	0.0	0.0	0.4	0.3	0.0	0.0
		v_p	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Total order	E_s	96.9	1.4	97.3	1.5	96,4	1.8	97.3	1.5	94.0	1.1
		v_s	3.2	6.6	2.7	13.1	6.0	10.7	3.1	4.5	7.3	5.0
		G_p	0.0	0.0	1.6	8.8	0.0	0.0	0.0	4.4	0.0	0.0
		v_p	0.0	0.0	1.3	8.8	0.0	0.0	0.6	4.1	0.0	0.0
Sand and Gravel	First order	E_s	98.2	5.8	98.7	4.8	98.2	5.2	98.6	4.8	98.7	4.8
		v_s	1.1	0.7	0.4	0.3	1.2	0.7	0.4	0.3	0.6	0.4
		G_p	0.0	0.0	0.2	0.2	0.0	0.0	0.3	0.2	0.0	0.0
		v_p	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Total order	E_s	98.9	0.7	99.4	0.4	98.7	0.7	99.4	0.4	99.4	0.4
		v_s	1.8	5.8	1.2	4.7	1.8	5.2	1.2	4.6	1.3	4.8
		G_p	0.0	0.0	0.7	4.7	0.0	0.0	0.8	4.8	0.0	0.0
		v_p	0.0	0.0	0.6	4.5	0.0	0.0	0.7	4.5	0.0	0.0

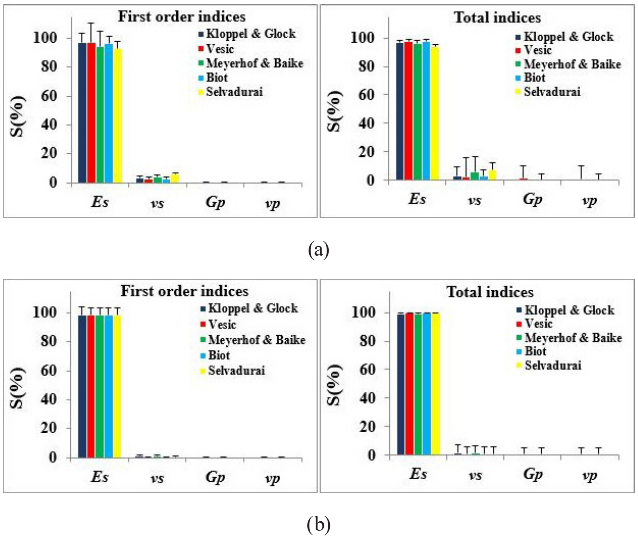


Fig. 7 Sobol indices of soil-pipe interaction system displacement; (a) Soft Clay soil; (b) Sand and Gravel soil

Similarly to the obtain results using FOSM method in [5], The effect of soil Young module's is clearly dominant in both soil types for all expressions, with the following total index values: (97.3; 97.3; 96.9; 96.4; 94)% for Biot, Vesić, Klöppel and Glock, Meyerhof and Baike, Selvadurai, respectively, for Soft Clay soil, and (99.4; 99.4; 99.4; 98.9; 98.7)% for Biot, Selvadurai, Klöppel and Glock, Meyerhof and Baike, and Vesić, respectively, for Sand and Gravel soil. The nature of the soil and the two types' varying rigidities account for a discrepancy in their indices. Also, the fluctuations attributed to Poisson's ratio have a negligible effects on pipe displacement, which confirm the discussions summarized by authors in [5]. The uncertainties resulting from the pipe's parameters have no bearing. It is seen that all of the expressions provide extremely close sensitivity indices, indicating that the random uncertainties associated to soil parameters affect the model's response rather than the expression choice. Therefore, in order to reduce uncertainty about the movement, it is necessary to reduce soil parameters uncertainty, particularly the soil modulus of elasticity. This makes it a crucial parameter to consider when assessing the performance and stability of pipes.

5 Probabilistic analysis

The log-normal statistical distribution model was selected to depict the soil reaction modulus and displacement probability curves of the SPI system based on the five semi-empirical formulas. The two factors that govern this model are its mean value (μ) and standard deviation (σ). The relationship below expresses its cumulative probability density:

$$logncdf\left(x|\mu,\sigma\right)=\frac{1}{2}\left[1+\operatorname{erf}\left(\frac{\ln\left(x\right)-\mu}{\sigma\sqrt{2}}\right)\right], \tag{22}$$

where x is the random variable for which the cumulative probability is calculated, and erf is the error function.

The mean, minimum, and maximum values of the probabilistic parameters are based on the normal distribution law, as can be observed in Table 1. 10,000 simulations have been run in order to determine the probability curves. The effect of soil and pipe parameter variability on soil reaction modulus and pipe displacement (deflection) is discussed in Section 5.

5.1 Soil reaction modulus

Using the five semi-empirical formulas, Fig. 8 examines the impact of pipe and soil parameter variability on the soil response modulus probability of the SPI system. The two distinct soil types were analyzed in the investigation.

As previously stated in Section 4, the random uncertainties of the soil parameters were the most prominent factor influencing the SPI system response, whereas the epistemic uncertainties resulting from the k calculation expressions had little effect. Clearly, the most comprehensive probability curves for k that provide higher values of this modulus for both soil types are those produced by Meyerhof and Baike, and Klöppel and Glock. Indicating that pipe characteristics have no effect on soil reaction modulus, lower values of k are seen for Biot and Vesić. The k modulus values for the Sand and Gravel soil are larger, though, because of the variations in stiffness and elasticity modulus between the two soil types. As an illustration, in the case of the Soft Clay soil and for a probability of 50%, k = (0.6; 0.8; 1; 1.5; 2.3) MN/m^3 and for the Sand and Gravel soil k = (27; 32; 38; 49; 83) MN/m^3 according to the expressions of Vesić, Biot, Selvadurai, Meyerhof and Baike, Klöppel and Glock respectively.

5.2 Displacement of SPI system

Fig. 9 shows the impact of epistemic uncertainties resulting from semi-empirical computation models of k on the displacement probability distribution function. It is evident that the choice of the relationships presented in Section 2.1.1 has a significant impact on the probability distribution. First, the displacement probability curves are inverted with respect to the soil reaction modulus. Additionally, displacements in Soft Clay soil are shown to be larger than in Sand and Gravel soil. This is because the two types of soil differ in their nature and stiffness. In contrast to the Sand and Gravel soil, which is an excellent soil for limiting settlements, the Soft Clay soil exhibits significant deformations. It should be noted that the curves produced by the Vesić and Biot expressions provide the biggest displacement values. Therefore, these displacement values are due to SPI in which is more noticeable in very soft soils than in hard soil. For example, for a probability of 50%, the displacements of SPI system resting in Soft Clay soil are (1.5; 2.4; 3.7; 4.4; 5.7) cm and for Sand and Gravel soil (0.045; 0.077; 0.10; 0.12; 0.14) cm for the expressions of Klöppel and Glock, Meyerhof and Baike, Selvadurai, Biot and Vesić respectively.

6 Conclusions

In this study, we investigate the influence of soil and pipe properties, along with their uncertainties, on the soil reaction modulus k and the structural responses (deflection) within the SPI system. The paper introduces the fundamental concept of soil reaction modulus and highlights key relationships for its determination. We explore five

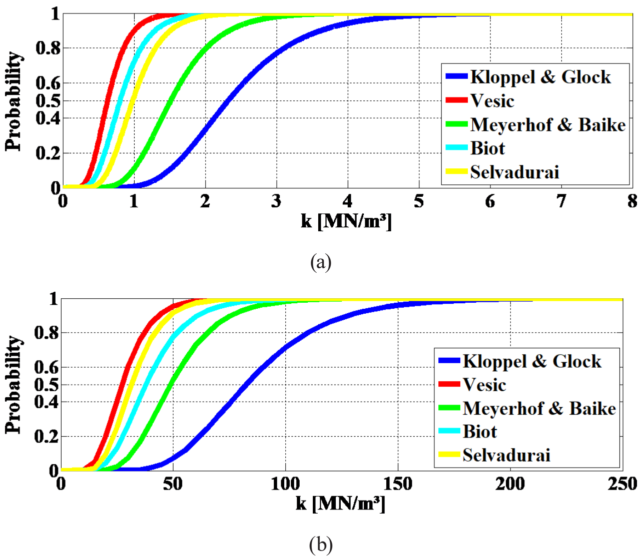


Fig. 8 The soil reaction modulus probability for: (a) Soft Clay soil; (b) Sand and Gravel soil

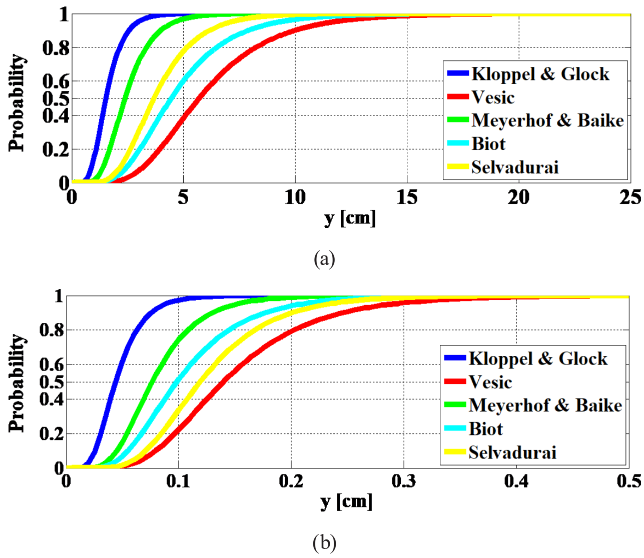


Fig. 9 Timoshenko displacement probability for: (a) Soft Clay soil; (b) Sand and Gravel soil

semi-empirical models for computing the modulus k by accounting for the inherent variability and measurement uncertainties associated with soil and pipe properties, including Young modulus, soil's Poisson ratio, pipe's Poisson ratio, and shear modulus of concrete pipe.

In order to assess the sensitivity of the SPI system, a global sensitivity analysis using the Sobol method is conducted, revealing the contribution of each input parameter within its range of variation on the uncertainty of output parameters (soil reaction modulus and displacement) for each semi-empirical expression of k. Probabilistic concepts prove to be a more suitable and sensitive approach for estimating uncertainty in the SPI system compared to deterministic assessment techniques.

The analysis categorizes input parameters based on their impact on the response output, distinguishing between deterministic and probabilistic considerations. Results, presented in terms of first order and total order indices, emphasize the substantial effects of soil modulus and Poisson's ratio uncertainties on the outputs for the five expressions and two soil cases. Conversely, the uncertainties associated with the Poisson ratio and shear modulus of the concrete pipe are deemed negligible due to their insignificant impact [5].

The sensitivity analysis indicates that the choice of the expression for k has minimal discernible effects on the SPI model's response, as evidenced by close values of Sobol sensitivity indices. The subsequent probability analysis of soil reaction modulus and displacement for both soil cases illustrates differences in values for the same probability when employing the five computation expressions of k.

Specifically, the expressions of Klöppel and Glock yield higher values for the soil's reaction modulus compared to Vesić, 2.3 MN/m^3, also, higher than 0.6 MN/m^3 for the case of Soft Clay, k = 83 MN/m^3 which is greater than 27 MN/m^3 for the case of Sand and Gravel soil, respectively. This discrepancy is more pronounced in Sand and Gravel soil, attributed to differences in rigidity and elasticity between the two soil types. However, during displacement analysis, an inverse order is observed in the probability curves associated with the semi-empirical expressions. Calculations using different expressions of k reveal that Vesić's expressions result in larger displacements for the SPI system compared to Klöppel and Glock, 5.7 cm larger than 1.5 cm for the case of Soft Clay soil and 0.14 cm larger than 0.045 cm for the case of Sand and Gravel soil, respectively.

The study further highlights that SPI system displacements are more significant in Soft Clay soil than in Sand and Gravel soil, owing to the Soft Clay's deformation capacity and the more perceptible SPI phenomenon in soft soils. The computation of k using five semi-empirical models emphasizes the challenge of selecting the appropriate expression, as the same expression may yield large values for k and small values for displacement, and vice versa. This variability in models introduces epistemic uncertainty into the simulations.

Finally, the main effects arise from uncertainties in Young modulus and Poisson's ratio of soil on the uncertainty of the coefficient of subgrade reaction. The uncertainties related to concrete pipe's shear modulus and its Poisson's ratio can be neglected in this current case study.

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