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Probabilistic Analysis of Shallow Foundations on c-φ Soils Using 2nd Order Response Surface Methods

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Abstract

The HL-Reliability Index is utilized to investigate the applicability of several linear and nonlinear response surface models based on design experiments techniques to evaluate the safety under random loading, against bearing capacity failure of shallow foundations resting on c-φ soils with multivariate correlated variables. The reliability results obtained using FORM/SORM coupled with these models, are checked by Monte Carlo simulation method. It is demonstrated that the application of these models significantly reduces the execution time and memory requirements, with the central composite design scheme being the most accurate. It is also concluded, that consideration of correlation, significantly affects the reliability index for large values of soil friction angle uncertainty. The reliability index is found to be highly sensitive to uncertainties of soil friction angle more than the cohesion and the loading which have approximately the same influence, especially for the case of lognormally distributed variables. In addition, the probabilistic results show that reliability index decreases substantially with the increase of the applied pressure, and that there is significant difference between the reliability indices values computed, based on the assumption of normal distribution as compared to lognormal distribution, especially for the lower ranges of applied loading.

Keywords

shallow foundations, soil variability, random loading, 2nd order response surfaces, central composite design

1 Introduction

Although most of research work on the behavior of shallow foundations is essentially of deterministic nature, there is a growing attention to the development and use of probabilistic methods to assess the reliability and performance of foundations against bearing capacity failure under uncertain soil and loading conditions. To accomplish this task, a reliability index criterion is usually utilized as a measure of system safety. Alternatively, a failure probability criterion can be also used as an indicator of system failure. Normally, a reliability index value in the range of 3.0 (i.e., probability of failure $P_f = 1.3 \cdot 10^{-3}$) to 4.0 (i.e., $P_f = 3.17 \cdot 10^{-5}$) is accepted for good performance of the system in a given environment and loading conditions.

Probabilistic investigations of shallow foundations on both cohesive soils (e.g., [1–3]) and cohesionless soils (e.g., [4, 5]) including granular materials (e.g., [6, 7]) have been performed by different researchers. Moreover, safety assessments of shallow foundations resting on c–φ soils have been carried out by various authors (e.g., [8–10] just to name a few). In this case, however, it is to be noted that contrary to sandy or clayey soils, characterized by only one strength parameter, the ultimate bearing capacity and hence the safety of shallow foundations, resting on c–φ soils, is essentially a function of its inherent resistance to both frictional and cohesive shear.

Reliability index can be computed by various probabilistic methods including essentially Point Estimation Methods PEM [11], First and Second Order Reliability Methods (FORM and SORM) [12], First and Second Order Second Moments methods (FOSM, SOSM) [13, 14] and MCS method [15]. Contrary to the above approximate methods, MCS results can be used to calculate not only the statistical moments but also the PDF of the output response. The method is generally more accurate than the above mentioned methods because it requires the PDF's of the system or component variables to be prescribed. However, it should be pointed out that, when using MCS or its variants [14] the reliability index cannot be computed directly

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and is generally evaluated indirectly from the probability of failure P_f as follows $\beta = \Phi^{-1}(1 - P_f)$ where Φ^{-1} is the inverse of the standard normal cumulative distribution.

Further, it is important to note that reliability index estimated using the FOSM or SOSM methods is not "invariant" as alternative but equivalent performance functions lead in general to different values of reliability index [13]. In this work, and for reliability evaluation purposes, use will however be made of Hasofer and Lind index β_{HL}, (defined in the standard normal space, as the shortest distance from the origin of the random variables to the limit state surface). This invariant definition is widely used in reliability studies and can be cast into a matrix formulation [16]. If need be, probability of failure P_f can be calculated using the formula $P_f = \Phi(-\beta_{HL})$. However, when using SORM, the following formula $P_f = \Phi(-\beta_{HL})(1 + \kappa_j\beta_{HL})$ should be used in order to account for the curvature of the response surface at the most probable design point MPP (see e.g., [13, 17] for more details). It should also be noted that for non-normal distributions, input soil and loading parameters need to be transformed into equivalent normal variables using the Rackwitz and Fiessler procedure [18].

When performing reliability studies of shallow foundations, it is to be noted that the choice of the deterministic model of ultimate bearing capacity (see e.g., [19–21]) in order to determine the reliability index of foundation is crucial. As a matter of fact, probabilistic analyses used in conjunction with Terzaghi's $N\gamma$ bearing capacity factor constitute a formidable numerical task because derivatives for variances by first- and second-order methods are laborious, not to say extremely difficult (e.g., [5, 22]).

On the contrary, the use of second order response surface models in establishing highly accurate functional relationships between dependent ultimate bearing capacity and input soil variables, allows to compute quickly and precisely these derivatives and hence the reliability index. These numerical aspects are advantageously utilized in the present work to efficiently extract the desired safety information thus reducing both CPU time and memory requirements as opposed to the excessive computational efforts required by MCS method.

The paper is organized as follows: Uncertainties of soil and loading characteristics are briefly described in Section 2. A brief background on the Response Surface Methodology is provided Section 3. The overall results of reliability analysis of foundation safety on c–φ soils with multivariate correlated variables are presented in Section 4, while the results of a sensitivity analysis of reliability index to statistical distributions and uncertainties of random soil and loading variables are examined in Section 5. Finally, the paper closes with a summary of the key findings and the main conclusions in Section 6.

2 Uncertainties of soil and loading characteristics

Soil and loading properties are characterized by several uncertainties. These uncertainties can significantly impact the safety of foundations and may lead in some instances to both foundation and structural failures [23].

Reliability analysis of foundations under random loading, on soils with multivariate correlated variables requires, among others, that coefficient of variation (CoV) of geotechnical and loading variables as well as correlation coefficient (ρ) between random pairs of input soil variables, to be known. These dimensionless parameters represent convenient measures of data dispersion around the sample mean, and correlation among pairs of input soil variables, respectively [13].

Among soil characteristics, angle of internal friction, cohesion and soil unit weight are the most frequently used random variables as they are specifically related to soil bearing capacity and hence to reliability analysis of shallow foundations. When sufficient data is not available, CoV values of these variables can be estimated based either, on published values accessible in specialized literature or on the classical "Three sigma rule" [24].

- Within the rather limited range of common values of the angle of friction φ for cohesive- frictional soils ($20° \leq \varphi \leq 40°$), CoV$\varphi$ varies typically between 2 and 5%. [25] and may be as large as 15% [23].
- For the effective cohesion, the coefficient of variation varies between 10 and 70% [8].
- For the soil unit weight, the coefficient of variation may be as large as 10% [26] or even larger [23].

In this paper, the illustrative values used for the coefficient of variation of input soil variables are as follows: CoVφ = 5–15 %; CoVc = 10–30%; CoVγ = 10%. The illustrative values of correlation coefficients (ρ_{ij},$i \neq$ j; i,j = φ, c, γ) used in the present study are summarized in Table 1 in accordance with typical values found in literature [27].

- As regards uncertainties in loading, the coefficient of variation of applied pressure q, is considered in this work, to vary from 10% to 30%. Further, it is assumed that there is no correlation between loading and soil variables.

Table 1 Correlation coefficient between pairs of input soil parameters

Coefficient of Correlation ($\rho_{ij}, i \neq j; i,j = \varphi, c, \gamma$)	$\rho_{\varphi c}$	$\rho_c \gamma$	$\rho_\varphi \gamma$
Case 1 Uncorrelated	0	0	0
Case 2 Weak correlation	-0.25	0.25	0.25
Case 3 Moderate correlation	-0.50	0.50	0.50
Case 4 Strong correlation	-0.75	0.75	0.75

3 Background on response surface methodology

In this work, several algorithms based on experimental designs to model linear and nonlinear response surfaces are investigated in the aim to calculate the reliability index. The basic idea of RSM methodology is to approximate the output response by a polynomial equation of low order using multiple regression analysis and approximate function relationship between dependent output $\boldsymbol{Y}$ (the vector of observed response) and input random coded variables x_i, $i = 1, \ldots, k$ with k number of random variables.

The following approximate explicit relationship has been employed in this work:

$$Y = b_0 + \sum_{i=1}^{k} b_i x_i + \sum_{i=1}^{k} b_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} b_{ij} x_i x_j + \varepsilon \,, \tag{1}$$

where Y is the observed response, b_0, b_i, b_{ii} and b_{ij} are regression coefficients and ε represents error involved in neglecting other sources of uncertainties. The regression coefficients bb_0, b_i, b_{ii} and b_{ij} can be determined efficiently by regression analysis using responses at some specific data points called *experimental sampling points* [23]. The number of regression coefficients is $p = 2k + 1$ for Eq. (1) without cross terms and $p = (k + 1)(k + 2)/2$ for Eq. (1) including cross terms.

The least squares method is used to determine the estimator of $\boldsymbol{b}$ as given by Eq. (2).

$$\hat{\boldsymbol{b}} = \left(\boldsymbol{x}\,\boldsymbol{x}^t\right)^{-1} \boldsymbol{x}^t \boldsymbol{Y} \,, \tag{2}$$

where $\boldsymbol{x}$ is the *experiment matrix* in coded space (see Appendix A) and $\boldsymbol{x}^t$ is its transpose.

The prediction model is then given by Eq (3):

$$\hat{Y} = \hat{b}_0 + \sum_{i=1}^{k} \hat{b}_i x_i + \sum_{i=1}^{k} \hat{b}_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \hat{b}_{ij} x_i x_j \,. \tag{3}$$

Since the natural variables X_i are usually expressed in different units, their effects are only comparable if they are converted in coded variables xi using the relationship $x_i = \frac{X_i - (\max(X_i) + \min(X_i))/2}{(\max(X_i) - \min(X_i))/2}$, with $\max(X_i) = \text{mean}(X_i) + h\sigma_i$ and $\min(X_i) = \text{mean}(X_i) - h\sigma_i$, where h is an integer and σ_i are the standard deviations of input variables (x_i). Eq (3) can then be used to finally write the 2nd order response surface prediction model in terms of natural variables. Further, in order to examine the adequacy of the fitted model, a normal probability plot should be approximately along a straight line. In addition, computed values of coefficients of multiple determinations (R^2) and adjusted R^2, also give information on the adequacy of the fitted model, where $R^2_{adj} = 1 - \frac{L-1}{L-p}(1 - R^2)$ with L, total number of observations [28].

Saturated Design (SD), Box-Behnken Design (BBD), Central Composite Design (CCD) are experimental designs schemes which are often used to generate experimental sampling points for fitting 2nd order surfaces. Excellent descriptions of these methods can be found in [28].

In order to optimize the response parameters, each parameter in the experimental design is studied at five different levels ($-\alpha$, -1, 0, +1, $+\alpha$). The choice of five levels for each variable is required by this design in order to explore the region of the response surface near the optimum. Saturated design (SD) and central composite design (CCD) are the most widely used experimental designs to model respectively second order response surfaces without and with cross terms.

A CCD scheme (illustrated in Fig. 1, for $k = 3$ variables) is composed of $N_f = 2^k$ factorial points, 2^k axial points ($\pm\alpha$, 0, 0, …, 0), (0, $\pm\alpha$, 0, …, 0), …, (0, 0, …, $\pm\alpha$) and a number of replications chosen at the center of the domain (0, 0, 0, …, 0) or central points, with $\alpha = (N_f)^{1/4}$.

As regards the bearing capacity problem, three independent input soil variables φ, c, γ are studied (see the table in Section 4.2). Therefore, for $k = 3$, it would take 8 cubic points, 6 axial points (with 2 axial points on the axes of each random variable at distance α from the central point) and 1 central point. This makes a total of 15 experiments necessary for a fully quadratic second order polynomial model. Using three variables $\alpha = 1.68$. The variables chosen in the context of this study are in coded and natural symbols: friction angle (x_1, X_1), cohesion (x_2, X_2) and soil unit weight (x_3, X_3). The bearing capacity based on shear failure criterion is the response variable.

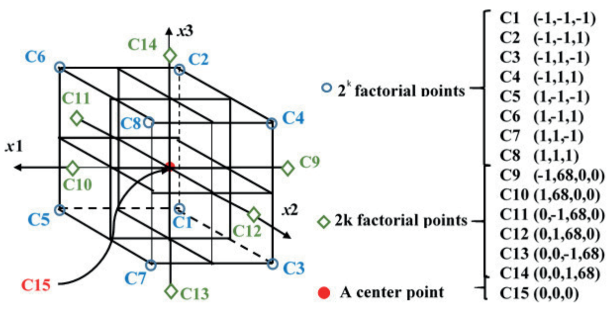


Fig. 1 Central Composite Design scheme (in coded space for $k = 3$)

4 Reliability analysis of foundation safety on soils with multivariate correlated variables

The performance function of shallow foundations resting on c–φ soils can be obtained using a bearing capacity formula. One of the most commonly used equations for bearing capacity analysis is Terzaghi's equation, with Meyerhof's approximations for bearing capacity factors N_c, N_q and N_γ all expressed in terms of friction angle [29]. Shape s_c, s_q and s_γ and depth d_c, d_q and d_γ factors are described in [21].

$$q_{ult} = cN_c s_c d_c + \gamma D_f N_q s_q d_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma \,, \tag{4}$$

where, q_{ult} is the ultimate bearing capacity, D_f the embedment depth and B is the width of the strip foundation.

In this work, a shallow strip foundation of width B =1.5 m at a depth D_f = 1.0 m, resting on a cohesive frictional soil is considered. The values of input soil and loading properties for conventional and reliability analysis are presented in Table 2.

Based on the mean values of input soil properties, the mean value of ultimate bearing capacity of foundation is calculated to be as 1388.53 kPa. For a safety factor equal to 3.0, the allowable bearing capacity $q_{allowable}$ of foundation soil is $q_{allowable}$ = 462.84 ≈ 460 kPa.

4.1 Comparison between probabilistic methods

Based on performance function given by Eq. (5), below

$$g\left(\varphi, c, \gamma, D_f\right) = q_{ult}\left(\varphi, c, \gamma, D_f\right) - q_{allowable} \,, \tag{5}$$

where $g(\varphi, c, \gamma, D_f) > 0$ denotes safety while $g(\varphi, c, \gamma, D_f) < 0$ represents unsafe condition.

Table 3 shows reliability index values using MCS and SORM for k = 4 normal random variables φ, c, γ, D_f. It is seen that value of reliability index β using SORM coupled with CCD experimental scheme for fitting fully quadratic response surface model (abbreviated hereafter as RSM-CCD) is the most accurate experimental design scheme, with comparable CPU times, when compared to SD and BBD schemes. In addition, the RSM-CCD results are in excellent agreement with those of Monte Carlo simulation technique contrary to results obtained from saturated design (RSM-SD with $(k + 1)(k + 2)/2$ and $(2k + 1)$ sample points). It is also noted that results based on experimental RSM-BBD are acceptable while RSM linear and RSM linear + interaction terms for fitting response surfaces with 2^k sample points should be discarded when compared to MCS results.

Table 2 Statistical properties of random input soil and loading variables

Input soil and loading variables	μ	σ	X_{min}*	X_{max}*	CoV (%)	Distribution
Cohesion (c), kPa	12	3.6	6.08	17.92	30	Normal
Friction angle (φ), °	33	1.65	30.29	35.71	5	Normal
Unit weight (γ), kN/m^3	15.8	1.58	13.20	18.40	10	Normal
Depth of foundation (D_f), m	1	0.1	0.84	1.16	10	Normal
Applied pressure (q), kPa	460	92	308.67	611.33	20	Normal

* The upper and lower limits X_{max} and X_{min} respectively are based on the assumption that input soil and loading parameters follow Normal distribution with probabilities of 5% and 95% being exceeded.

Table 3 Reliability index against bearing capacity failure using MCS, RSM methods for k = 4 random variables with the following statistical characteristics (μ_φ=33°– CoVφ = 5%; μ_γ=15.8 kN/m^3– CoVγ = 10%; μ_c=12 kPa – CoVc = 30%; μ_{D_f}= 1 m – CoV$_{D_f}$= 10%)

Method: Design scheme:	MCS	RSM Linear	RSM Linear +interaction	RSM Saturated Design SD	RSM Saturated Design SD	RSM Box-Behnken designs (BBD)	RSM Central composite design CCD
N° experemental smpling points:		2^k	2^k	$2k + 1$	$(k + 1)(k + 2)/2$	$2k(k – 1) + 1$	$2^k + 2k + 1$
β Error	3.69	3.05	3.20	3.31	3.98	3.85	3.71
	-	17.34%	-13.20%	-10.43%	7.94%	4.32%	0.61%
CPUtime(sec)	1796.07	0.023	0.351	0.9596	1.0061	1.0451	1.0728

Linear 2^k: $g = b_0 + \sum_{i=1}^{n} b_i X_i + \epsilon$

Linear +interaction 2^k: $g = B_0 + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} b_{ij} X_i X_j + \epsilon$

SD$(2k + 1)$: $g = b_0 + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n} b_{ii} X_i^2 + \epsilon$

BBD + CCD + SD: $g = B_0 + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n} b_{ii} X_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} b_{ij} X_i X_j + \epsilon$

It is also significant that the CPU time required for generating the reliability index by MCS using Meyerhof's equation (1796.07 sec) was 1674 times that needed for RSM-CCD (1.0728 sec), using a laptop with the following characteristics: Intel® Core™ i5 CPU M 520 @ 2.40 GHz. On the basis of these results, we will consider in what follows only the CCD experimental scheme (unless indicated otherwise) for performing reliability analyses.

4.2 Results from Response Surface Method obtained from Meyerhof's equation

A preliminary sensitivity calculation reliability index shows that for the range of variation of the input variables φ, c, γ, D_f, the bearing capacity problem with $k = 4$ variables can be reduced essentially to problem with $k = 3$ variables φ, c, γ using MCS method with a percentage error on β not exceeding 4%. On this basis only the soil variables φ, c, γ will be considered in the safety assessment of the bearing capacity problem using Meyerhof's equation.

Table 4 shows a single replicate central composite design for constructing response surface for ultimate bearing capacity using fully complete quadratic response surface model based on 15 combinations of input soil variables and corresponding output response q_{ult} obtained from Meyerhof equation.

Based on Meyerhof's formula Eq. (4) and CCD scheme, the 2nd order response surface equation for bearing capacity is found to be as:

$$q_{ult} = 14402 - 882.8\varphi - 197.0\gamma - 96.5c + 13.445\varphi^2 \\ -0.124\gamma^2 - 0.024c^2 + 7.726\varphi\gamma + 4.434\varphi c - 0.0001\gamma c. \quad (6)$$

By plotting the parity curves (see Fig 2(a)), giving the predicted values (Eq. (6)) versus of the experimental values (Table 4), it can be seen that results obtained from the 2nd order response surface model are in excellent agreement with those of design experimental results. The approximately straight line in the normal probability plot shown in Fig. 2(b) and the computed values of R^2 and R^2_{adj} equal to 99.92% and 99.85% respectively, confirm the adequacy of the 2nd order RSM-CCD fitted model noted that these values of R^2 and R^2_{adj} are very close to 100% indicating that most of the variability in bearing capacity is explained by the 2nd order, fully complete quadratic response surface model. Using second order second moment SOSM [16], the mean (σ_{qult}) and standard deviation (μ_{qult}) values of bearing capacity (i.e., q_{ult}) obtained are 1412.63 kPa and 363.37 kPa, respectively.

Table 4 Single replicate central composite design CCD showing 15 combinations of input soil variables (k = 3) and corresponding bearing capacity obtained from Meyerhof's equation

No Exp	Combination			Variables			q_{ult} (kPa
				φ (°)	c (kPa)	γ (kN/m³)	
1	–	-	–	30.28	6.06	13.19	691.20
2	–	-	+	30.28	6.06	18.41	873.66
3	–	+	–	30.28	17.94	13.19	1142.40
4	–	+	+	30.28	17.94	18.41	1324.86
5	+	–	–	35.72	6.06	13.19	1391.72
6	+	–	+	35.72	6.06	18.41	1793.56
7	+	+	–	35.72	17.94	13.19	2129.46
8	+	+	+	35.72	17.94	18.41	2531.31
9	–α	0	0	28.42	12	15.8	820.29
10	+α	0	0	37.58	12	15.8	2525.58
11	0	–α	0	33	2.01	15.8	907.74
12	0	+α	0	33	21.99	15.8	1869.31
13	0	0	–α	33	12	11.42	1163.70
14	0	0	+α	33	12	20.18	1613.35
15	0	0	0	33	12	15.8	1388.53

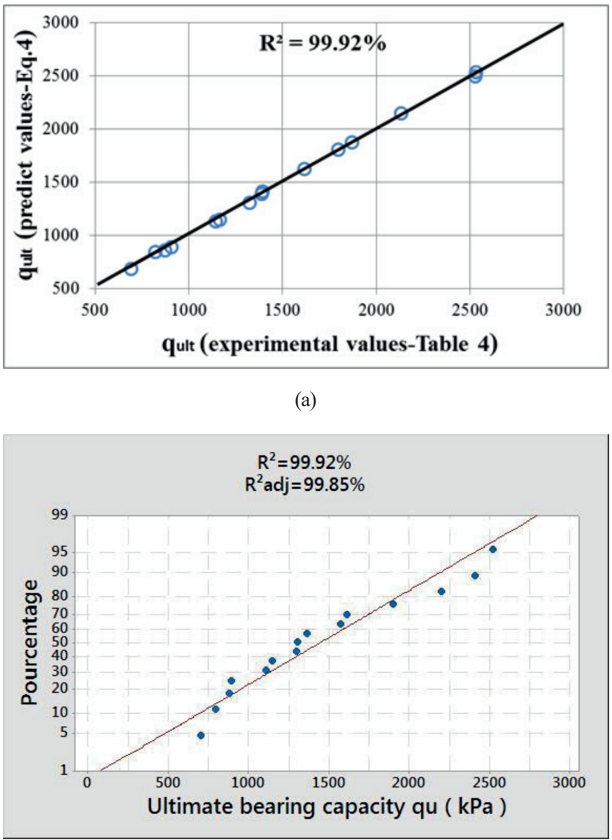


Fig. 2 (a) Comparison of experimental and predicted values of q_{ult}; (b) Normal probability plot for q_{ult} using RSM-CCD model

A 3D representation for q_{ult} using Eq. (4) is shown in Fig. 3(a) while contour plots for q_{ult} obtained from RSM-CCD and Eq. (4) are presented in Fig. 3(b). It is clearly observed that for the range of expected variation of input soil parameters considered in the present study, the results of RSM-CCD are practically coincident with the contour plots obtained from Meyerhof's equation, and that the nonlinear fully complete quadratic RSM-CCD of ultimate bearing capacity is highly accurate.

4.3 Effect of correlation of soil parameters on reliability index against bearing capacity failure

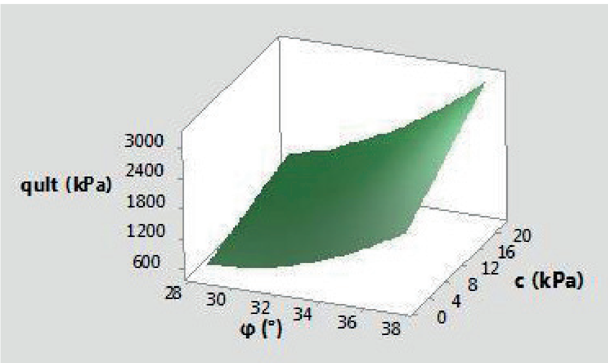
To illustrate the effect of correlation on the reliability index, Table 5 presents the results of the reliability index β_{HL} calculated using SORM for the above selected values of statistical parameters and for various values of the correlation coefficient $\rho = 0$, 25, 50 and 75% corresponding to case 1, case 2, case 3 and case 4, respectively, assuming normal and lognormal distributions of input soil variables.

It is observed that the variations in coefficient of correlation have a considerable effect on the reliability index, particularly when the correlation coefficient is large (ρ = 0.75; case 4). For example, for CoVφ = 10% and ρ = 0.75 (case 4) assuming correlated normal input soil variables, $\beta_{HL} = 3.29$ (i.e., $P_f = 0.0501\%$) whereas $\beta_{HL} = 2.71$ (i.e., $P_f = 0.3364\%$) when $\rho = 0$ (case1). This represents an increase of 21.4 % in β_{HL} (i.e., a decrease of 85% in P_f). For the same values of uncertainties of input soil variables but for the case of lognormal distribution, the increase in β_{HL} is equal to 21.85% (i.e., a decrease of 98% in P_f). Further, it is clear that the reliability index decreases substantially with increasing values of CoVφ for both normal and lognormal distributions. Also, these results show that probability of failure is a more sensitive indicator of the effect of correlation than reliability index.

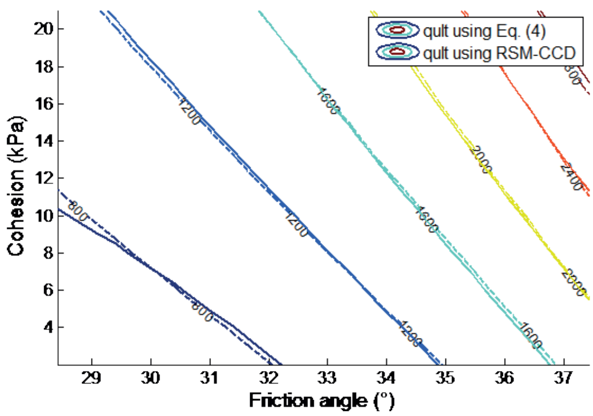
Similarly, from Fig. 4, it is seen that for the case of normal distribution (ND) of input soil variables, reliability index slightly increases for increasing values of $|\rho_{ij}|$, but decreases significantly with increasing values of CoVφ. It also can be noted that consideration of correlation between input soil parameters marginally affects the reliability index values for the lower values of CoVφ. Similar trends are also observed for the case of lognormal distribution (LND) of input soil variables, but with higher reliability index values.

Table 5 Reliability index β_{HL} failure assuming normal (ND) and lognormal distributions of uncorrelated and correlated input soil parameters ($\mu_\varphi = 33°$; $\mu_\gamma = 15.8$ kN/m^3, CoVγ = 10%, $\mu_c = 12$ kN/m^3, CoVc = 30%)

CoV$_\varphi$ (%)	Case 1		Case 2		Case 3		Case 4	
	ND	LND	ND	LND	ND	LND	ND	LND
5	3.93	4.97	4.02	4.97	4.04	5.05	4.06	5.11
7	3.41	3.96	3.56	4.06	3.70	4.23	3.81	4.54
10	2.71	3.02	2.87	3.14	3.06	3.33	3.29	3.68
15	1.95	2.13	2.05	2.23	2.18	2.38	2.33	2.64



(a)



(b)

Fig. 3 (a) 3D representation for q_{ult} using Eq. (3); (b) Contour plot for q_{ult} obtained from RSM-CCD model and Eq. (3)

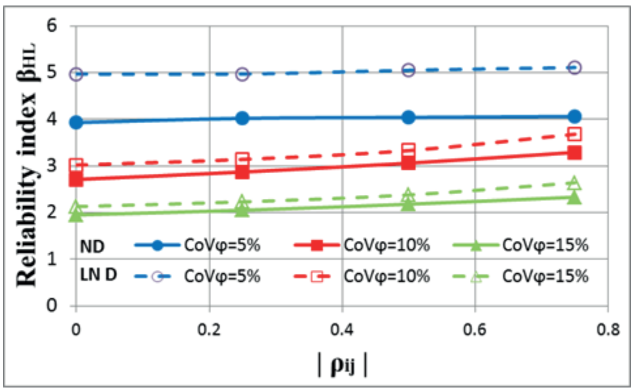


Fig. 4 Reliability index of ultimate bearing capacity for normal (ND) and lognormal (LND) distributions of input soil variables ($\mu_\varphi = 33°$ – CoVφ = 5–15%; $\mu_\gamma = 15.8$kN/m^3 – CoVγ = 10%; $\mu_c = 12$ kPa – CoVc = 30%)

5 Sensitivity analyses of reliability index

5.1 Sensitivity of reliability index to normal and lognormal distributions of soil and loading variables

The main results of the study of the influence of random applied pressure q for two statistical distributions (normal and lognormal) on the safety of the foundation are summarized in Table 6. The Hasofer–Lind reliability index β_{HL}, the corresponding design points (φ^*, c^*, γ^*, q^*) for different values of random applied pressure varying from small values up to ultimate applied pressure q_{ult}, are presented using Eq. (7).

$$g(\varphi, c, \gamma, q) = q_{ult}(\varphi, c, \gamma) - q \tag{7}$$

It is clearly observed from Table 6 that the reliability index decreases with the increase in the applied pressure until it vanishes for an applied pressure equal to the mean value of ultimate bearing capacity. This case corresponds to a deterministic state of failure for which safety factor $F = 1$ obtained for the mean values of the random variables. The corresponding failure probability is equal to 50%.

Note also that for the lower applied pressures there is a significant difference between the β_{HL} values obtained when using normal and lognormal statistical distributions of input soil and loading variables. However, this difference reduces for the higher range of applied pressure until the mean value of ultimate bearing capacity. It is further noticed that MPP values of applied pressure q^* on the foundation and the reliability index β_{HL} are significantly higher for lower range of applied pressure when assuming lognormal distribution of input soil and loading proprieties.

Table 6 Reliability index β values, corresponding design points and failure probabilities from ultimate bearing capacity criterion for different applied pressures (($\mu_\varphi = 33°$ – CoVφ = 5%; $\mu_\gamma = 15.8$ kN/m^3 – CoVγ = 10%; $\mu_c = 12$ kPa – CoVc = 30%; CoVq = 20%)

Applied pressure q (kPa)	σ_q	φ^*	c^*	γ^*	q^*	β_{HL}
Normal variables						
300	60	29.03	0.76	13.94	436.81	4.70
460	92	29.74	4.91	14.58	651.39	3.57
600	120	30.44	7.16	14.94	809.39	2.75
800	160	31.31	9.22	15.28	998.89	1.82
1000	200	32.02	10.52	15.51	1155.47	1.08
1200	240	32.58	11.40	15.68	1285.79	0.48
1388.53	277.71	33.00	15.80	12.00	1388.53	0.00
Lognormal variables						
300	60	29.20	7.28	14.36	685.66	5.23
460	92	30.15	8.26	14.77	817.36	3.72
600	120	30.78	8.94	15.01	916.57	2.80
800	160	31.49	9.75	15.27	1042.85	1.81
1000	200	32.07	10.43	15.46	1157.13	1.06
1200	240	32.57	11.02	15.61	1263.13	0.46
1376.73	275.35	33.00	15.80	12.00	1376.73	0.00

5.2 Sensitivity of reliability index to reduction of number of random variables

Table 7 presents results of sensitivity analysis of reliability index to successive reductions of number of random variables. It is seen that the 4-variable (φ, c, γ, q) problem can be reduced to a 3-variable problem including shear strength and loading parameters (φ, c, q) with equally important sensitivities: –0.58, –0.55 and 0.60, respectively, and a percentage error on β_{HL} less than 3%. We also observe that for the case of 2 random variables φ and q can be discarded in practice. The sensitivity factors α_{X_i} ($X_i = \varphi, c, \gamma, q$) convey the relative importance of the random variables in affecting reliability and can be calculated using Eq. (8)

$$\alpha_{X_i} = -\left.\frac{\partial g}{\partial X_i}\right|_{X^*} \Bigg/ \sqrt{\sum_{i=1}^{n} \left.\frac{\partial g}{\partial X_i}\right|_{X^*} \sigma_{X_i}{}^2}\,, \tag{8}$$

where X^* is design point.

5.3 Sensitivity of reliability index to uncertainties in loading and soil shear strength variables

To study the effects of soil shear strength variability and loading randomness on Reliability index, Fig. 5 shows the Reliability index versus the coefficients of variation of shear strength and loading variables. The value of the safety factor is taken equal to 3 ($\mu_q = 460$ kPa). The results show that the reliability index is highly influenced by the coefficient of variation of friction angle more than the cohesion and the loading which have approximately the same influence. This means that the accurate determination of these parameters is key in obtaining meaningful probabilistic results. Similar trends (not shown in the figure) are also observed for lognormal random variables but with significantly higher β_{HL} values.

Table 7 Sensitivity analysis of reliability index to reduction of number of variables ($\mu_\varphi = 33°$ – CoVφ = 5%; $\mu_\gamma = 15.8$ kN/m^3 – CoVγ = 10%; μ_c = 12 kPa – CoVc = 30%; $\mu_{Df} = 1$ m – CoV$_{Df}$ = 10%; $\mu_q = 460$ – CoVq = 20%)

Number of variables	α_φ	α_q	α_c	α_γ	β_{HL}	error (%)
SORM 4variables (φ, c, γ, q)	-0.55	0.58	-0.55	-0.22	3.57	
SORM 3 variables (φ, c, q)	-0.58	0.60	-0.55	-	3.67	2.80%
SORM 2 variables (φ, q)	-0.70	0.72	-	-	4.58	28.29%

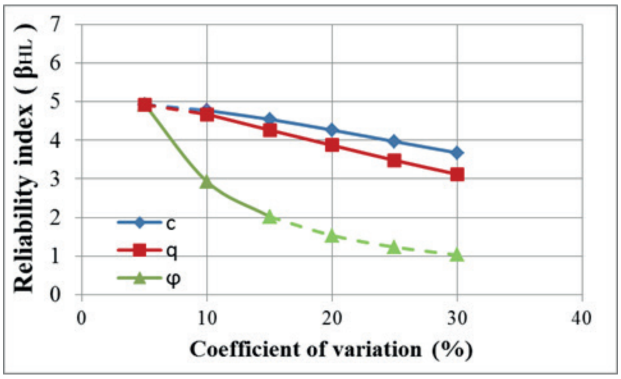


Fig. 5 Reliability index versus different levels of uncertainties using normal distribution (μ_φ = 33°, μ_c = 12 kPa, μ_q = 460 kPa)

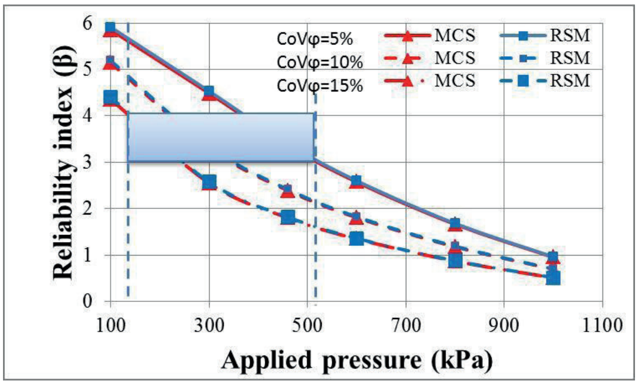


Fig. 6 Results of the reliability analysis using RSM-CCD model and MCS, versus applied pressure for different CoVφ (μ_φ = 33°; μ_γ = 15.8 kN/m^3 – CoVγ = 10%; μ_c = 12 kPa – CoVc = 30%; CoVq = 20%)

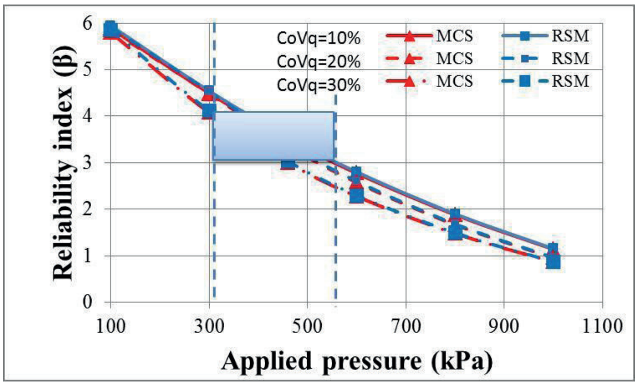


Fig. 7 Results of the reliability analysis using RSM-CCD model and MCS, versus applied pressure for different CoVq (μ_φ = 33° – CoVφ = 5%; μ_γ = 15.8 kN/m^3 – CoVγ = 10%; μ_c = 12 kPa – CoVc = 30%)

The probabilistic results presented in Fig. 6, show that the allowable pressure on the foundation varies from 354 kPa to 515 kPa for CoVφ = 5%, from 216 kPa to 351 kPa for CoVφ = 10% and from 130 kPa to 235 kPa for CoVφ = 15% in order to achieve reliability index β_{HL} values in the range of 3 to 4, as indicated in the shaded region. It is also seen that allowable pressure calculated from Meyerhof's classical formula for ultimate bearing capacity for a safety factor equal to 3.0 (i.e., q_{all} = 460 kPa) is satisfactory only in the case of CoVφ = 5% as it is associated with required range of reliability index β(3–4).

Similarly, it can be shown in Fig. 7 that the allowable pressure on the foundation Meyerhof's classical formula for ultimate bearing capacity for a safety factor equal to 3.0 is in the present case satisfactory in all cases of CoVq (10%–30%), as it is associated with required range of reliability index β(3–4).

Further, it can be noticed that the probabilistic results obtained from RSM-CCD model are practically coincident with those obtained from MCS method. However, it should be noted that the number of simulations required by RSM- CCD algorithm ($2^k + 2k + 1$) which for k = 4 variables (φ, c, γ, q) leads to 25 simulations as opposed to 150000 Monte Carlo simulations necessary to achieve a stable value of CoV$_{P_f}$ = 5% considered in the present study. It follows that a considerable advantage of using the RSM- CCD algorithm is in terms of execution CPU time and memory requirements.

6 Conclusions

The work presented in this paper investigated essentially the applicability of several 2nd order Response Surface Models in conjunction with Meyerhof's deterministic model, to evaluate the safety under random loading, against bearing capacity failure of shallow foundations resting on c–φ soils with multivariate correlated variables. A comparison of reliability results obtained using MCS technique with those of 2nd Order Response Surface Models based on design experiments coupled with SORM has been performed. Also, the implications of possible cross-correlation between pairs of input soil parameters on foundation safety against bearing capacity failure have been examined for both normal and lognormal distributions. Moreover, the effects of soil variability and loading randomness on foundation safety have been analyzed.

From the results presented in this study, the following conclusions may be drawn:

- Results from experimental design techniques show that CCD is more accurate (with practically comparable CPU times) than BBD and SD when compared to MCS results. Values of reliability index of linear and linear + interaction terms showed marked deviations from Monte Carlo simulation results, suggesting that these models should be discarded in practice.

- It is also shown, that consideration of correlation significantly affects the reliability index for the large values of soil friction angle uncertainty. Contrary to sandy soils, the hypothesis of commonly used assumption of uncorrelated input parameters is found to be conservative comparatively to that of correlated parameters.
- The reliability index is found to be highly sensitive to uncertainties of soil friction angle more than the cohesion and the loading which have approximately the same influence, especially in the case of lognormal variables. This means that the accurate determination of these uncertainties is key to obtain meaningful probabilistic results.
- The probabilistic results show that reliability index decreases importantly with the increase of the applied pressure, and that there is significant difference between the reliability indices computed, based on the assumptions of normal and lognormal distributions, especially for the lower ranges of applied loading.

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Appendix A

Experiment matrix $\boldsymbol{x}$ for $k = 3$ coded variables.

$$\begin{matrix} & x_1 & x_2 & x_3 & x_1^2 & x_2^2 & x_3^2 & x_1x_2 & x_1x_3 & x_2x_3 \end{matrix}$$

$$\boldsymbol{x} = \begin{pmatrix}
1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 \\
1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 \\
1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1.68 & 0 & 0 & 2.82 & 0 & 0 & 0 & 0 & 0 \\
1 & 1.68 & 0 & 0 & 2.82 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & -1.68 & 0 & 0 & 2.82 & 0 & 0 & 0 & 0 \\
1 & 0 & 1.68 & 0 & 0 & 2.82 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & -1.68 & 0 & 0 & 2.82 & 0 & 0 & 0 \\
1 & 0 & 0 & 1.68 & 0 & 0 & 2.82 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$