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RESEARCH ARTICLE

An Estimation of the Learning Curve Effect on Project Scheduling with Working Days Calculation

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Abstract

In this paper, a learning curve effect on project duration is shown – calculating with working days only - and estimated with the help of simulated examples. Although learning is an essential part of our life, traditional scheduling techniques cannot efficiently handle the learning curve effect. It is assumed that the duration of impending repetitive activities are shorter due to the learning curve effect if the gap between consecutive activities is small enough. Taking into account the effects of the learning curve (or experience curve), it is possible to better predict project duration thus saving time and money. This effect normally results in shorter project duration. Although the effect is a "simple" calculation, it leads to an exponential time algorithm if the learning effect is applied to traditional project scheduling techniques like CPM, or Precedence Diagramming Method. In this paper, an integer programming model is developed, and an efficient algorithm is used to establish project duration. (This paper is an extended version of Levente Malyusz: Learning curve effect on Project Scheduling, Procedia 2016.)

Keywords

learning curve, project scheduling, project duration

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1 Introduction

In practice, project scheduling methods suffer from a lack of precision; consequently, it is a significant challenge to create a realistic and usable project schedule. It is difficult and time-consuming to estimate time, assign resources, determine interdependencies between tasks, and manage changes. It is, therefore, important to identify and investigate the differences between the practice and theory of scheduling methods (Francis et al., 2013). In a construction project, the general contractor distributes the job among subcontractors. It is a common observation that subcontractors rarely start their work at the earliest possible time. It is obvious that they can reduce their costs if they work continuously.

According to learning theory, when numerous similar or nearly identical tasks are performed, the necessary effort is reduced with each successive task (Oglesby et al., 1989; Drewin, 1982; Teplitz, 1991; Everett and Farghal, 1994; Everett and Farghal, 1997; Lutz et al., 1994; Lam et al., 2001; Couto and Teixeira, 2005). Learning curve theory can be applied to predicting cost and time, generally in units of time, to complete repetitive activities (Malyusz and Pém, 2014; Hasanzadeh et al., 2016). Based on the theory, there are two different methods for calculating the activity time of repetitive activities: unit time and cumulative average time. The unit time method means that the time of a doubled unit equals the time of the unit times the slope of the learning curve. The cumulative average method means that the cumulative average time of a doubled unit equals the cumulative average time of the unit times the slope of the learning curve. This was used in the original formulation of the learning curve method, referred to as Wright's model, in Wright's famous paper on the subject (Wright, 1936). Several researchers have suggested that Wright's model is the best model available for describing the future performance of repetitive work (Everett and Farghal, 1994, Couto and Teixeira, 2005). In the exponential average method (Malyusz and Pém, 2013), $\alpha = 0.5$ yielded the most accurate predictions. Of course, there is no consensus on which model provides the best fit and predictability for construction data (see Srour et al., 2015). It means that more theoretical and experimental investigations are necessary to adjust a model according to the real problems.

In construction project management, the proper scheduling of project is an essential problem. Estimation of an activity's time is a crucial part of the schedule. Learning curves impact on an activity's time and can be calculated in recent management software, which can readily handle resources (Hajdu, 1993; Hajdu, 2015). The objective of this paper is to show an effect of the learning curve that can cause changes in project duration. Moreover, we estimate these changes in the project duration with or without this effect. This occurs when the same construction team performs similar activities continuously, so after finishing the predecessor activity, the successor can immediately start.

There is little information in the literature about the use of learning curves in scheduling, although it seems that the principle of learning curves gathers ground in the scheduling of repetitive construction operations (Hinze and Olbina, 2008, Fini et al., 2015). In Zahran et al., 2016, the learning curve effect on linear scheduling method is discussed.

2 Learning curve

Learning curve theory applies to the prediction of the cost or time of future work, assuming repetitive work cycles with the same or similar working conditions regarding technology, weather, and workers, without delay between two consecutive activities. The direct labour required to produce the (x + 1)st unit is always assumed to be less than the direct labour required for the xth unit. The reduction in time is a monotonically decreasing function, an exponential curve, as described in Wright's 1936 paper.

Wright's linear log x, log y model is as follows:

$$\ln y = \ln a + b \ln x; \vee y = ax^{b} = ax^{\log_2 r} \qquad (1)$$

where x is the cycle number; y is the time required to complete cycle x in labour hours/square meter; a is the time required to complete the first cycle; b is a learning coefficient, and r is the rate of learning. For example, if r = 0.9 (90%), then b = –0.151 see Fig. 1. Wright discovered that when the labour cost decreases at a constant rate, that is, the learning rate, the production/cycles doubles. So, learning rate is the constant rate with which labour time/cost decreases when the production/cycles doubles in a linear log x, log y model. This feature of the learning rate comes from the logarithms nature and is true only in the linear log x, log y model.

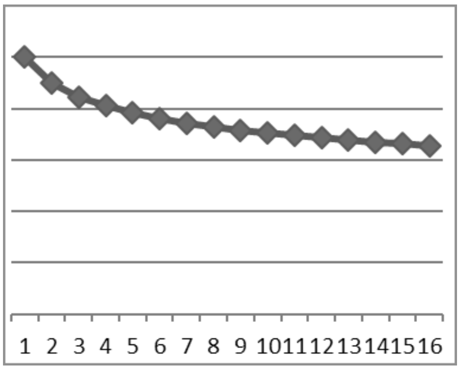

Fig. 1 Learning Curve

In the construction industry, the learning rate is between 85–95%. Taking an example with 90% of the learning rate, in this case, if a job is 10 days, a repetition of that is 8 days if the working conditions are similar. The learning curve effect does not always apply, of course. It flourishes where certain conditions are present. It is necessary for the process to be a repetitive one. Also, there needs to be a continuity of workers without any abrupt stops during the production process. When the learning curve effect, on occasions, comes to an abrupt stop, graphically, the curve jumps up (Feriyanto, 2015).

3 Project Scheduling

In this paper, we follow the concept of activity on node network using the Precedence Diagramming Method (PDM). Although there are four relationships that can be defined between activities, for the sake of simplicity, we use only the Finish Start relationship and its variants, namely: FS0, maxFS1, max FS0. The maximal precedence relationship describes the maximum allowable time between the start/finish point of the preceding and the start/finish point of the succeeding activity. The concept of non-linear activity-production-time functions was introduced by Hajdu, 2013. Mathematically, in a given directed graph with integer numbers on arrows, where positive circles are allowed, we find the longest path.

3.1 An example on reducing project duration

The simulated example project can be seen in Fig. 2. Activity B and E are worked by the same group of workers. B and E are originally a 10 days job. Each relationship is FS0. According to the results of the time analysis, B ends on day 15; E starts on day 18, so there are 2 days between them. In this case, the activity time of B and E is 10 days. If there is a one day gap between the two jobs, let the second job be 9 days. Each activity starts on its earliest date.

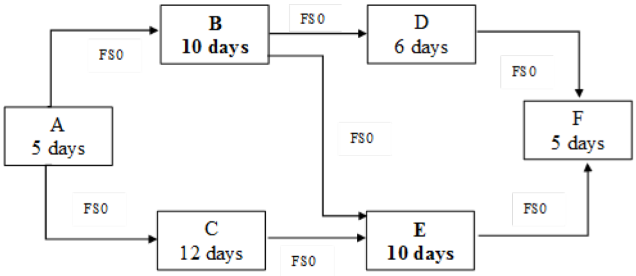

Fig. 2 Project duration is 32 days

In Figure 2, the traditional representation of this problem is shown. Minimum project duration is 32 days. Now let us consider the effect of the learning curve on activity time E.

Activity B and E are similar jobs, and they are carried out by the same construction team.

If after finishing B, E immediately (the following morning) starts; in this case, to complete it, E needs less time -that is 8 days (see Fig. 4).

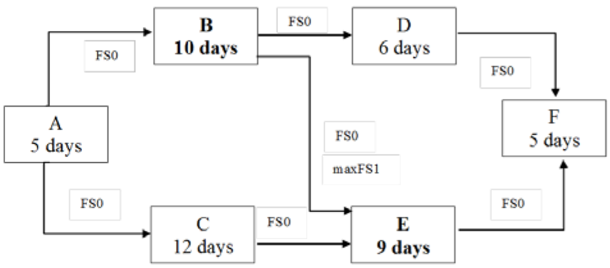

Fig. 3 Project duration is 31

In this case, the minimum project duration is 30 days. If there is one day between the finishing of B and starting of E, then 9 days is necessary to complete E (see Fig. 3). In this case, the minimum project duration is 31 days. Otherwise 10 days is the minimum time of activity E.

Table 1

starting E-finishing B	2 or more	1	0
activity time of E	10	9	8

On modifying the example by adding a maxFS1 relationship between B and E, a fter calculating, B ends on day 16, E starts on day 18, so there is 1 day between them (day 17). See Fig. 3. Since maximum relationship can cause a cycle in the network, it is not sure that a feasible solution exists.

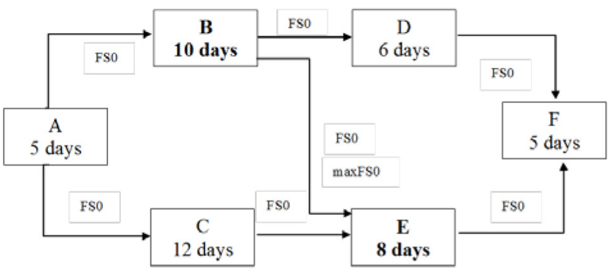

Fig. 4 Project duration is 30 days

3.2 Example on increasing project duration

In the next example, we will see that if B and E are closer to each other and the learning curve effect is taken into account, the project duration is increased.

Learning curve effect on project scheduling with reverse critical activity. According to the results of the time analysis, B ends on day 15, E starts on day 18, so there are 2 days between them.

In Fig. 5, a traditional representation of this problem is shown. Minimum project duration is 41 days.

In the next example, the effect of the learning curve on activity time E is taken into account.

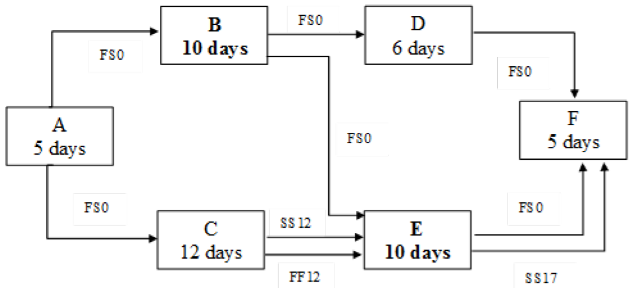

Fig. 5 Example with reverse critical activity

Activity B and E are similar jobs, and they are carried out by the same construction team.

If there is one day between the finishing of B and starting of E, then 9 days are necessary to complete E. In this case, minimum project duration is 42 days (see Fig. 6). If after B finishes, E starts immediately (the following morning), in this case, to complete E needs less time that is 8 days. In this case, the minimum project duration is 43 days. Otherwise 10 days is the minimum time of activity E with a project duration of 41 days.

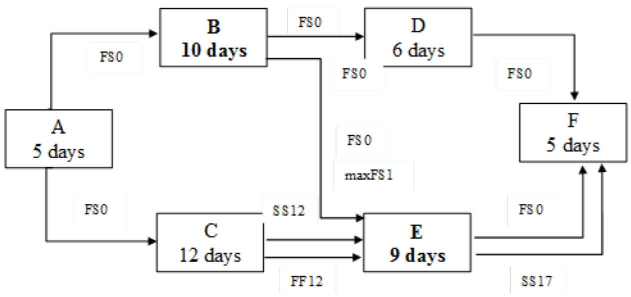

Fig. 6 Example with reverse critical activity

3.3 Integer programming model

This problem leads to an integer programming model. It can be written by modifying the classic integer programming model of project scheduling.

Let 1, 2,…, n be the indices of the activities, two integer variables are assigned to each activity, namely x_{2i-1} and x_{2i}, representing the starting and finishing time of activity i, respectively. If b_i denotes the duration of activity i, then the constraint $x_{2i} - x_{2i-1} = b_i$ should be added to the model. The precedence conditions can be expressed as inequalities, for example, the condition $x_{2j-1} - x_{2i} \geq 0$ means that activity j can be started only after activity i is finished. Our goal is to minimise the total time of the project, so the objective function is x_{2n}, the finishing time of the last activity.

Taking the learning curve effect into consideration means that there will be activities with a shorter duration than the original *b*, and this shortening depends on the finishing time of a previous activity.

Let us consider the example in Section 3.1 and suppose that k and l are two similar activities that should be carried out by the same construction team, for example, activity B and E in Fig. 2. According to Table 1, if l's original duration is 10 days, then it can be shortened to 9 or 8 days, depending on the finishing time of activity *k*.

For every such (*k*, *l*) pair we should add the following constraints to our model:

$$\text{If } x_{2l-1} - x_{2k} = 0 \text{ then } x_{2l} - x_{2l-1} = 8, \quad (2)$$

$$\text{if } x_{2l-1} - x_{2k} = 1 \text{ then } x_{2l} - x_{2l-1} = 9, \quad (3)$$

$$\text{if } x_{2l-1} - x_{2k} \geq 2 \text{ then } x_{2l} - x_{2l-1} = 10. \quad (4)$$

These constraints can be easily linearized by adding new binary variables. For every (k, l) pair, where the duration can be shortened, we introduce three new binary variables. For ease of understanding, if only one of these constraints is considered and the new variables are denoted with y_1, y_2 and y_3, their index represents which of the three constraints is active; for example, $y_1 = 0$ if constraint (2) is satisfied. Since only one of the three constraints can be satisfied at once, it is necessary to ensure that only one of the y_i's can be zero at once:

$$y_1 + y_2 + y_3 = 2$$

To linearize the constraints (2)–(4), the following eleven inequalities are added:

$$x_{2l-1} - x_{2k} \geq 0, \quad (5)$$

$$x_{2l} - x_{2l-1} \geq 8, \quad (6)$$

$$x_{2l-1} - x_{2k} \leq N\, y_1, \quad (7)$$

$$x_{2l} - x_{2l-1} - 8 \geq -N\, y_1, \quad (8)$$

$$x_{2l-1} - x_{2k} - 1 \leq N\, y_2, \quad (9)$$

$$x_{2l-1} - x_{2k} - 1 \geq -N\, y_2, \quad (10)$$

$$x_{2l} - x_{2l-1} - 9 \leq N\, y_2, \quad (11)$$

$$x_{2l} - x_{2l-1} - 9 \geq -N\, y_2, \quad (12)$$

$$x_{2l-1} - x_{2k} - 2 \geq -N\, y_3, \quad (13)$$

$$x_{2l} - x_{2l-1} - 10 \leq N\, y_3, \quad (14)$$

$$x_{2l} - x_{2l-1} - 10 >= -N\, y_3. \quad (15)$$

N is a sufficiently large constant, meaning that if $y_i = 1$ for some value of i, then the corresponding inequality is redundant. For example, if $y_1 = 0$, because of (5) and (7) $x_{2l-1} - x_{2k} = 0$ holds, so activity *l* starts the day after activity k finishes. In this case, according to Table 1, the duration of activity l should be 8 days, indeed, (6) and (8) imply that $x_{2l} - x_{2l-1} = 8$. If $y_1 = 1$, and we have chosen N properly, (7) and (8) are redundant constraints.

3.4 Numerical results

To test the algorithm, we generated different random networks. Assuming that the activities can be organised to rows with equal lengths, there are MS+2 activities in each network, where M denotes the number of rows, S denotes the number of activities in a row, and there is a starting and a finishing activity, the first and the last one. In every row, the neighbouring activities relate to an FS0 relationship. The durations are uniformly generated random numbers. The pairs representing the learning curve effect are randomly chosen activities from different rows; K denotes the number of this kind of relationships. We also connected L pairs of activities from different rows with FS0 relationships, but in these cases, the learning curve effect is not applied (different kind of jobs, not the same construction team).

XPRESS-Mosel Optimization Software was used to implement the model (Xpress user guide, 2002). The numerical results are shown in Table 2. The notations in the table are the following:

- M: the number of rows in the random network,
- S: the number of activities in a row of the random network,
- K: the number of relationships with a learning curve effect (possible reduction),
- L: the number of relationships between different lines of the random network, without a learning curve effect,
- Without LCE: total project duration (in workdays) without considering the learning curve effect,
- With LCE: total project duration (in workdays) considering the learning curve effect.

Table 2 The learning curve effect on random project networks

M	S	K	L	Without LCE (workdays)	With LCE (workdays)	Reduction (%)
50	100	100	100	544	540	0.74%
50	100	200	200	591	583	1.35%
50	100	300	300	657	647	1.52%
50	100	500	500	682	645	5.43%
50	200	100	100	438	434	1.95%
50	200	200	200	587	575	2.23%
50	200	300	300	673	653	4.19%
50	200	500	500	732	696	5.49%

The project durations were calculated with and without taking into consideration the learning curve effect and the results compared. The total project durations with and without the learning curve effect can be seen in the "Without LCE" and the "With LCE" columns of Table 2.

Running times of the model can be seen in Table 3. The integer programming approach is much more efficient than the algorithm described in the previous paper because, in this case, we only have to solve one integer programming problem instead of the 3^K problems described in the previous article. For all the test problems in Table 2, the optimal solution is reached in less than 15 seconds.

Table 3 Running times

M	S	K	L	Without LCE (sec)	With LCE (sec)
50	100	100	100	0.422	0.696
50	100	200	200	0.576	0.687
50	100	300	300	0.626	0.737
50	100	500	500	0.501	12.655
50	200	100	100	1.149	1.173
50	200	200	200	1.175	1.283
50	200	300	300	1.172	1.343
50	200	500	500	1	5.689

4 Conclusions and further investigations

In this paper, a learning curve effect on project duration is shown. This effect can reduce project duration in a range of 1%–5%. Numerous project networks were constructed randomly to measure this effect. In these generated projects, more activities have this learning effect. With an integer programming model, we efficiently solved the problem for large test instances. Further investigation is necessary to measure the effect on real networks and to make other more sophisticated generated models for the learning curve effect in projects where not only activities are repetitive, but also substructures of a project can be the same. More investigation is necessary for the field of renovation and maintenance of historic buildings (Kutasi and Vidovszky, 2010).

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