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THE LOAD-CARRYING CAPACITY FOR SOME TUBULAR REACTORS AND STRESS CONCENTRATION

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Abstract

The purpose of this paper was to evaluate the state of stress and the stress concentration in the main critical junctions of a tubular reactor. Additional two numerical analyses were developed to identify the critical junctions of reactor due to the main loads: temperature gradient and internal pressure. One analysis is based on the extension of classical thin shell theory and the flexibility matrix method and the second on the finite element method (FEM), by the package COSMOS/M Designer II. Comparative experimental study based on recording the strains at selected surface positions, for different values of the reaction loads, was done using strain gauges. The analyses reveal a reasonable accuracy of the results and accurate positioning of the critical junctions. The loads are variable, so that the study may give a primary estimation on the fatigue design and analysis.

Keywords: numerical analysis, tubular reactor, the state of stress, stress concentration factors, flexibility matrix method, strain gauges.

1. Introduction

Chemical tubular reactors are apparatus characterised by a great diversity of constructive forms. About 6–8% of chemical industrial technologies is realised in these kinds of apparatus. They have different constructions which depend on the type of chemical reactions, flow capacity, technological parameters, etc. Our study deals with tubular reactors for liquid phase reactions. These reactions are well known as strongly exothermal and having a dynamic behaviour. Liquid phase reactions are sensitive to all working parameters especially to the rise of the temperature of the cooling liquid, when the reaction can run away and the pressure in the reactor can rapidly increase, having destructive effects and the structure of the reactor must resist these loads. There are two principal types of tubular reactor (*Fig.1*): tubular reactor with tubular fascicle in instalments and tubular reactor with central tube in instalments. The length of the tubular reactor can vary from a few meters to hundred meters, so they often have a modular construction. Although the mechanical construction of these reactors is relatively simple, due to the dynamic behaviour of the chemical reaction, this structure may be subjected to cyclic fatigue loading through the variation of the chemical reaction parameters: pressure and temperature.

This paper highlights the highest stressed zones and the stress concentration in a tubular reactor using a classic theoretical analysis; a numerical study based on finite element method (FEM) and strain gauge measurements. In order to facilitate the strain gauge measurement at high temperature, the numerical analysis and experimental study were done using a particular equivalent tubular reactor for ethoxylated products. In this reactor the liquid phase reaction has the following steady parameters: maximum temperature $T_{\max} = 120$ °C and the pressure $p = 0.4...0.8$ MPa. Taking into account the principle of superposition in the analysis results, for this study, only the linear elastic behaviour of the material was considered.

2. Theoretical Analysis

Analysing the drawings of these tubular reactors (*Fig. 1*), we can establish major structural discontinuities at the junctions between the reactor's elements – changes of the geometrical profile and the variations of the elements' thickness. It is well known that junction D (*Fig. 2a*) is a critical area in which important stress concentration is developed (Rose, 1962; Cioclov, 1983 and Melerski, 1991). Degradation effects induced at these critical areas are extremely dangerous when the external loads are variable and have a major effect on the corrosion resistance. *Table 1* presents the main geometrical characteristics of the tubular reactor elements required for strength calculation. The structural elements of the tubular reactor are subjected to the main reaction parameters: internal pressure p and temperature gradient ΔT.

Table 1. The main geometrical characteristics of the reactor elements

No	Element	Geometrical simplex	Value	Type of element
1	Plate cover	β_i	$0.257 < 0.3$	'Moderate' rigid ring
2	Outer cylinder	β	$0.12 < 0.2$	Shell of revolution
4	Inner cylinder	β	$0.066 < 0.2$	Shell of revolution

$\beta = h/R$;
$\beta_i = h_i/R_m$;
R – medium radius of the shell;
R_m – medium radius of the ring cover;
h and h_i – shell and cover thickness.

It is well known that the classical shell theory is suitable for shells called 'thin', having simplex order $h/R \leq 0.1$, but it is valid with reasonable accuracy of the results for shells called 'moderate' (Steele 1974; Im, 1986; Ranjan, 1980 and Hutchins, 1973) having simplex order $h/R \leq 0.2 \ldots 0.33$. Several authors

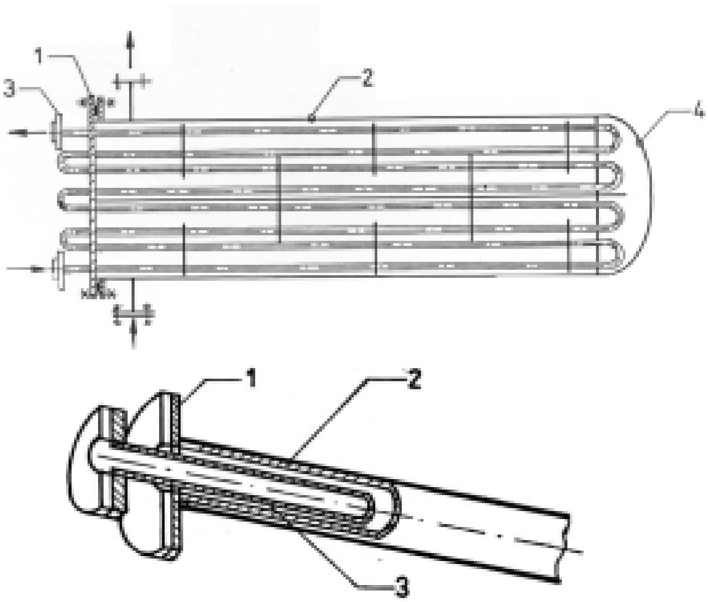

Fig. 1. The main constructive types of tubular reactors. a. Tubular reactor: 1-plate cover, 2-cylindrical shell, 3-pipe column, 4-spherical closure. b. Double-pipe tubular reactor: 1-plate cover, 2- outer pipe, 3-inner pipe

(PAVEL, 1998 and HUTCHINS, 1973) have examined, on the basis of the classical shell theory, some shells having simplex order $h/R \leq 0.5$ and produced reasonable results, the differences did not exceed 7–12%. In this case, it is necessary to develop a higher order theory capable of accurately describing the elastic behaviour. Most components of these technological structures, under conditions of axial symmetry, may be considered typical 'thin or moderate', having a simplex order $h/R \leq 0.1 \ldots 0.33$. As we have presented above, in this situation, it is possible to make an investigation based on the classical shell theory (IM, 1986; BERDICHEVSKY, 1992 and ANGHEL, 1997) for stress state and displacements estimation. In these conditions, the theoretical analysis represents an approximate analytical method developed on the basis of the classical shell theory and flexibility matrix method (the influence coefficients method). The approaches for these methods are cumbersome, so that, based on previous considerations and concrete relations established in the literature (CIOCLOV, 1983; CONSTANTINESCU and TACU, 1979; MELERSKI, 1991, 1992 and ANGHEL, 1996,1998), only the final equations for internal efforts and stresses, for 'moderate' shell of revolution, are presented in *Table* 2 and 3.

A polynomial function of degree 4 for the distribution of the temperature along the wall of the outer shell was considered, based on the correlation with

Table 2. Internal efforts and stresses in a shell of revolution under internal pressure

No.	Effort or stress	Symbol	Formula
1	Contour shearing force	Q_0	$-0.778\, p\sqrt{Rh}$
2	Contour couple	M_0	$0.303\, p\, Rh$
3	Meridional couple	M_x	$Q_0 F_4/\beta + M_0 F_2$
4	Hoop couple	K_x	μK_x
5	Shearing force	T_x	$2\beta Q_0 F_1 + 2\beta^2 R M_0 F_3$
6	Meridional stress	σ_1	$S_x/h \pm 6M_x/h^2$
7	Hoop stress	σ_2	$T_x/h \pm 6K_x/h^2$
$D = Eh^2/12(1-\mu^2)$;		$k = 1.285/\sqrt{Rh}$	
$F_1 = e^{-kx}\cos kx$;		$F_2 = e^{-kx}(\cos kx + \sin kx)$	
$F_3 = e^{-kx}(\cos kx - \sin kx)$;		$F_4 = e^{-kx}\sin kx$	
μ – Poisson ratio			

Table 3. Thermal efforts and stresses

No.	Effort or stress	Symbol	Formula
		Shell of revolution	
1	Contour shear force	Q_0	$6\alpha k^3 a_3 DR$
2	Contour couple	M_0	$2\alpha k^2 a_2 DR$
3	Meridional couple	M_x	$\alpha k^2 DR$
4	Shear force	T_x	$24\alpha a_4 DR^2$
		Plate cover	
6	Radial stress	σ_3	$(-Q_0 R_2^2/r^2 - Q_0)/(\beta^2-1)h$ $-E\alpha\int rT(r)\,\mathrm{d}r$ $+E\alpha(1/R_1^2 - 1/r^2)\int rT(r)\,\mathrm{d}r/(\beta^2-1)$
7	Hoop stress	σ_2	$(-Q_0 R_2^2/r^2 - Q_0)/h(\beta^2-1)$ $+E\alpha\int rT(r)\mathrm{d}r/R^2$ $+E\alpha(1/R_1^2 + 1/r^2)\int rT(r)\mathrm{d}r/(\beta^2-1)$ $-E\alpha T(r)$

experimental measurements of the temperature (*Table* 6) and well known statements (CONSTANTINESCU, 1979 and FETT, 1986):

$$T(x) = \sum_{i=0}^{4} a_i x^i. \tag{1}$$

A logarithmic temperature distribution in radial direction has been considered for plate cover (PAVEL and POPESCU, 1998 and JINESCU, 1984). The liquid phase reaction is exothermal and the thermal flux is from the inner surface to the outer surface of the plate cover, so that the distribution of the temperature in radial direction is:

$$T(r) = T_i - \Delta T \frac{\ln \beta_r}{\ln \beta}, \tag{2}$$

where $\beta = R_e/R_i$; $\beta_r = R_2/r$; $\Delta T = T_i - T_e$ and $T_i > T_e$.

Considering an elastic behaviour of the material, the state of stress in the shell and in the plate cover may be calculated separately for the internal pressure and for the temperature gradient and then, using the superposition of effects, the global thermo-mechanical stress may be calculated.

3. Numerical Analysis

The FEM analysis of the stress distribution in the same equivalent reactor was done. Due to the axial symmetry of the model, only a half of the axial section was analysed (*Fig. 1b*). The numerical analysis was carried out by COSMOS /M Designer II, using triangular axisymmetric solid elements with six nodes. *Fig. 2d* shows the equivalent model for FEM analysis and the boundary conditions. In the FEM analysis the elements and the nodes employed were 2217 and 5064, respectively. The principal and von Mises equivalent stresses were calculated.

4. Stress Concentration

In order to evaluate the theoretical effect of stress concentration in the area of constructive junctions, *D* (*Fig. 2*), which are the major structural discontinuity, we will simplify the analysis taking into consideration plane stress which is close to the axisymmetric thin shell – the main structure of the tubular reactors. Based on well-known statements, the area of the elastic and elastoplastic domain of stress can be satisfactorily approximated using Neuber's formula (RENERT, 1982):

$$\alpha^2 = \alpha_\sigma \cdot \alpha_\varepsilon, \tag{3}$$

where α is the general stress concentration factor considering a linear elastic behaviour of the material, α_σ is the pure stress concentration factor and α_ε the pure strain concentration factor. According to the usual design standards and other statements, we will consider a modified Neuber's formula taking into consideration a major theoretical dangerous load, which produces 0.2% maximum plastic strain in the most stressed zone, named 'critical area'. On the other hand, for elastic and elasto-plastic domain of stress and for materials generally used in tubular reactor construction, having technical yield stress $\sigma_{0.2} = (250 - 450)$ MPa (*Table 4*), according to RENERT, 1982, the stress concentration factor may be estimated using a modified and simplified formula:

$$\alpha = 1.0717\sqrt{\alpha_\sigma(\alpha_\sigma + 2.15)}, \tag{4}$$

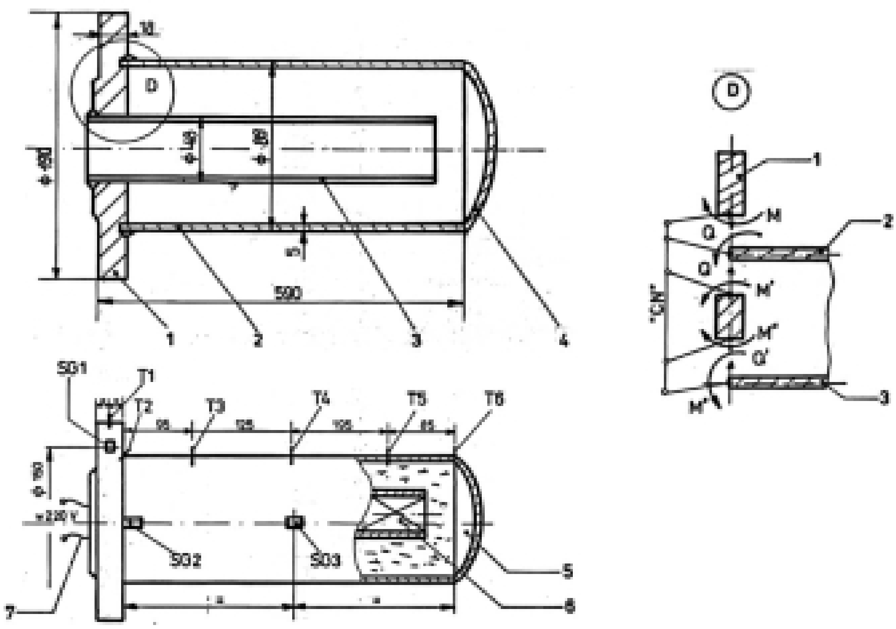

Fig. 2. Experimental tubular reactor. a. A sketch of the cross-section of the tubular reactor: 1-plate cover, 2-cylindrical shell, 3-central pipe, 4-spherical closure. b. Actual location of the strain gauges and thermocouples: 5-oil bath, 6-electric resistance, 7-electric cables, c. Conventional discretising junction area and replacement by a discrete system of structural elements: 1-plate structural element, 2 and 3-cylindrical structural element. d. FE model and boundary conditions

where the stress concentration factors, α_σ, can be substituted by the Kellog–Win stress concentration factor, α_{kw}, which can be evaluated using a 'pressure area' method (RENERT, 1982). Based on previous statements, the maximum elastic stress concentration factor, according to PAVEL and POPESCU, 1998 and CIOCLOV, 1983, may be simplified in the form as:

$$\alpha_{k\sigma} = \frac{\sigma_{ech\,\max}}{\sigma_{\mathrm{nom}}}, \tag{5}$$

where $\sigma_{ech\,\max}$ is the von Mises equivalent stress in the junction and σ_{nom} is the nominal normal stress according to the Laplace formula. For an axisymmetric cylindrical shell this stress is:

$$\sigma_{\mathrm{nom}} = \max(\sigma_1, \sigma_2) = \frac{pD}{2h}. \tag{6}$$

For a hasty assessment of the stress concentration factor, a modified formula can be used (JINESCU, 1984 and RENERT, 1982):

$$\alpha_\sigma = \frac{\sigma^*_{\max}}{\sigma_{\max}}, \tag{7}$$

where $\sigma_{\max}$ is the maximum value for the nominal or equivalent stress in the area without stress concentration and $\sigma^*_{\max}$ the effective value for the stress in the critical area with stress concentration. If the values of stresses are in the elastic domain, where ($\sigma_{\max}$ and $\sigma^*_{\max}$) $< \sigma_{0.2}$, the previous formula may be considered in a simplified form:

$$\alpha \cong \alpha_\sigma = \frac{\sigma^*_{\max}}{\sigma_{\max}}. \tag{8}$$

For safe operation of the tubular reactor, we can consider the following relationship between the stress concentration factors:

$$\alpha_{k\sigma} \geq [\alpha, \alpha_\sigma]. \tag{9}$$

5. Numerical Applications

The quantitative analysis of the stress distribution and stress concentration will be realised considering seeding condition and the development of the chemical reaction in steady state conditions, characterised by two working parameters: maximum temperature $T_{\max} = 120$ °C and variable internal pressure $p = 0.4 \ldots 0.8$ MPa (CARLOGANU, 1980). If the reaction runs out of control (for example disturbances in the cooling system) and the working parameters exceed 200 . . . 240°C for temperature and 1.6 MPa for internal pressure, the chemical reaction runs away with a strong and quick increase of the pressure over 1.6 MPa. Therefore we considered these parameters as limiting values for the normal working conditions in the reactor analysis. Based on experimental measurements of the temperature using thermocouples, with ±1 °C accuracy, the temperature distribution in the shell and in the plate cover was calculated using *Eqs.* (1) and (2). The polynomial coefficients for an algebraic polynomial of degree four are presented in *Table* 5, for two cases.

Analytical, numerical (FEM) and experimental results for the state of stress and the stress concentrations, corresponding to case 1, 3 and 6 only (*Table* 6) are presented graphically and in tables. For simplicity, the results are presented only for the von Mises stresses. Analysing *Figs. 3* and *4* we can state that the most stressed area is the inner surface of the welding joint between the plate cover and cylindrical shell. In steady-state conditions the maximum equivalent stress is $\sigma_{\max} = (50 - 70)$ MPa, but this stress increases to $(54 - 76)$ MPa at the limit of stable operation (*Table* 6), so that the state of stress is still under the allowable stress of the material $\sigma_a = 150$ MPa.

According to our expectations and other assessments (ROSE, 1962; CIOCLOV, 1983 and MELERSKI, 1991, 1992), the maximum analytical nominal and equivalent

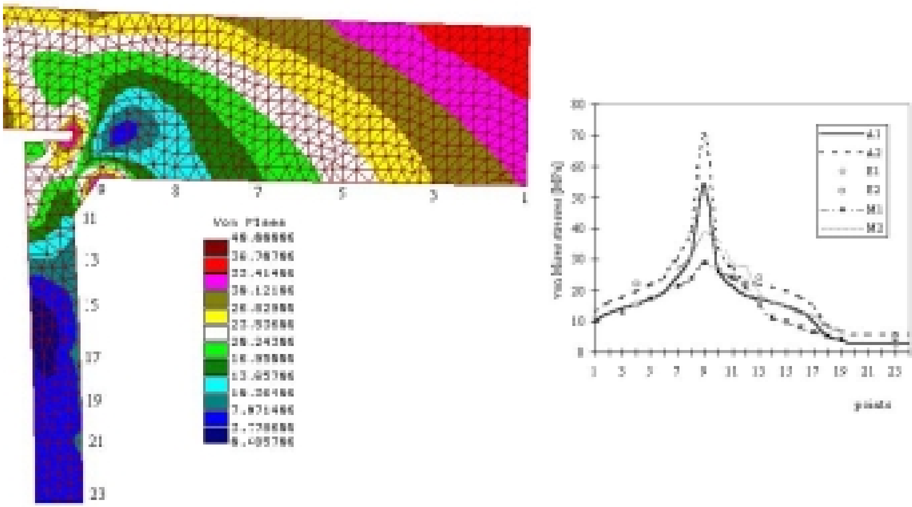

Fig. 3. The distribution of the equivalent stresses in junction *D*. a. FE von Mises stresses and deformed shape around the critical junction case 3. b. Analytical results along the outer surface: A1 - case 1, A2 - case 3; Finite element method: M1 - case 1, M2 - case 3; Experimental: E1 - case 1, E2 - case 3

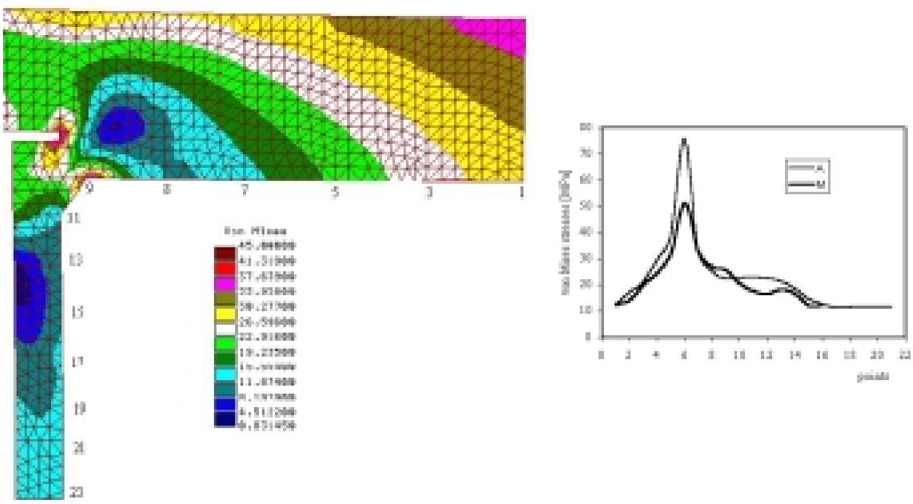

Fig. 4. Equivalent stresses in junction *D* for stationary loading conditions – case 6 (*Table 6*). a. FE von Mises stresses and deformed shape around the critical junction, b. A - analytical results along the outer surface, M - finite element method

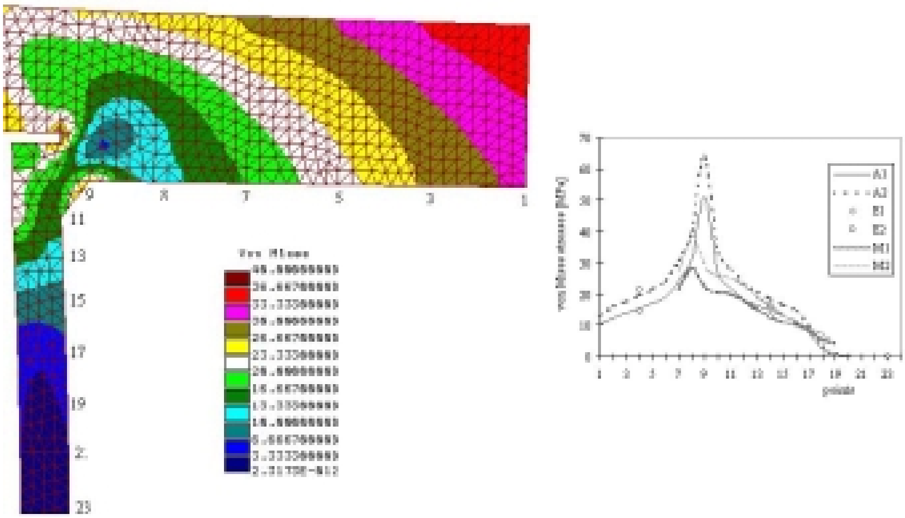

Fig. 5. Equivalent stresses due to temperature only, in junction *D*. a. FE von Mises stresses and deformed shape around the critical junction case 3. b. Equivalent stresses along the outer surface due to temperature only (*Table 6*): Analytical results: A1 - case 1, A2 - case 3; Finite element method: M1 - case 1, M2 - case 3; Experimental: E1 - case 1, E2 - case 3

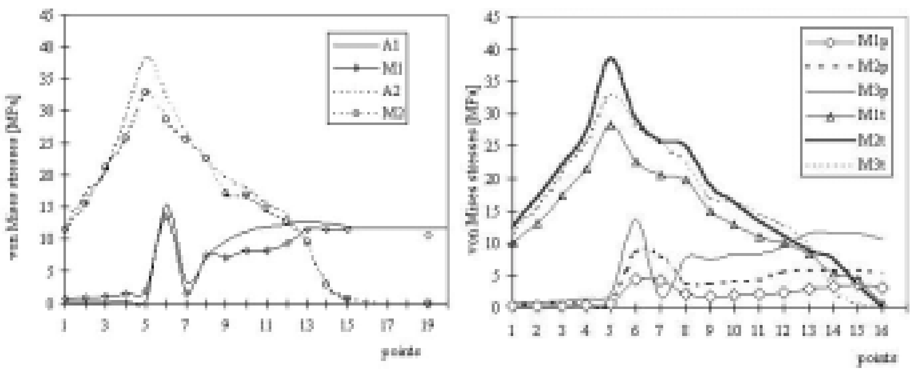

Fig. 6. The variation of the equivalent stresses along the outer surface of junction D. a. Equivalent stresses in steady loading conditions - case 3 (*Table 6*): A1, M1-analytical and FEM values due to pressure; A2, M2 - analytical and FEM results due to temperature. b. FEM equivalent stresses done by distinct loads: M1p, M2p, M3p, - values from pressure and M1t, M2t, M3t, - values from temperature, corresponding to cases 1, 2 and 3 (*Table 6*)

stresses in the junction exceed numerical (FEM) or experimental results by more than 30%–50%. A first agreed and well known justification is the assumption that the connections between structural elements at their mid planes lead to an overestimation of the shell bending moments and an underestimation of the plate moments. *Fig.* 5 shows the pure thermoelastic stress distribution, when the internal pressure is zero, marking the same joint with maximum equivalent stress as in *Fig.*3. According to the values of the thermoelastic equivalent stresses, we can assume that, for the welding joint between the plate cover and cylindrical shell of the tubular reactor, the temperature gradient has a major role in the stress distribution. The analysis of *Fig.* 6 turns into certainty the fact that for this modelled tubular reactor the thermal gradient has the major weight in the stress state in the welding joint between the plate cover and cylindrical shell. In the limiting operation conditions (case 6 – *Table* 6), the maximum equivalent stress is less than in transitory reaction conditions (case 5 – *Table* 6) – possibly due to zero temperature gradient along the cylindrical shell. As the maximum thermoelastic equivalent stress, $\sigma_{t\,\max}$, due to temperature gradient, ΔT, is localised in the welding joint (*Fig.*2*b*), the maximum equivalent stress, $\sigma_{e\,\max}$, due to internal pressure, p, reaches another local maximum in the tubular reactor. This value does not exceed the maximum value in the welding joint. Analysing *Figs.* 3–6 we can conclude that the length of the critical stress concentration zone increases with the internal pressure, p, and temperature gradient, ΔT. Due to the high rigidity of the plate cover compared to the cylindrical shell rigidity, the stress concentration factor will be calculated using the nominal stress in the shell.

Based on the modelled values of the stresses, the stress concentration effects identified in the junction between the plate cover and cylindrical shell are strong (*Table* 7), even if the proper values of the stress are relatively small. Because the maximum value of the stress concentration factor respects the inequality (9) and the important consolidation due to the dimensions of the plate cover, the actual limits of the maximum stress level are still in the linear elastic domain, offering a good reserve of load-carrying capacity.

Table 4. Material properties

Element	Material*	$\sigma_{0.2}$ [MPa]	σ_r [MPa]	KCU [J/cm^2]	σ_a [MPa]
Shell	OLT 45	250	440–550	40	150 ... 160
Plate cover	R 44	250	430–540	47	150 ... 160

* Proceeding from STAS 8183–80 and 8183/1–80 at ambient temperature;
$\sigma_a = \min\{\sigma_{0.2}/c_c;\ \sigma_r/c_r\}$ allowable stress at working temperature;
$c_c = 1.5$ safety factor for technical yield stress; $c_r = 2.4$ safety factor for breaking strength, proceeding from JINESCU, 1984 and PAVEL, 1998.

Table 5. The polynomial coefficients

Case	a_0	a_1	a_2	a_3	a_4
1	$-8.526631e-10$	$1.569466e-06$	$9.878861e-04$	$2.143444e-01$	$6.399971e+01$
3	$1.718707e-09$	$2.448055e-06$	$1.302311e-03$	$2.705599e-01$	$8.699941e+01$

Table 6. Modelling parameters for equivalent tubular reactor

Case No.	p [MPa]	Experimental temperature measured by thermocouples [°C]					
		T1	T2	T3	T4	T5	T6
1	0.4	45	64	76	76	68	66
2	0.6	59	78	92	92	87	80
3	0.8	63	87	102	104	97	91
4	1.0	70	92	109	110	103	96
5	1.2	77	96	114	114	105	98
6*	1.6	200	220	223	223	223	223

* Conventional values considered as limit for a steady running of the reactor.
All other values of the temperature were experimentally determined on the model of the equivalent tubular reactor (*Fig. 2a*).

Table 7. Stress concentration factors

Case	$\sigma^{**}_{e\,\max}$ [MPa]	σ_{nom} [MPa]	α^{***}_{kw}	$\alpha_{k\sigma}$	α_σ	α
3*	38.9	6.72	1.90	5.8	1.71	2.96
6*	51.4	13.44	1.90	3.8	3.10	2.96

* according to *Table 4*;
** results proceeding from *Figs. 3–4*;
*** analytical results according to PAVEL, 1998.

6. Experimental Studies

The aim of the experiments was to determine the state of stress corresponding to various loading cases in some characteristic locations and to supply numerical values for a primary validation of analytical and numerical (FEM) results. All experiments were done on an equivalent reactor (*Fig. 2a* and *b*) considering plane state of stress, in the following conditions:

- strain gauges with thermal compensation between 120 – 160 °C, type 3/120XY11, Hottinger Baldwin Messtechnik, with $R = 120\ \Omega$ (and $k =$

1.98 ± %1, glued in the two principal, hoop and meridional, directions using thermal resin;

- a strain gauge apparatus having 6 channels, type N-2302 IEMI Bucharest, carrying frequency 5000 Hz, internal calibrating device, analog indicator with 0.3–0.5% accuracy;
- thermocouples with ±1 °C accuracy for measuring temperature in six points, $T_1, \ldots, T_6$ (*Fig. 2a*).

In order to obtain different values for the internal pressure, p, and temperature, T, the reactor was filled with oil and an electric resistance was placed in the central pipe (*Fig. 2b*). The electric resistance was controlled using an electronic thermoregulator coupled to the six thermocouples, $T_1, \ldots, T_6$, having ±1 °C accuracy. Using the experimental principal strains, ε_1 and ε_2, the principal stresses σ_1, σ_2 and von Mises stresses were calculated. *Figs. 3* and *4* show the analytical, FEM and experimental equivalent stresses.

7. Conclusions

The first important conclusion is a reasonable correlation between analytical, numerical (FEM) and experimental results. Excepting the inaccuracies between the maximum analytical stresses and numerical (FEM) results, in the junction D, the established average differences between the three methods, $\pm(2-12)\%$, are usually accepted. Numerical and experimental results for the usual operation of the equivalent tubular reactor for ethoxylated products show a state of stress under the allowable stress of the material and an important reserve of load-carrying capacity. Taking into consideration the duration of the chemical reaction, which varies between 45 and 75 minutes (Carloganu, 1980), during the working life of the real tubular reactor (9–12 years) this loading due to internal pressure and temperature gradient may be considered as oligocyclical, the number of loading cycles is $N < 10^5$. The variable loading establishes a state of stress under the allowable stress of the material $\sigma_a(\sigma_c)$ so it will lead to a favourable consequence, the increase of the fatigue endurance limit (Pavel 1998). This favourable consequence may determine a simultaneous dropping of the corrosion resistance.

Based on these comparative analyses of the mechanical stress due to internal pressure and pure thermoelastic stress in the welded junction plate between cover and cylindrical shell (*Fig. 2b*, 4–5) we can conclude the predominant effect of the thermoelastic stress.

Based on the reasonable correlation between analytical, numerical (FEM) and experimental results and the high cost of materials – strain gauges, glue and strain gauge measuring instrument for experiments at high temperature – we can consider the numerical analysis as an alternative useful method for the mechanical design of these reactors.

Appendix

Other notations:

h	–	thickness;
R	–	radius;
R_m, $R_{\max}$, $R_{\min}$	–	average, maximum, and minimum radius;
$\beta = R_{\max}/R_{\min}$	–	geometrical simplex;
L	–	length of structural element.

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